HARMONIC FUNCTIONS FOR SEA-SURFACE TEMPERATURES AND SALINITIES, KOKO HEAD, OAHU, 1956-69, AND SEA-SURFACE TEMPERATURES, CHRISTMAS ISLAND, 1954-69

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ABSTRACT

Harmonic functions have been fitted to time-series, sea-surface temperatures and salinities in order to facilitate studies of the oceanographic climate near Hawaii and Christmas Island. The manner in which Fourier analysis has been adapted to this application has been described. The standard errors of estimate for Koko Head temperatures and salinities are less than 0.26° C and less than 0.05‰, respectively. The standard errors of estimate for Christmas Island temperatures are approximately 60 % above those for the Koko Head temperature. The expected values of the Koko Head temperature and salinity functions have an uncertainty of ±0.1° C and ±0.015‰, respectively, when samples are obtained twice weekly. Error terms of the Christmas Island temperatures, with daily sampling, are on average 0.07° C. Harmonic analysis spanning the entire sampling duration shows that long-term variations in the Christmas Island temperature and Koko Head salinity are larger than the seasonal variations. Seasonal variations in the Koko Head temperatures are dominant and longer term variations small. The results of the harmonic analyses are presented in the appendixes: (1) a listing of coefficients that define the Koko Head temperature and salinity functions for each year and the Christmas Island temperature functions for each quarter of each year, (2) graphs of the fitted curves together with the observed values for each year.

In this paper harmonic functions are presented of sea-surface temperatures and salinities that have been regularly measured near Koko Head, Oahu (lat. 21°16′ N., long. 157°41′ W.) since 1956 and at Christmas Island (lat. 1°51′ N., long. 157°23′ W.) since 1954 (Fig. 1).

Sea-surface temperatures and salinities change in response to, and therefore reflect, sea-air interaction processes (heat exchange, evaporation minus precipitation) and oceanographic processes (advection, diffusion). For example, the mean sea-surface temperature for a month at Koko Head provides a measure of the mean heat content of the water near the surface. Thus, if the mean temperature for March is above that for February, then meteorological and oceanographic processes must have taken place to raise the mean heat content of

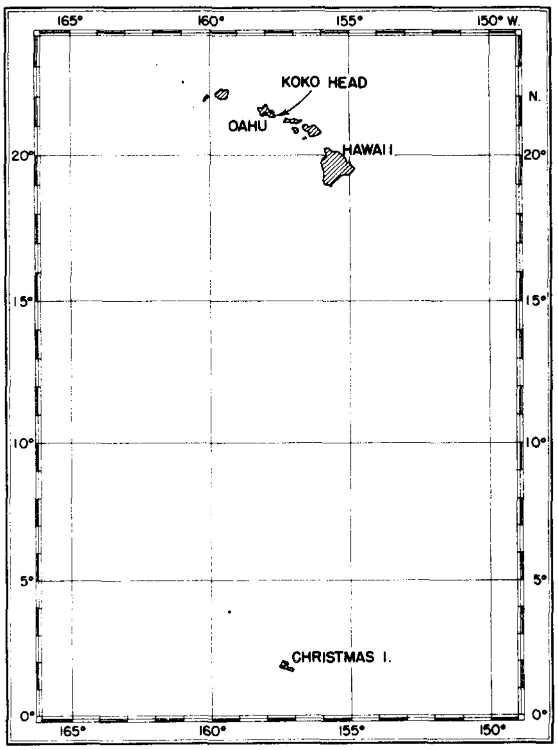

FIGURE 1.—Location of Koko Head, Oahu and Christmas Island.

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the surface water in March above that in February. This concept was used in studies of the Hawaiian oceanographic climate (Seckel, 1962, 1969) and has been applied to Hawaiian fishery problems (Seckel and Waldron, 1960; Seckel, 1963).

Rigorously, the theory of distribution of properties in the sea states that the change of sea-surface temperature during a time interval, say from the first day of one month to the first day of the next month, is equal to the integral of all meteorological and oceanographic processes affecting the temperature during the time interval:

$$\theta_b - \theta_a = \int_a^b (\text{all processes})\, dt.$$

θ_a is the temperature at the beginning and θ_b is the temperature at the end of the interval. In application, the choice of θ_a and θ_b presents the following problems: The difference in the observed temperatures at times a and b also reflects the effect of short-term variability ("noise") that is not of interest in monitoring the large-scale events. If one uses monthly mean temperatures in the heat budget equation that include observations made 15 days before and after times a and b, then the change of temperature incorporates the effect of processes that lie outside the interval of interest. Although mean values usually provide an adequate measure of the temperature change during given time intervals, the true change of temperature can be obscured. One can overcome the problems caused by the two unsatisfactory methods of obtaining measures of the temperature change by finding suitable functions that filter out undesirable short-term variability without obscuring the basic temperature and salinity trends.

Techniques that can be used in the smoothing of time series data have been reviewed by Holloway (1958) and usually involve moving averages of the data to which weighting factors have been assigned.

Curve fitting provides another method of approach. A useful technique that has been used in this report, is to obtain an analytic expression for the temperature and salinity as a function of time by Fourier analysis. The Fourier series is efficiently, and therefore inexpensively, derived by computer. Efficiency is furthered in that graphs can be produced by automatic plotter. The Fourier series provides a least-squares fit of the observed values. It permits filtering of undesired variability, facilitates statistical evaluation of the data, and—within limits—provides insight into the properties of the distribution.

These advantages will become apparent in the following sections of this report. The results of the analyses for each year of observation are presented in the appendix in both tabular and graphical form.

THE FOURIER METHOD

Fourier series are well known, widely applied, and adequately described in texts of advanced calculus. A good description can be found in Sokolnikoff (1939) where the derivation of the Fourier coefficients by least-squares method is also presented.

The temperature or salinity is expressed as a function of time, t, in the Fourier series:

$$S_n(t) = \frac{A_0}{2} + \sum_n (A_n \cos n\omega t + B_n \sin n\omega t), \quad n = 1,2,3, \ldots k$$

where $\omega = \frac{2\pi}{T}$, and T is the fundamental period. For example, if harmonic analysis is to be performed on data collected for a duration of 1 year, T would be 365 days.

The Fourier series contains the coefficients A_0, A_n, and B_n that are given by the Fourier integrals

$$A_n = \frac{2}{T} \int_0^{t=T} F(t) \cos(n\omega t)\, dt, \quad n = 0,1,2, \ldots k,$$

and

$$B_n = \frac{2}{T} \int_0^{t=T} F(t) \sin(n\omega t)\, dt, \quad n = 1,2,3, \ldots k.$$

The coefficient A_0 is the special case of A_n with $n = 0$. In our application $F(t)$ is the temperature or salinity at the time t. Of course, the functional relationship between temperature and time or salinity and time is not known so that

$F(t)$ is the observed temperature or salinity at the time t. Furthermore, $F(t)$ is known only at finite intervals of time so that the above Fourier integrals must be obtained by numerical integration. This integration, approximating the area under the curves $F(t)$ cos $(n\omega t)$ and $F(t)$ sin $(n\omega t)$, is performed by summing areas of rectangles with height $G(t)$ cos $(n\omega t)$ or $G(t)$ sin $(n\omega t)$, and with width Δt, the sampling interval.

The finite difference form of the Fourier integrals is

$$A_n = \frac{2}{T} \sum_{i=1}^{m} G(t)_i \cos(n\omega t)\Delta t_i, \quad n = 0,1,2, \ldots k,$$

and

$$B_n = \frac{2}{T} \sum_{i=1}^{m} G(t)_i \sin(n\omega t)\Delta t_i, \quad n = 1,2,3, \ldots k.$$

The number of samples in the interval $t = 0$ to $t = T$ is $m + 1$,

$$\Delta t_i = t_i - t_{i-1},$$

and $\quad G(t)_i = \frac{1}{2}[F(t_i) + F(t_{i-1})], \quad i = 1,2,3, \ldots m.$

The time used to evaluate the geometric factor is $\frac{1}{2}(t_i + t_{i-1})$. Other schemes of obtaining the best estimate of $G(t)$ cos $(n\omega t)$ during the interval Δt can be used but would not significantly affect the results in our application (see Kaplan, 1953: p. 168-172).

Library programs for the evaluation of Fourier coefficients by computer usually require that the sampling interval, Δt, be constant. Since this condition is not necessarily met in our application, a more flexible computer program was written to evaluate the coefficients. In this program the sampling interval may vary, and the number of samples for the basic period of analysis need not be the same in each application.

The Fourier coefficients evaluated in the above manner enable us to describe analytically the temperature or salinity as a function of time. If we wish to go further and gain insight into the properties of the temperature or salinity distribution, it is more useful to express the Fourier series as a sum of cosines:

$$S_n(t) = \frac{A_0}{2} + \sum_n C_n \cos \omega(nt - \alpha_n), \quad n = 1,2,3, \ldots k.$$

The transformation is accomplished by the use of the trigonometric indentities

$$A_n = C_n \cos \omega\alpha_n,$$

$$B_n = C_n \sin \omega\alpha_n,$$

$$C_n = \pm(A_n^2 + B_n^2)^{1/2},$$

and $\quad \omega\alpha_n = \arctan \frac{B_n}{A_n}.$

In the application described in this report the fundamental period in the Fourier series is the sampling duration or any portion of this duration that may be arbitrarily chosen; the amplitudes and phase angles do not necessarily coincide with natural variations in temperature or salinity; and the harmonic functions have no predictive value.

In some cases, such as the Koko Head temperatures with a well-defined annual cycle, the fundamental period of the Fourier series derived for each year approximates the annual cycle. At Christmas Island, however, an annual temperature cycle is not always clearly apparent. Despite the fact that choice of the fundamental period may be arbitrary and may not coincide with a naturally occurring period, the spectrum is resolved beyond the first few harmonics. For example, if the fundamental period, $n = 1$, is 12 months then the period of the first harmonic, $n = 2$, is 6 months. A naturally occurring 9 months cycle in the observations would in this case not be resolved. As n increases, however, resolution improves to 4, 3, 2.4, 2, etc., months.

The highest harmonic, or n-value, to which harmonic analysis can be carried, is limited by the number of observations. In the ideal case and when samples are equally spaced in time, there must be at least $2n$ observations, i.e., at least two samples per cycle. In nature, where we are dealing with noncyclical variations and unequal spacing of samples a sinusoidal curve cannot be resolved with only two samples, and a minimum of four or, better, six samples is required to achieve good resolution. For example, sea-surface temperatures are to be monitored and the fundamental period of observations is to be 12 months. Resolution of a 1-month cycle ($n = 12$), requires four samples per month, or sampling once per week.

APPLICATION OF THE FOURIER METHOD

In practice, the Fourier method described above must be adapted to each specific application. In addition to the minimum number of samples necessary in order to attain a desired resolution another restriction applies to variations in the sampling interval. Although the computer program used to obtain the results of this paper allows a varying sampling interval, thus accepting a sequence with missing observations, the sampling interval can be allowed to vary only within limits. For example, at least four samples per month are necessary to resolve a monthly cycle. This cycle will, however, not be resolved if the samples are taken on four consecutive days, rather than being evenly distributed throughout the month. It is also possible to aid the harmonic analysis in rapid convergence to its best fit with the observed values by adjusting the fundamental period of analysis and by performing some preliminary operations which are described below.

APPLICATION TO KOKO HEAD SEA-SURFACE TEMPERATURES AND SALINITIES

The sampling station is located near Koko Head at the exposed, eastern shore of Oahu so that the sea-surface temperatures and salinities measured there reflect open-ocean conditions. The salinities appear to be affected by runoff only on rare occasions of heavy rainfall. Both the temperatures and salinities are based on bucket samples. The salinity is determined in the Hawaii Area Fishery Research Center, Honolulu.

Before 1961 samples were collected at weekly intervals and subsequently twice weekly, usually on Tuesday and Friday mornings. Occasionally sampling has been missed. The computer program must therefore accept data with an irregular sampling interval.

The basic period for analysis has been chosen to be 1 year. Harmonic analysis began with the first sample and ended with the last sample of the year. The sampling time, in days and months, was converted to days of the year beginning with the first of the year.

Owing to a longer term trend, the value of a property at the beginning is not necessarily the same as at the end of an annual cycle. In the case of Koko Head salinities and Christmas Island temperatures, it will be seen later that an annual cycle is, in fact, not always apparent. The noncyclic trend during the analysis period can be obtained by linear approximation. Rapid convergence to the best fitting function can then be achieved by performing the harmonic analysis on the residuals of the observed values from a linear fit.

In our application the first observed value, $F(t_0)$, and the last observed value, $F(t_l)$, for the period were used to obtain the linear equation

$$S' = F(t_0) + bt$$

where $$b = \frac{F(t_l) - F(t_0)}{t_l - t_0}.$$

The residuals, $R_m = F(t_m) - [F(t_0) + bt_m]$, $m = 0,1,2, \ldots l$, were used to obtain the Fourier coefficients. The Koko Head temperatures and salinities for each year are then expressed by the function

$$S = K + bt + \sum_{n=1}^{k} C_n \cos\omega(nt - \alpha_n)$$

where $$K = F(t_0) + \frac{A_0}{2} \text{ and } \omega = \frac{2\pi}{T}.$$

The phase angles and coefficients for each of the years 1956-69 of the sea-surface temperatures are listed in appendix A Table 1, and of the sea-surface salinity are listed in appendix A Table 2.

The functions for each year together with the observed values of the sea-surface temperature and salinity have been drawn by automatic plotter and are presented in appendix B.

Quality control of the data was achieved by two passes of the data through the computer. First, the fitted graphs and plots of the observed values as well as listed deviations of observed values from the functions that resulted from the first computer analysis were used to reject obviously erroneous observations. The analysis was then repeated without the rejected observations. The tabulations in appendix A and figures in appendix B are the result of the second pass through the computer. The rejected values are plotted and identified in the figures of appendix B.

APPLICATION TO CHRISTMAS ISLAND SEA-SURFACE TEMPERATURES

The Christmas Island sea-surface temperatures are measured with a bucket thermometer each morning (about 0900 local time) in the channel leading from the open sea to the lagoon. The differences between open sea and lagoon water temperatures have not been determined. It is reasonable to assume that these temperatures differ, and so introduce variability in the observed temperature as tidal currents in the channel change from day to day. The tide-induced variability will, however, not be reflected by a harmonic function where the resolution of the highest harmonic is longer than 1 month. Although the sampling site is not ideal, the observed temperatures are believed to reflect, with some bias induced by lagoon temperatures, the changes of sea-water temperature from month to month.

The procedure to obtain functions of the Christmas Island temperatures was the same as that used for the Koko Head temperatures and salinities with the exception that a different fundamental period was chosen. In contrast to Koko Head where an annual cycle dominates the sea-surface temperature, longer term changes dominate the temperature at Christmas Island. The basic temperature pattern at Christmas Island also changes from year to year. For these reasons a duration of 120 days was chosen as fundamental period and Fourier analysis was performed, as before, on the residuals of the observed values from a linear fit.

For each year, the 120-day periods followed in sequence with an overlap of 30 days. The periods ran from the first day of the year to day 120, from day 91 to day 210, from day 181 to day 300, and from day 271 to day 390, extending 25 days into the following year. In this manner rapid convergence of the harmonic function to the best fit was obtained.

With daily sampling and a fundamental period of 120 days, harmonic analysis could be carried to the harmonic $n = 30$, but to do so would introduce variability that we wish to smooth out. Although a resolution of 1 month requires harmonic analysis to $n = 4$ only, the analysis was arbitrarily carried out to $n = 7$, resolving a period of 16 days.

The resulting phase angles and coefficients for 1954-69 of the sea-surface temperature are listed in appendix C. The functions for each year together with the observed values have been drawn by automatic plotter and are presented in appendix D.

Quality control procedures were identical to those for the Koko Head analyses. Relatively large data gaps occurred at Christmas Island in 1964, 1967, and 1968. Because some observations were available during each of the 120-day periods in question, harmonic analysis produced coefficients that enabled drawing of curves in appendix D although there were no data. These curves were not erased since it is instructive to see what harmonic analysis will do when faced with insufficient data.

DISCUSSION OF RESULTS

In this paper we are concerned with the derivation and presentation of harmonic functions of regularly observed sea-surface temperatures and salinities at fixed stations rather than with oceanographic interpretations. In the discussion of the results we will, therefore, concern ourselves primarily with the quality of fit of the functions. We will also briefly discuss some properties of the temperature and salinity distributions that are reflected by the functions and, finally, show functions spanning the entire time of observations.

QUALITY OF FIT

A superficial inspection of the figures in appendixes B and D shows that the harmonic functions follow the trend of the observed values very well. Closer inspection, however, reveals that there are cases where the fitted curves depart from the observed trend. An example occurred when the Koko Head salinity function (appendix B) for 1956 fluctuated about the observed values from day 145 to day 180. The fluctuations were caused by a data gap between these days. A 15-day data gap is too large when harmonic analysis resolves a period of 1 month. Another example of deviations occurred in the Christmas Island temperature function (appendix D) for 1968 between day 240 and day 275. Again, a 30-day data gap is too large when harmonic analysis resolves a period of 19 days.

These examples illustrate that the sampling interval in harmonic analysis may vary only within limits and that the interval of permissible sampling gaps depends upon the period resolved by the analysis. In cases such as were cited, where the fundamental period of analysis is much longer than the sampling gap, it is possible to constrain the harmonic function by inserting "dummy" values based on linear interpolation of the last sample before, and the first sample after, the data gap.

There are cases where the fitted curve fails to follow the observed trend. When the deviations from the fitted curves are relatively large, there is a tendency to reject the observed values during quality control procedures, blaming the deviations on erroneous sampling. Temperature deviations of this type occurred at Koko Head during days 65 to 90 of 1967. First the observed temperatures fell to 0.6° C below the fitted curve and then rose abruptly 1.3° to 0.6° C above the fitted curve. Erroneous sampling is ruled out since more than one sample was involved in establishing the trend that was abruptly broken and, in addition, the salinity showed similar variability during the same time interval. First the observed salinity rose to 0.15‰ above the expected value and then dropped abruptly 0.37‰ to 0.16‰ below the expected value. In the Hawaiian region the temperature increases and the salinity decreases southward. Thus, northward-southward displacements of the water that would result in the observed temperature and salinity changes were the probable cause for the large deviations rather than sampling error.

In order to assess the quality of fit quantitatively, we will consider several aspects of the standard error of estimate (root mean square deviations of the observed from the expected values). This statistical parameter is listed in three tables for each function, with harmonic analysis carried out for the fundamental period, the first harmonic, the second harmonic, etc. ($n = 1,2,3,\ldots$). Table 1 applies to Koko Head temperatures, Table 2 to Koko Head salinities, and Table 3 to Christmas Island temperatures.

In each case the listed standard error of estimate decreases or reaches a constant value with increasing n. The fit of the function therefore improves or levels off as the analysis is carried out to higher harmonics. Exceptions to this trend occurred in 1956, 1959, and 1961 when the standard errors of estimate for the Koko Head salinity functions (Table 2) increase as the highest n values are reached. Prior to May 1961, only four or five samples per month were obtained at Koko Head and therefore the highest n value permitted by the sampling frequency had been reached. In addition, sampling gaps occurred in 1956, as mentioned before, and in 1961 between days 220 and 241.

The fit of the Koko Head temperature functions (Table 1) improves most rapidly during the first few harmonics and with analysis carried out to $n = 6$, the standard error of estimate is near or below 0.3° C. With analysis carried out to $n = 13$, the standard error of estimate is below 0.2° C for all years excepting 1963 and 1965-68.

Greatest improvement of fit for the Koko Head salinity functions (Table 2) does not always occur during the first few harmonics but continues as analysis is carried beyond $n = 6$. In 1960, for example, the standard error of estimate with analysis to $n = 1$, $n = 6$, and $n = 13$, is 0.090‰, 0.075‰, and 0.038‰, respectively. The standard error of estimate at the n value of best fit in Table 2 is below 0.04‰ except

TABLE 1.—Standard error of estimate (° C) for each annual temperature function at Koko Head, 1956-68, with harmonic analysis carried out in sequence to $n = 1, 2, 3, \ldots$ and 13.

	N-VALUES												
YEAR	1	2	3	4	5	6	7	8	9	10	11	12	13
1956	0.21	0.19	0.17	0.16	0.16	0.14	0.14	0.14	0.13	0.13	0.12	0.12	0.11
1957	0.36	0.29	0.24	0.24	0.23	0.23	0.23	0.21	0.20	0.20	0.19	0.19	0.18
1958	0.31	0.29	0.24	0.24	0.22	0.22	0.22	0.22	0.22	0.20	0.20	0.18	0.17
1959	0.41	0.38	0.29	0.29	0.26	0.24	0.24	0.24	0.22	0.21	0.20	0.20	0.19
1960	0.30	0.27	0.24	0.24	0.23	0.23	0.22	0.22	0.19	0.19	0.17	0.16	0.15
1961	0.47	0.37	0.35	0.33	0.32	0.31	0.31	0.28	0.24	0.23	0.20	0.18	0.17
1962	0.32	0.25	0.25	0.25	0.23	0.21	0.21	0.21	0.20	0.19	0.19	0.19	0.17
1963	0.30	0.29	0.28	0.28	0.27	0.26	0.23	0.23	0.21	0.21	0.21	0.21	0.21
1964	0.29	0.29	0.28	0.25	0.25	0.24	0.23	0.21	0.20	0.18	0.17	0.17	0.16
1965	0.49	0.45	0.38	0.36	0.34	0.30	0.28	0.27	0.27	0.27	0.27	0.26	0.26
1966	0.43	0.32	0.32	0.30	0.30	0.27	0.27	0.27	0.26	0.26	0.26	0.26	0.26
1967	0.44	0.40	0.34	0.33	0.32	0.27	0.26	0.25	0.25	0.24	0.24	0.24	0.23
1968	0.37	0.32	0.28	0.28	0.28	0.27	0.26	0.25	0.24	0.24	0.23	0.23	0.23

TABLE 2.—Standard error of estimate (‰) for each annual salinity function at Koko Head, 1956-68, with harmonic analysis carried out in sequence to $n = 1, 2, 3, \ldots$ and 13.

	N-VALUES												
YEAR	1	2	3	4	5	6	7	8	9	10	11	12	13
1956	0.045	0.034	0.032	0.031	0.031	0.030	0.030	0.029	0.028	0.025	0.025	0.028	0.030
1957	0.068	0.057	0.054	0.054	0.045	0.041	0.039	0.037	0.034	0.034	0.032	0.030	0.030
1958	0.069	0.066	0.059	0.059	0.056	0.053	0.053	0.052	0.049	0.048	0.041	0.036	0.036
1959	0.174	0.099	0.076	0.075	0.074	0.073	0.069	0.064	0.060	0.058	0.054	0.053	0.054
1960	0.090	0.083	0.081	0.078	0.077	0.075	0.069	0.063	0.056	0.053	0.050	0.042	0.038
1961	0.064	0.061	0.051	0.047	0.043	0.038	0.037	0.036	0.033	0.030	0.027	0.028	0.028
1962	0.049	0.046	0.046	0.044	0.043	0.041	0.037	0.037	0.036	0.035	0.034	0.034	0.034
1963	0.054	0.053	0.052	0.046	0.045	0.045	0.043	0.041	0.037	0.037	0.034	0.033	0.033
1964	0.086	0.078	0.069	0.063	0.061	0.059	0.052	0.052	0.051	0.044	0.044	0.043	0.038
1965	0.094	0.085	0.078	0.072	0.072	0.071	0.066	0.064	0.058	0.056	0.054	0.048	0.045
1966	0.044	0.043	0.042	0.037	0.037	0.034	0.034	0.033	0.033	0.030	0.030	0.029	0.028
1967	0.079	0.078	0.074	0.072	0.068	0.061	0.055	0.054	0.052	0.051	0.050	0.046	0.044
1968	0.060	0.052	0.051	0.046	0.042	0.040	0.038	0.038	0.035	0.035	0.033	0.033	0.033

TABLE 3.—Standard error of estimate (° C) for each quarterly temperature function at Christmas Island, 1954-68, with harmonic analysis carried out in sequence to $n = 1, 2, 3, \ldots$ and 7.

		N-VALUES						
YEAR	QUARTER	1	2	3	4	5	6	7
1954	1	0.44	0.41	0.41	0.41	0.32	0.30	0.29
	2	0.36	0.35	0.33	0.33	0.32	0.30	0.30
	3	0.51	0.50	0.50	0.50	0.47	0.45	0.44
	4	0.39	0.38	0.37	0.33	0.30	0.30	0.29
1955	1	0.30	0.28	0.27	0.26	0.26	0.26	0.25
	2	0.29	0.29	0.28	0.28	0.28	0.27	0.26
	3	0.34	0.33	0.33	0.33	0.32	0.32	0.32
	4	0.46	0.45	0.43	0.42	0.40	0.39	0.39
1956	1	0.40	0.38	0.38	0.37	0.36	0.35	0.35
	2	0.52	0.50	0.48	0.48	0.46	0.45	0.45
	3	0.48	0.47	0.45	0.44	0.43	0.41	0.41
	4	0.38	0.38	0.36	0.36	0.36	0.32	0.32
1957	1	0.48	0.46	0.45	0.44	0.43	0.43	0.43
	2	0.61	0.54	0.54	0.54	0.53	0.51	0.51
	3	0.44	0.44	0.43	0.43	0.40	0.39	0.38
	4	0.40	0.39	0.35	0.34	0.33	0.30	0.28
1958	1	0.26	0.25	0.24	0.24	0.24	0.23	0.23
	2	0.33	0.33	0.33	0.32	0.31	0.30	0.29
	3	0.37	0.35	0.30	0.29	0.28	0.28	0.28
	4	0.32	0.31	0.28	0.27	0.26	0.25	0.25
1959	1	0.41	0.34	0.30	0.28	0.28	0.27	0.27
	2	0.40	0.38	0.36	0.35	0.34	0.34	0.33
	3	0.48	0.39	0.36	0.34	0.32	0.32	0.32
	4	0.43	0.39	0.36	0.32	0.31	0.29	0.29
1960	1	0.30	0.29	0.27	0.26	0.26	0.25	0.25
	2	0.35	0.33	0.32	0.31	0.31	0.31	0.31
	3	0.32	0.31	0.30	0.27	0.26	0.26	0.25
	4	0.39	0.32	0.29	0.26	0.26	0.24	0.23
1961	1	0.36	0.34	0.34	0.32	0.31	0.30	0.27
	2	0.34	0.34	0.31	0.27	0.26	0.26	0.25
	3	0.36	0.29	0.29	0.28	0.27	0.25	0.25
	4	0.26	0.24	0.22	0.22	0.20	0.19	0.18
1962	1	0.39	0.34	0.30	0.30	0.30	0.27	0.27
	2	0.38	0.33	0.31	0.30	0.29	0.27	0.25
	3	0.26	0.24	0.21	0.20	0.20	0.19	0.19
	4	0.30	0.26	0.25	0.24	0.24	0.24	0.24
1963	1	0.36	0.36	0.34	0.34	0.32	0.32	0.28
	2	0.46	0.38	0.31	0.30	0.28	0.27	0.21
	3	0.36	0.29	0.29	0.27	0.26	0.25	0.24
	4	0.30	0.29	0.28	0.26	0.26	0.25	0.25
1964	1	0.32	0.32	0.31	0.30	0.29	0.29	0.28
	2	0.37	0.31	0.30	0.29	0.28	0.28	0.27
	3	0.34	0.31	0.29	0.29	0.28	0.27	0.26
	4	0.28	0.27	0.26	0.23	0.22	0.21	0.21
1965	1	0.30	0.29	0.28	0.27	0.26	0.25	0.25
	2	0.36	0.32	0.29	0.29	0.29	0.29	0.29
	3	0.53	0.52	0.45	0.42	0.39	0.38	0.37
	4	0.37	0.32	0.31	0.30	0.30	0.30	0.29
1966	1	0.42	0.39	0.35	0.34	0.33	0.30	0.29
	2	0.42	0.35	0.35	0.34	0.34	0.31	0.30
	3	0.60	0.51	0.48	0.46	0.43	0.42	0.40
	4	0.68	0.68	0.53	0.52	0.51	0.51	0.47
1967	1	0.42	0.40	0.34	0.34	0.34	0.32	0.32
	2	0.40	0.39	0.36	0.36	0.36	0.36	0.35
	3	0.39	0.38	0.37	0.36	0.34	0.33	0.33
	4	0.32	0.30	0.28	0.27	0.27	0.27	0.26
1968	1	0.42	0.37	0.36	0.33	0.33	0.33	0.32
	2	0.37	0.35	0.31	0.31	0.31	0.30	0.30
	3	0.28	0.29	0.29	0.30	0.31	0.28	0.28
	4	0.28	0.27	0.26	0.24	0.23	0.21	0.21

in 1959 and 1965 when it is 0.054‰, and 0.045‰, respectively.

At Christmas Island (Table 3), the average standard error of estimate at $n = 4$ (resolution of 1 month) is near 0.33° C and therefore about 60 % higher than that for the Koko Head temperatures. As previously mentioned, high temperature variability is to be expected at the Christmas Island sampling site.

A standard error of estimate based on all samples used to obtain a function obscures the month-to-month changes in variability that may have occurred. At Koko Head the month-to-month changes in temperature variability as reflected by the standard error of estimate for each month ranges from 0.05° to 0.45° C, the same values for the Koko Head salinities range from 0.006‰ to 0.136‰, and those for Christmas Island temperatures range from 0.17° to 0.66° C. Assuming that sampling error remains constant, the range of variability reflects changes in oceanographic conditions.

The standard error of estimate computed from the temperature and salinity observations of each month also reflects sampling quality in that low values indicate the residual variability in the ocean plus sampling error. For the Koko Head temperature, low values of the monthly standard error of estimate are near 0.1° C and for the Koko Head salinity they are near 0.02‰. The sampling error is therefore within ±0.1° C for the temperature and ±0.02‰ for the salinity. These are the limits to be expected when bucket sampling of the temperature and salinity is carefully done.

Finally, how is the quality of fit affected by sampling frequency and how reliable are the expected values that may be obtained from the harmonic functions? The constraint imposed by the sampling frequency on the resolution that may be attained by harmonic analysis has already been discussed. The present question concerns improvement of fit when the sampling frequency is increased above the minimum requirements.

At Koko Head the sampling frequency was increased from once to twice weekly in 1961. No significant change can be seen in the standard errors of estimate listed in Tables 1 and 2 as a result of doubling the sampling frequency. This observation is consistent with results obtained from oceanographic data collected at Ocean Weather Station "P" in the Gulf of Alaska. Tabata (1964: Table 8) lists the monthly mean value and the standard deviation of the temperature at 10-m depth based on data obtained twice daily, data obtained every second, third, fourth, fifth, sixth, and seventh day of July 1959 and May 1961. For July 1959 the mean temperatures range from 10.70° to 10.81° C and the standard deviations range from 0.60° to 0.76° C. For May 1961 the mean temperatures range from 5.84° to 5.90° C and the standard deviations range from 0.39° to 0.46° C.

In May 1961 Koko Head temperatures and salinities were sampled on 25 days. The mean of all temperature observations was 24.67° C with standard deviation 0.27° C. The mean of temperatures taken every fifth day was 24.58° C with standard deviation 0.39° C. The mean of all salinity observations was 34.759‰ with standard deviation 0.051‰. The mean of salinities taken every fifth day was 34.772‰ with standard deviation 0.058‰. The temperature results from Koko Head are comparable to those from Ocean Weather Station "P" in that mean values and standard deviations based on different sampling frequencies fall within approximately the same range. The standard errors of estimate for the May 1961 Koko Head temperatures and salinities, based on the harmonic functions with resolution of 1 month, are lower than the standard deviations, namely, 0.25° C and 0.027‰, respectively. The standard errors of estimate as well as the standard deviations do not change significantly when the sampling frequency is increased above the required minimum to attain a desired resolution by harmonic analysis.

Increasing the sampling frequency does, however, improve the confidence limits of a mean value or the expected value of a harmonic function. A good measure of the confidence limits of a mean value is the standard error of the mean (the standard deviation divided by the square root of the number of samples). Returning to Tabata's table the standard error of the mean for July 1959 is for twice daily sampling every day 0.086° C, and for twice daily sampling every seventh day 0.253° C. For the same sampling frequencies in May 1961 the standard errors of the mean are 0.053° and 0.15° C, respectively. For the May 1961 Koko Head temperatures the standard error of the mean is 0.055° C with 25 samples and 0.16° C with sampling every fifth day. The standard error of the mean for the May 1961 Koko Head salinities is 0.010‰ with 25 samples and 0.024‰ with sampling every fifth day. On the basis of these considerations, the expected values obtained from the temperature functions have an uncertainty of ±0.10° C, and those from the salinity functions have an uncertainty of ±0.015‰ when samples are obtained twice weekly.

At Christmas Island temperatures are sampled daily rather than twice weekly as at Koko Head. In consequence, despite the larger variability, expected values obtained from the harmonic functions have approximately the same uncertainty as those obtained from the Koko Head harmonic functions. This statement is confirmed by considering the error terms that can be obtained by taking the difference of the expected values at the midpoint of the 30-day overlap portion of the Christmas Island temperature functions (see appendix D). On average this error term is 0.07° C and ranges from 0 to 0.26° C.

SOME PROPERTIES OF THE TEMPERATURE AND SALINITY DISTRIBUTIONS

Although the harmonic functions are merely analytic expressions of the temperature and salinity as a function of time, they do provide, to some extent, insight into the nature of the distributions. For instance, the monthly standard error of estimate, mentioned in the previous section, provides a measure of the month-to-month changes in variability. At Koko Head there is no seasonal pattern in this variability of the temperature; however, there is a seasonal pattern in the variability of the salinity. The monthly standard errors of estimate of the salinity function with harmonic analysis carried out to $n = 13$, are listed in Table 4.

TABLE 4.—Standard error of estimate (‰) for each month, 1956-68, of the Koko Head salinity. Harmonic analysis is carried out to $n = 13$.

	MONTH											
YEAR	1	2	3	4	5	6	7	8	9	10	11	12
1956	0.030	0.017	0.027	0.048	0.064	0.052	0.024	0.008	0.012	0.014	0.014	0.015
1957	0.049	0.013	0.021	0.015	0.030	0.034	0.029	0.017	0.036	0.031	0.018	0.034
1958	0.006	0.041	0.052	0.049	0.059	0.026	0.028	0.028	0.013	0.023	0.044	0.022
1959	0.049	0.035	0.044	0.136	0.040	0.036	0.054	0.023	0.047	0.032	0.023	0.035
1960	0.042	0.032	0.018	0.019	0.056	0.043	0.075	0.035	0.033	0.014	0.014	0.024
1961	0.036	0.019	0.017	0.019	0.027	0.054	0.011	0.020	0.025	0.021	0.023	0.023
1962	0.054	0.040	0.064	0.021	0.013	0.023	0.025	0.033	0.031	0.031	0.018	0.027
1963	0.029	0.026	0.018	0.073	0.045	0.036	0.021	0.025	0.020	0.022	0.032	0.036
1964	0.031	0.033	0.031	0.030	0.029	0.019	0.035	0.050	0.053	0.052	0.024	0.036
1965	0.044	0.053	0.059	0.092	0.032	0.043	0.034	0.016	0.018	0.033	0.019	0.019
1966	0.026	0.016	0.011	0.014	0.022	0.021	0.011	0.012	0.016	0.033	0.065	0.036
1967	0.026	0.029	0.097	0.055	0.050	0.015	0.019	0.021	0.017	0.031	0.029	0.056
1968	0.034	0.024	0.057	0.041	0.040	0.019	0.035	0.026	0.018	0.021	0.016	0.038

In each year excepting 1957, 1964, and 1966, highest variability occurred during the first 7 months of the year. In 1957 a seasonal pattern was not clearly apparent and in 1964 and 1966 highest variability occurred during the last 5 months of the year. Although the seasonal pattern of variability has not been examined in detail, it is consistent with the results of previous studies (Seckel, 1962, 1969). First, Hawaii is located in the vicinity of a relatively high salinity gradient that delineates the boundary of the North Pacific Central Water. Thus, the salinity measured at the Koko Head sampling station is sensitive to variations in the location of this water type boundary. Secondly, northward displacement of water (warm advection) tends to occur during the first 7 months of the year. In consequence the water type boundary that generally lies south of the Koko Head sampling station during autumn and winter is brought to within the vicinity of the sampling station. The months with higher variability tend to be associated with declines in the Koko Head salinity.

Insight into the nature of the distributions is also obtained by examining the spectra of the harmonic functions. It is evident from the figures in appendix B, that considerable temperature and salinity variability at Koko Head occurs with timespans of 35 to 60 days. Rather than showing the amplitudes for each harmonic of every function, the 13-year mean of the absolute magnitude of amplitudes for each harmonic of the Koko Head temperature and salinity functions is presented in Figure 2.

For both the temperature and the salinity, the amplitude of the annual cycle ($n = 1$) is largest. The amplitudes then decline rapidly with increasing harmonics to $n = 5$. In the case of the temperature, a slight increase in amplitude occurs at $n = 6$ and $n = 9$. Similar small increases in amplitudes occur in the case of the salinity at $n = 7$ and $n = 9$. The increased amplitudes at $n = 6$ and $n = 7$, resolving 60- and 52-day periods, reflect the climatic signals described by Seckel (1962, 1969). The increased amplitude at $n = 9$, resolving a 41-day period, reflects shorter term variability that may be due to large geostrophic eddies with dimensions near 200 km (Wyrtki, 1967) or eddying flow near the Hawaiian Islands.

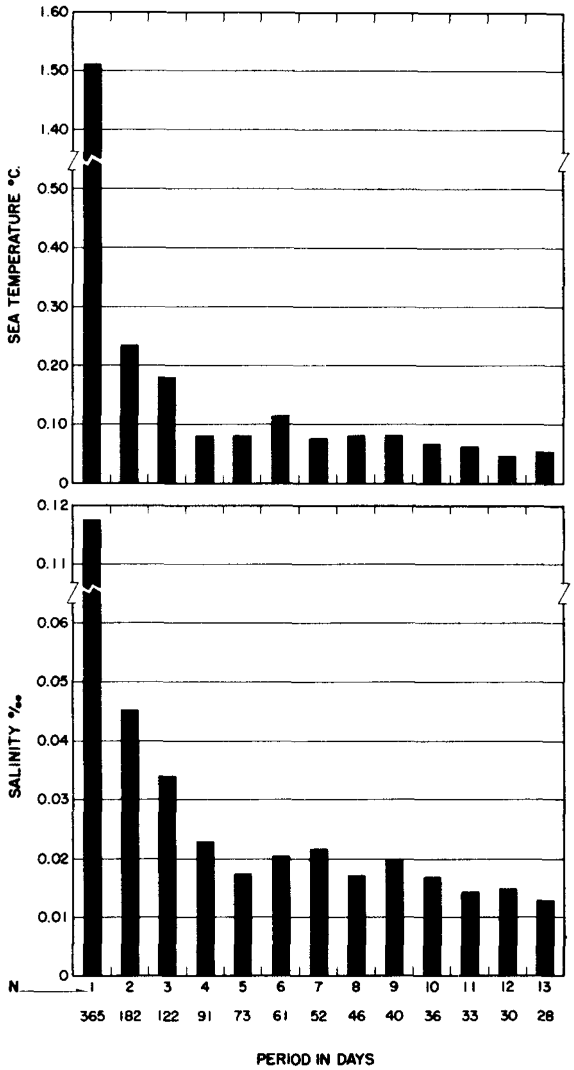

FIGURE 2.—Mean magnitude of amplitudes for each harmonic of the Koko Head temperature and salinity functions, 1956-69.

LONG-TERM HARMONIC FUNCTIONS

Long-term harmonic functions with the fundamental period spanning the entire duration of observations, can be obtained by the method described before in this paper. Temperatures and salinities were used as computed for the

1st and 16th of each month from the harmonic functions whose phase angles and coefficients are tabulated in appendixes A and C. Harmonic analysis was carried to $n = 42$ for the Koko Head temperature and salinity, and $n = 48$ for the Christmas Island temperature, giving in each case a 4 months' resolution. The fitted curves resulting from this analysis are shown in Figure 3, together with the values that were used as input data. Clearly the annual cycle forms the dominant signal in the Koko Head temperature curve. In the Koko Head salinity and Christmas Island temperature curves longer term changes are more pronounced than the annual cycle.

The relatively large deviations of the input data from the long-term function are to be expected. The figures of appendixes B and D show that variations with a duration of less than 4 months can be relatively large and are not resolved by the long-term analyses made.

The spectra of the long-term harmonic functions for the Koko Head temperatures and salinities and the Christmas Island temperatures are shown in Figure 4.

As is also apparent from Figure 3, the spectrum of the Koko Head temperature function is distinct from those of the Koko Head salinity and Christmas Island temperature functions. In the former the 12-month period has the most pronounced amplitude, but in the latter two, although the annual period has a large amplitude, the amplitudes of longer period changes are large and for some periods exceed those of the annual period.

CONCLUSION

The results of this paper show that sea-surface temperatures and salinities regularly monitored at island sampling stations can be expressed by harmonic functions of time. Advantages of analytic expressions for the temperature and salinity were cited in the introduction. Important applications will be in climatic oceanography where one may wish to filter out undesired "background noise." At Christmas Island, for example, the short-term variability with a duration of 1 month or less can be filtered out by using only the harmonic terms to $n = 3$ in the quarterly functions. At Koko Head, the variability with duration of less than 50 days, that may be due to large geostrophic or island-induced eddies, can be filtered out by using only the harmonic terms to $n = 7$ in the annual functions.

We mentioned in the introduction that the rates of change of temperature reflect the climatic processes of change and that distortions or aliasing may occur when monthly mean temperatures are used to compute the change of a property. Consider, for example, the Christmas Island temperatures from March to May 1968 (appendix D, days 61 to 152). In Table 5 are listed the monthly mean observed temperatures, the month-to-month changes of mean temperature, the expected temperatures from the harmonic functions for the 16th of each month (computed with harmonic terms up to $n = 4$), and the month-to-month changes of expected temperatures. It is clear from this illustration that the use of mean values would result in an underestimate of the rise in temperature from March to April, and would obscure the decline in temperature from April to May. The example is not isolated and other instances can be found in both the Koko Head and the Christmas Island data.

TABLE 5.—Month-to-month temperature differences using mean observed temperatures and expected temperatures from the harmonic function, Christmas Island, March to May 1968.

Date	Mean temperature	Change of mean temperature	Expected temperature	Change of expected temperature
	° C	° C	° C	° C
March 1968	25.1		25.1	
		0.9		1.2
April 1968	26.0		26.3	
		0.2		−0.3
May 1968	26.2		26.0	

The results also aid in the choice of an optimum sampling frequency. Both the desired confidence limit and the desired resolution must be considered. If the harmonic functions are to be used in monitoring the oceanographic climate as is the case of those presented in this paper, then the limits of about ±0.1° C for the expected temperature value and ±0.02‰ for the expected salinity value are adequate. As-

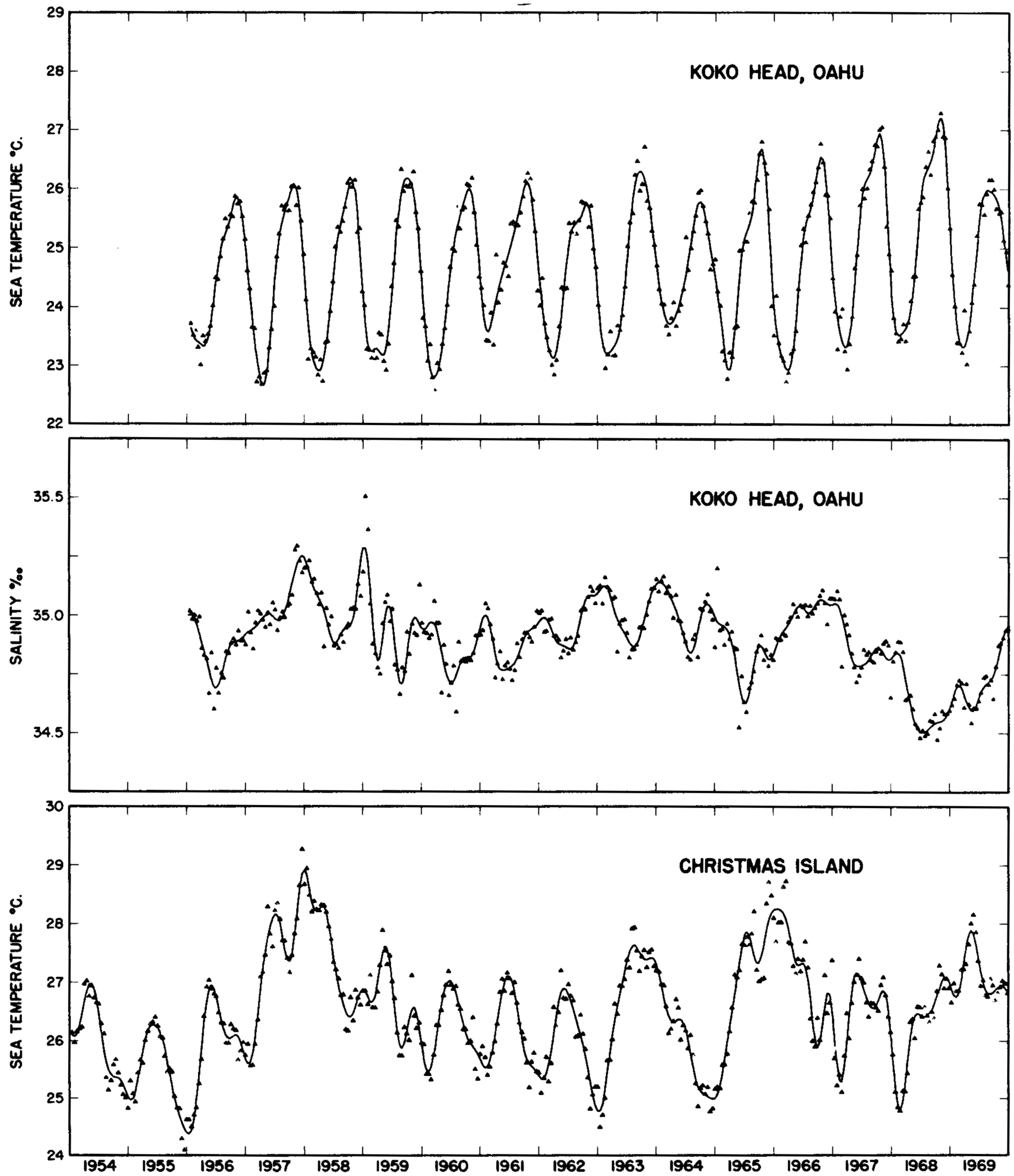

FIGURE 3.—Fitted curves with 4 months' resolution of the Koko Head temperature, 1956-69, Koko Head salinity, 1956-69, and the Christmas Island temperatures, 1954-69. Input data are indicated by small triangles.

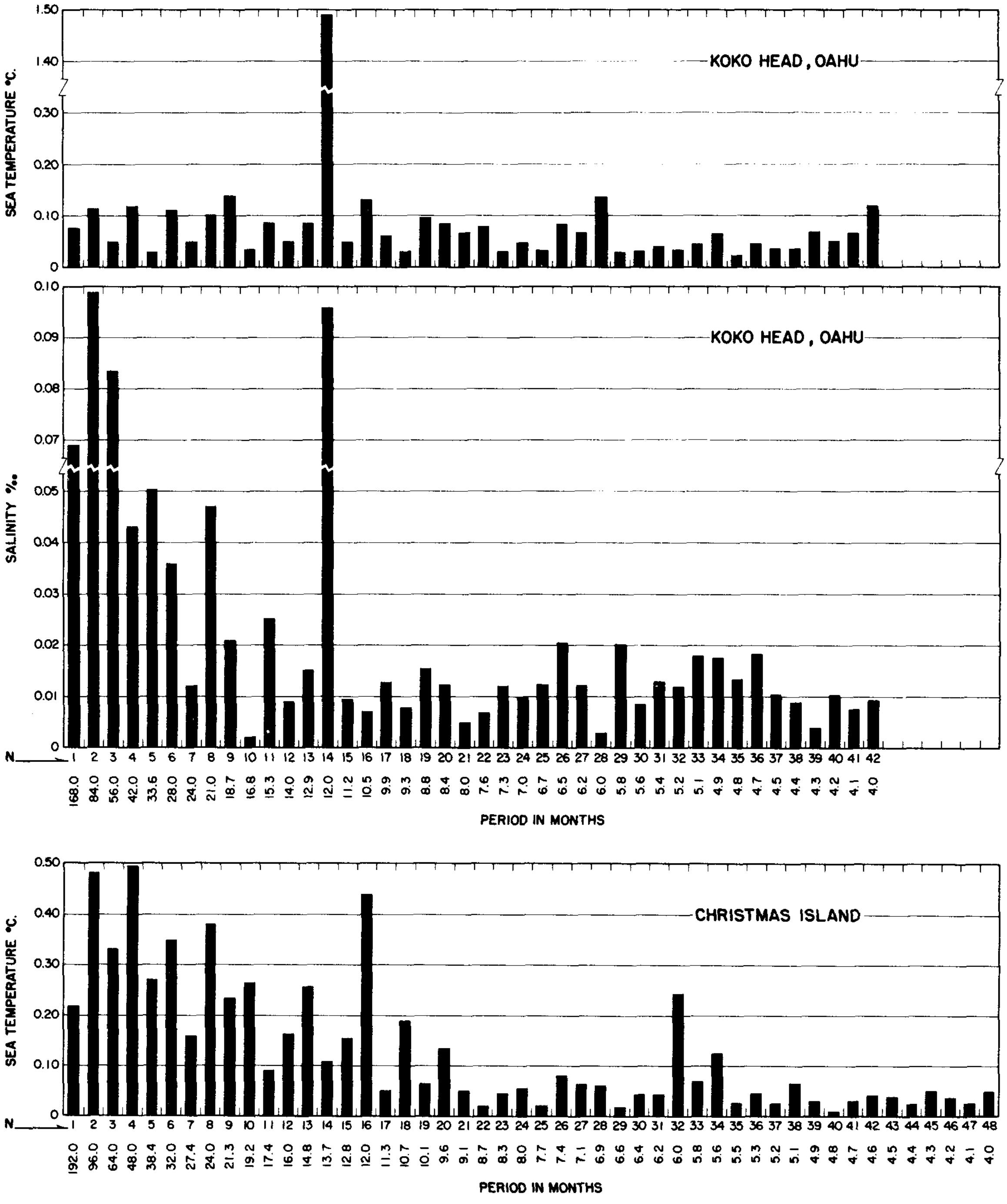

FIGURE 4.—Spectra of the long-term harmonic functions for Koko Head temperatures, 1956-69, Koko Head salinities, 1956-69, and Christmas Island temperatures, 1954-69.

suming that temperature and salinity samples are of Koko Head quality, then for a resolution of 1 month, weekly sampling is sufficient. Occasionally, however, a scheduled sample is not taken or an erroneous value must be eliminated. In such cases sampling gaps would become too large for the desired resolution. Undesirable sampling gaps can be avoided by doubling the minimum sampling frequency.

The simplicity and economy of deriving harmonic functions by computer are of practical value, particularly in the analysis of data sampled automatically. By this method large quantities of data can be brought into useful form rapidly.

The results of this paper, based on manual sampling, are useful in the investigations of changes with a duration of more than 1 month. Automated sampling would broaden the spectrum and permit analyses of shorter term variations such as diurnal changes, changes of tidal period, and other changes with durations of less than 1 month.

Automated sampling would also improve the quality of data since instruments can be placed in locations where undesirable variability is minimized and where manual sampling is difficult. At Koko Head, for example, samples are obtained from an exposed rock ledge where the island effects on the temperature and salinity are small. At Christmas Island, however, the sampling site is convenient and the best obtainable for manual sampling, but it is not the best in terms of monitoring open-ocean temperatures. This shortcoming is often also the case when temperatures and salinities are measured at tide stations located in protected bays or harbors.

The value of regularly monitoring the sea-surface temperatures and salinities has been demonstrated in many instances. For example, empirical relations between Koko Head temperatures and salinities and the availability of skipjack tuna to the Hawaiian fishery have been demonstrated (Seckel, 1963). Bjerknes (1969) has shown the relationship between anomalously high equatorial sea-surface temperatures using primarily Canton Island observations, and the intensification of the North Pacific westerlies and trades. This relationship must, in turn, affect temperatures and salinities in the North Pacific.

In view of these factors, serious consideration should be given to the establishment of automated sampling stations at selected islands in the Pacific. The derivation of harmonic functions, as demonstrated in this paper, would make reduction of data into usable form simple and economical and so facilitate the study of processes which govern the climate in both ocean and atmosphere.

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APPENDIX A

Sea-surface temperatures and salinities, Koko Head, Oahu, 1956-69: Phase angles and coefficients for harmonic functions

$$S = K + bt + \sum_{n=1}^{k} C_n \cos\omega(nt - \alpha_n) ,$$

$$\omega = \frac{2\pi}{365} \text{ days}^{-1} ,$$

t is the time in days beginning with the first day in each year.

APPENDIX A TABLE 1.—Phase angles and coefficients for sea-surface temperatures, Koko Head, 1956-69.

PHASE ANGLES IN DAYS

	N-VALUES												
YEAR	1	2	3	4	5	6	7	8	9	10	11	12	13
1956	73.88	69.97	10.42	-49.85	52.41	-25.23	70.98	-[illegible]7.01	-55.97	19.33	-75.22	-52.34	46.35
1957	76.76	-72.88	30.75	58.99	8.81	-38.20	35.79	73.88	47.14	15.87	-36.68	-16.51	-8.30
1958	62.38	-6.51	-54.94	48.26	-34.27	-45.62	-51.72	-71.25	-46.58	-51.12	-10.61	-2.52	3.53
1959	65.59	-49.40	8.61	-1.48	56.56	-5.01	-70.18	17.77	71.89	-16.60	-27.85	-66.12	-50.78
1960	64.16	74.63	-62.69	37.38	-23.54	16.29	-89.39	23.77	-52.95	32.76	54.93	24.86	74.57
1961	53.19	15.31	-22.50	7.72	39.01	16.65	-51.44	-67.83	-2.26	49.83	88.27	-35.38	27.19
1962	66.15	-74.11	80.89	66.44	-7.78	-13.61	51.28	88.26	20.81	-30.04	29.78	41.65	-26.88
1963	63.76	-68.73	38.52	-27.45	-63.66	79.24	70.43	8.76	27.22	18.36	-76.61	35.19	-25.46
1964	73.31	-23.47	15.92	-57.15	-83.11	54.99	61.66	54.83	48.58	-0.50	51.76	3.89	-85.70
1965	64.85	45.82	-83.31	81.93	40.40	87.16	1.79	4.19	-21.73	-57.28	58.34	27.01	-90.88
1966	72.24	85.17	-29.44	82.13	-85.83	10.42	8.38	31.25	-0.28	-55.09	-24.99	65.54	-64.62
1967	60.48	66.72	-70.68	-9.91	83.89	84.03	-81.16	69.38	-9.63	-62.49	-84.89	-59.30	61.41
1968	65.18	71.07	-5.56	-19.37	26.00	-1.69	9.55	-41.85	-4.66	70.29	32.63	-19.50	-50.35
1969	58.78	-16.32	-68.76	90.78	49.97	12.98	89.93	40.17	-0.78	43.89	-88.96	-54.80	76.11
MEAN	66.13	62.61	-34.59	-9.48	68.75	40.82	29.88	86.20	-16.08	23.05	79.63	19.30	-87.99

APPENDIX A TABLE 1.—Phase angles and coefficients for sea-surface temperatures, Koko Head, 1956-69.—*Continued.*

			AMPLITUDES												
			N-VALUES												
YEAR	K	b	1	2	3	4	5	6	7	8	9	10	11	12	13
1956	24.2857	0.0014	-1.1174	-0.1340	-0.1099	-0.0633	0.0255	-0.1008	-0.0407	0.0206	0.0575	0.0346	-0.0565	0.0313	-0.0228
1957	25.1584	-0.0039	-2.0983	0.3004	-0.2188	0.0195	-0.1006	-0.0499	-0.0256	-0.1151	-0.0761	-0.0508	0.0691	0.0278	0.0400
1958	24.2433	0.0011	-1.4988	-0.1047	-0.2459	-0.0062	-0.0965	0.0225	-0.0385	-0.0154	0.0513	0.1033	-0.0374	-0.0878	-0.0477
1959	24.3697	0.0008	-1.5707	-0.2354	-0.3358	0.0470	0.1414	-0.1173	-0.0240	0.0239	0.0798	0.0968	-0.0969	0.0389	-0.0391
1960	24.3239	0.0008	-1.4963	-0.1570	-0.1641	0.0825	0.0383	-0.0437	0.0810	-0.0558	0.1175	0.0393	0.0931	-0.0677	0.1034
1961	24.7453	0.0006	-1.0587	-0.4112	-0.1832	0.1314	0.0498	0.1401	-0.0574	0.1761	0.1798	0.1026	0.1073	-0.1105	-0.0403
1962	24.7987	-0.0014	-1.4437	0.2802	0.0761	0.0502	-0.1250	0.1465	0.0561	-0.0798	0.0590	-0.0587	0.0302	0.0352	0.1037
1963	24.3471	0.0019	-1.4430	-0.1124	-0.1025	0.0501	0.0916	-0.0979	0.1870	0.0645	-0.1092	-0.0450	-0.0409	0.0344	0.0424
1964	24.7972	-0.0008	-1.0946	-0.0931	0.0691	0.1611	-0.0573	-0.0956	-0.0675	-0.1461	0.0639	-0.1162	0.0872	-0.0194	-0.0702
1965	24.8006	-0.0003	-1.6891	-0.2896	-0.3339	0.1715	0.1857	0.2120	0.1332	0.1040	0.0290	0.0596	0.0518	-0.0716	0.0622
1966	25.0047	-0.0014	-1.9152	-0.4056	-0.0944	-0.1330	0.0300	-0.1687	0.0236	0.0548	0.0825	-0.0161	0.0254	-0.0170	0.0253
1967	25.2850	-0.0008	-1.8725	-0.2850	-0.3005	-0.1125	-0.0966	-0.2437	0.1082	-0.0694	0.0794	-0.0341	0.0586	0.0555	-0.0572
1968	24.8546	0.0028	-1.5364	-0.2820	-0.2155	-0.0568	-0.0194	-0.0275	0.1192	-0.1263	0.0787	-0.0445	-0.0137	0.0294	-0.0817
1969	24.7701	0.0003	-1.3374	0.2035	-0.0760	-0.1235	0.1544	0.1096	0.1056	-0.0626	0.0594	0.1396	-0.1064	-0.0373	-0.0016
MEAN	24.6989	0.0001	-1.5040	-0.1458	-0.1291	0.0147	0.0220	-0.0164	0.0215	-0.0335	0.0461	0.0098	0.0324	-0.0135	-0.0013

APPENDIX A TABLE 2.—Phase angles and coefficients for sea-surface salinities, Koko Head, 1956-69.

PHASE ANGLES IN DAYS

YEAR	N-VALUES												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1956	-14.52	-68.27	-89.13	-4.08	52.05	15.36	16.54	-2.09	-40.95	62.40	24.44	-7.01	-8.85
1957	-25.52	67.90	6.11	22.47	4.85	-25.00	-8.60	-19.40	28.27	-82.38	73.57	83.14	34.59
1958	25.99	46.89	57.99	50.30	15.25	-88.16	17.82	-10.99	20.65	-34.78	43.80	-31.33	-5.52
1959	-57.46	-68.08	18.94	-71.46	59.29	-35.64	-64.98	-19.12	-72.96	71.02	3.20	-8.24	28.66
1960	15.99	-61.03	81.05	56.42	-14.00	43.21	-58.49	-64.87	-87.20	62.22	71.57	4.93	-16.77
1961	-17.27	48.97	45.52	-26.18	2.69	13.58	2.15	-50.58	-38.94	82.68	-85.76	70.83	72.92
1962	-37.91	35.77	-1.00	-88.77	23.96	-32.29	-56.02	43.47	21.90	-52.99	24.88	-39.69	0.0
1963	17.33	-33.19	-66.85	53.84	13.09	-61.03	86.91	-72.22	12.17	2.06	-56.52	27.79	-39.95
1964	28.39	-19.79	-82.91	-82.99	53.70	-69.42	55.05	51.23	-17.95	8.29	-67.42	-80.98	-1.92
1965	-39.50	1.54	-49.50	-29.90	-31.57	-81.95	9.24	1.57	-56.55	90.65	42.78	-22.53	9.48
1966	-51.26	10.69	59.98	14.96	-10.51	78.69	26.93	-21.00	-86.73	30.96	10.93	-40.71	64.32
1967	-50.66	-58.57	35.70	38.98	-76.06	-77.32	-70.26	-26.09	17.82	-57.29	-13.46	26.17	44.48
1968	12.29	85.76	80.38	-11.84	-1.90	-31.29	-19.27	-49.39	-5.37	43.75	-66.43	70.00	18.69
1969	-9.16	30.08	30.65	-63.98	-47.50	47.99	74.49	66.13	-85.89	48.21	84.68	-58.03	-60.81
MEAN	-11.17	47.12	63.72	46.06	26.70	29.49	39.79	-16.95	-79.12	-76.28	-75.40	13.10	-5.70

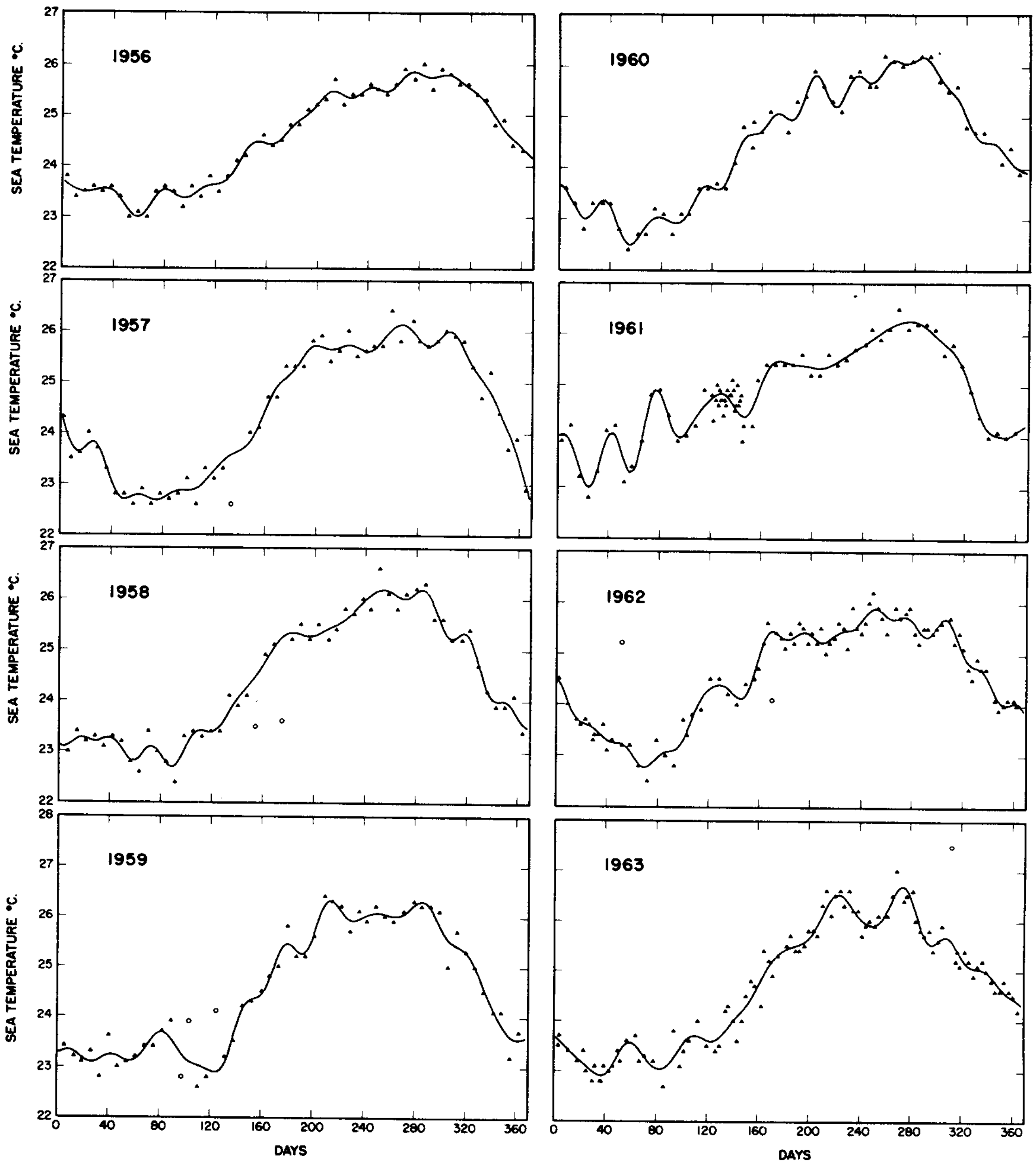
APPENDIX A TABLE 2.—Phase angles and coefficients for sea-surface salinities, Koko Head, 1956-69.—*Continued.*

			AMPLITUDES												
			N-VALUES												
YEAR	K	b	1	2	3	4	5	6	7	8	9	10	11	12	13
1956	34.8727	-0.0001	0.1261	-0.0420	-0.0164	-0.0071	0.0038	0.0099	0.0043	0.0167	-0.0176	-0.0258	0.0240	-0.0133	0.0270
1957	34.9345	0.0006	0.0604	-0.0544	-0.0305	-0.0029	0.0407	0.0242	0.0183	-0.0156	0.0195	-0.0079	-0.0127	0.0122	-0.0037
1958	34.9491	0.0004	0.1650	0.0324	0.0412	0.0088	0.0262	-0.0244	0.0063	0.0213	0.0244	-0.0130	-0.0292	0.0160	-0.0042
1959	35.2530	-0.0018	0.1996	0.2016	0.0884	0.0168	-0.0140	0.0120	-0.0272	-0.0340	-0.0271	0.0179	0.0254	-0.0191	0.0106
1960	34.8357	0.0000	0.1089	-0.0434	-0.0269	-0.0287	0.0239	0.0188	-0.0380	0.0378	-0.0373	-0.0198	-0.0187	0.0283	-0.0172
1961	34.8545	0.0001	0.1147	0.0368	0.0375	-0.0172	-0.0194	-0.0324	0.0162	-0.0191	-0.0179	-0.0177	0.0117	0.0064	-0.0151
1962	34.9396	0.0001	0.1084	-0.0230	-0.0116	0.0211	0.0035	0.0190	0.0254	0.0076	-0.0103	0.0106	-0.0051	0.0055	-0.0064
1963	34.9780	0.0001	0.1388	0.0067	-0.0164	-0.0371	0.0161	-0.0111	-0.0215	0.0162	0.0226	-0.0113	0.0197	0.0122	-0.0072
1964	34.9727	0.0001	0.1178	-0.0503	-0.0503	-0.0416	0.0209	-0.0158	0.0394	-0.0022	-0.0174	0.0316	-0.0025	-0.0142	0.0272
1965	34.9953	-0.0010	0.1566	-0.0564	0.0477	0.0412	0.0056	0.0163	0.0328	-0.0185	0.0356	0.0195	-0.0188	0.0331	0.0252
1966	34.8927	0.0006	-0.0401	-0.0131	0.0156	0.0271	0.0083	-0.0177	-0.0011	-0.0088	-0.0077	0.0161	0.0043	0.0084	-0.0114
1967	35.0184	-0.0009	0.1037	-0.0109	-0.0366	0.0248	-0.0310	-0.0417	-0.0368	-0.0184	-0.0169	0.0167	0.0091	0.0237	0.0179
1968	34.7135	-0.0006	0.1219	0.0424	0.0134	-0.0306	-0.0245	0.0191	0.0160	-0.0055	-0.0195	0.0044	0.0147	0.0050	-0.0022
1969	34.5912	0.0007	0.0821	0.0190	-0.0441	-0.0157	0.0044	-0.0243	-0.0200	0.0172	-0.0054	-0.0222	-0.0044	0.0103	0.0052
MEAN	34.9143	-0.0001	0.0972	-0.0051	0.0071	-0.0013	0.0064	0.0013	0.0070	-0.0026	-0.0065	0.0022	0.0073	0.0067	0.0040

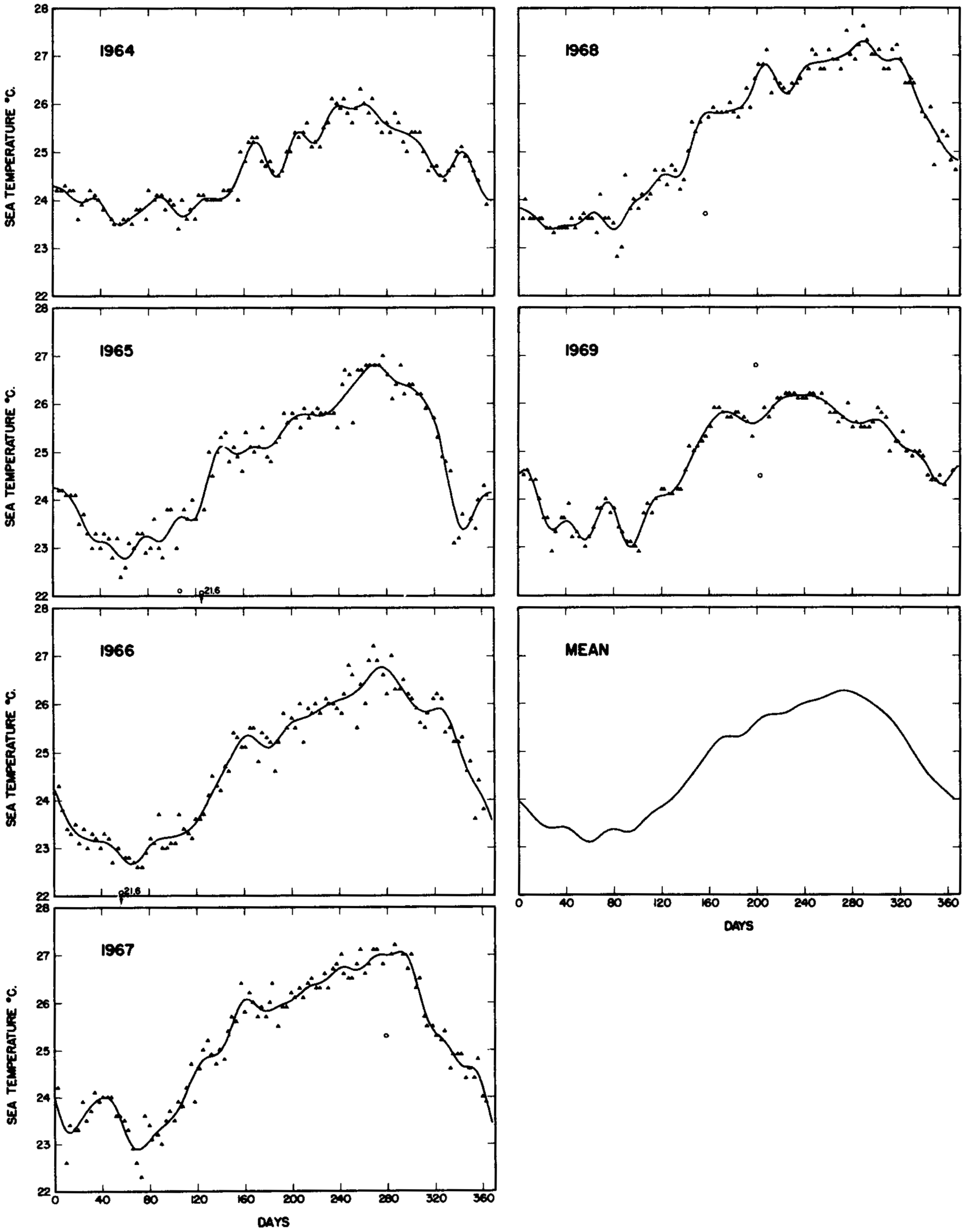
APPENDIX B

Sea-surface temperatures and salinities, Koko Head, Oahu, 1956-69: Fitted curves with observed values for each year.

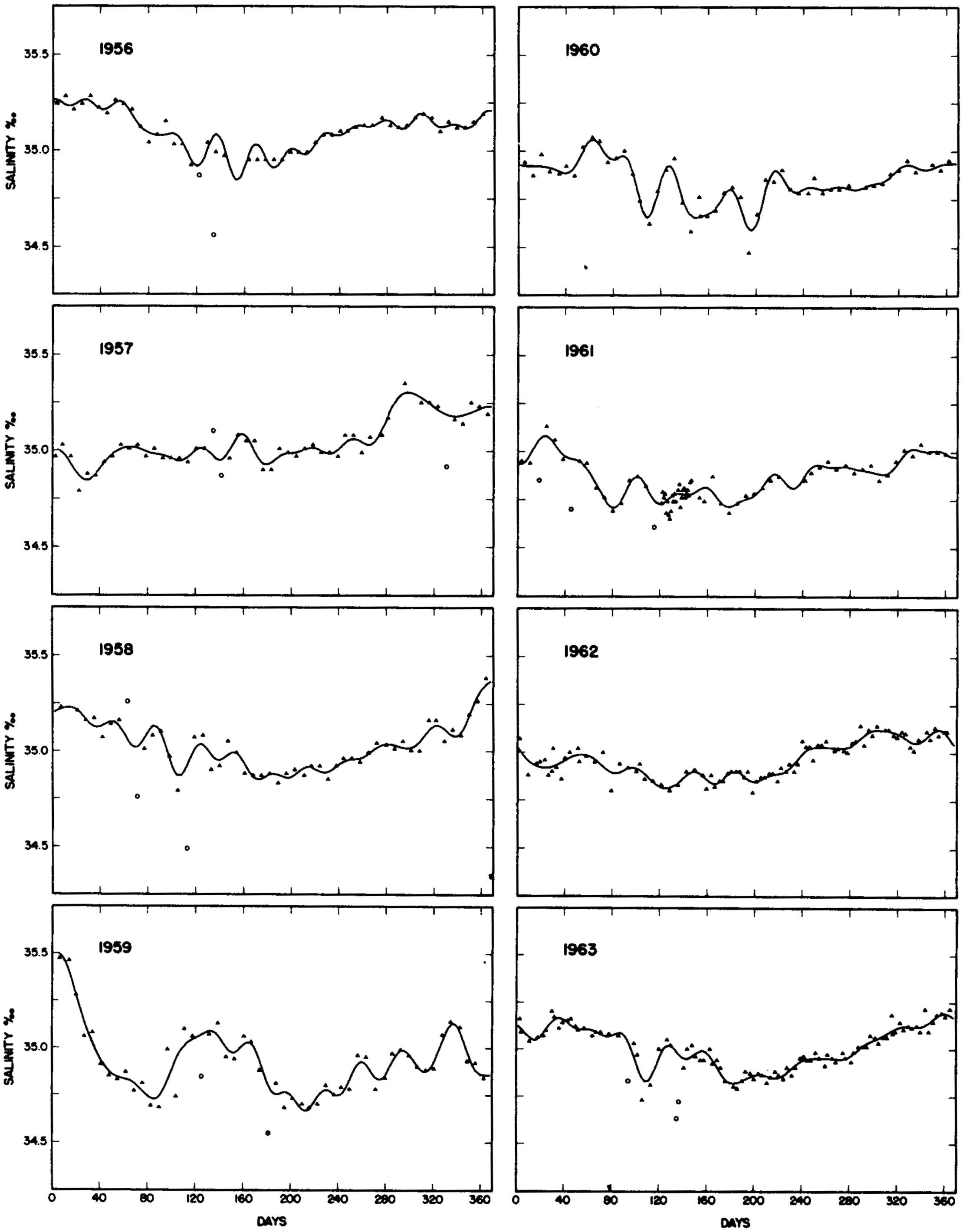
Note: Circled observations have not been used in the harmonic analysis.



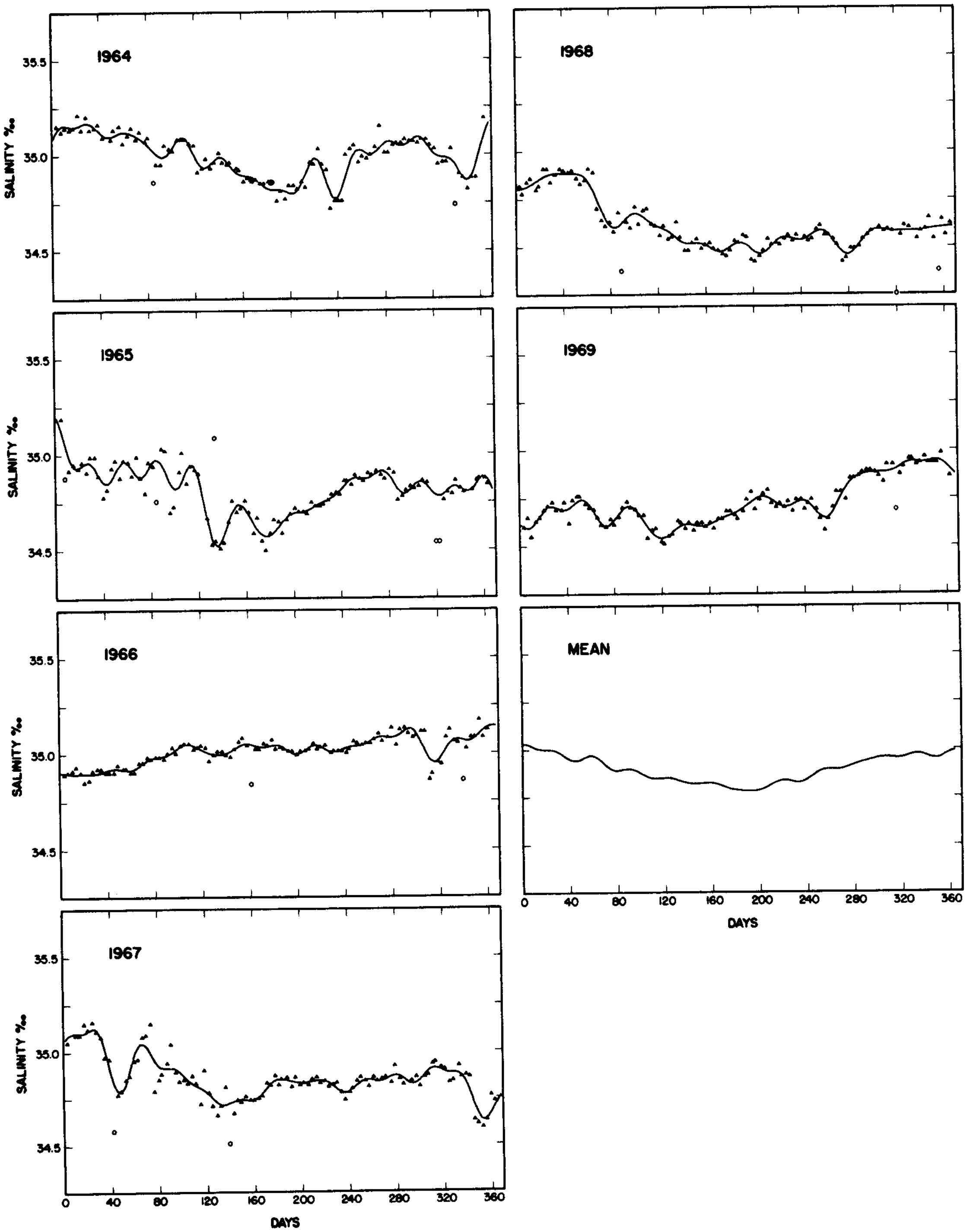
APPENDIX B FIGURE 1.—Sea-surface temperatures, Koko Head, 1956-69.



APPENDIX B FIGURE 1.—Sea-surface temperatures, Koko Head, 1956-69.—*Continued.*



APPENDIX B FIGURE 2.—Sea-surface salinities, Koko Head, 1956-69.



APPENDIX B FIGURE 2.—Sea-surface salinities, Koko Head, 1956-69—*Continued.*

APPENDIX C

Sea-surface temperatures, Christmas Island, 1954-69. Phase angles and coefficients for harmonic functions for each quarter of each year:

Days 1 to 120 = First quarter,
91 to 210 = Second quarter,
181 to 300 = Third quarter,
271 to 390 = Fourth quarter, extending 25 days into new year,

$$S = K + bt + \sum_{n=1}^{k} C_n \cos\omega(nt - \alpha_n) ,$$

$$\omega = \frac{2\pi}{120} \text{ days}^{-1} ,$$

t is the time in days beginning with the first day of each quarter.

Note: Mean values do not include phase angles and coefficients for the third and fourth quarters of 1967.

APPENDIX C TABLE 1.—Phase angles and coefficients for sea-surface temperatures, Christmas Island, 1954-69.

PHASE ANGLES IN DAYS

		N-VALUES						
YEAR	QU.	1	2	3	4	5	6	7
1954	1	29.96	-14.98	0.68	22.60	-16.29	0.45	-28.43
	2	21.92	24.40	-25.77	15.21	23.02	24.75	8.06
	3	-19.46	21.45	-25.73	-10.27	-14.81	-6.52	-20.70
	4	-15.22	27.64	6.23	-9.84	25.21	27.55	-26.39
1955	1	-25.04	-14.36	-19.66	-25.05	10.26	1.57	-17.43
	2	-1.34	-22.96	16.15	-12.62	-27.83	-6.10	24.53
	3	-28.76	23.27	29.69	-21.77	-18.26	28.12	10.11
	4	-5.04	-11.73	-11.08	5.52	9.94	-2.89	-1.09
1956	1	-15.33	29.40	18.77	24.08	27.16	17.61	-20.53
	2	25.55	29.94	-23.88	27.18	-22.20	-10.40	-16.16
	3	-26.54	21.39	-22.54	13.46	20.29	29.39	-17.29
	4	-28.99	16.56	-15.87	-0.61	6.14	-27.68	1.48
1957	1	-20.33	9.65	-3.33	-13.39	-22.18	24.07	4.11
	2	12.45	3.80	-21.56	22.60	5.19	18.04	16.55
	3	12.50	-8.47	27.08	22.42	5.63	4.72	-24.37
	4	1.99	-11.91	-1.14	15.68	5.08	-17.77	-21.22
1958	1	-15.56	-9.55	16.21	-19.82	-16.02	-9.08	8.91
	2	-25.92	-8.31	-7.41	26.09	-26.16	-28.63	17.52
	3	17.11	-4.98	25.83	9.96	6.39	-15.05	-24.35
	4	-25.45	16.23	-11.73	19.38	7.25	16.83	20.96
1959	1	-11.25	18.21	25.30	27.40	2.42	-3.45	-27.76
	2	-4.58	-18.05	-2.17	-6.57	15.76	-29.98	28.66
	3	-13.92	-7.90	26.00	-0.14	15.14	3.09	21.42
	4	-25.63	-23.22	-13.08	18.13	-10.51	-22.26	28.72
1960	1	-21.36	3.16	-22.67	15.60	-1.99	22.75	21.91
	2	14.95	-26.09	18.68	6.43	23.12	-22.78	28.33
	3	21.14	17.18	10.04	7.87	6.28	20.84	-2.55
	4	-0.89	0.26	-28.94	9.46	17.99	23.48	-26.06
1961	1	-4.50	2.29	14.03	9.01	9.50	-26.75	14.56
	2	-9.38	-18.74	15.60	10.81	5.59	-18.31	-27.53
	3	-27.87	-27.65	27.26	26.89	21.47	-10.55	-13.26
	4	-19.95	-17.14	-3.05	0.39	-13.80	-7.92	24.95
1962	1	10.13	-24.96	-16.24	29.64	4.40	16.34	-1.98
	2	-10.33	-1.79	27.51	28.58	-0.81	8.88	-15.25
	3	18.78	20.95	-6.12	-29.13	-2.99	-1.52	2.56
	4	29.70	7.75	14.37	-13.15	-25.15	-4.47	13.65
1963	1	18.62	14.96	21.66	21.78	-21.72	-8.72	-19.16
	2	-25.86	-23.08	-21.84	-25.95	-12.90	-7.23	-3.15
	3	27.97	-11.92	17.92	-10.01	20.37	-9.25	15.06
	4	5.53	-27.74	-4.23	12.23	13.77	6.00	-14.49
1964	1	10.51	23.82	-4.95	-11.60	-22.13	13.52	9.47
	2	29.78	-24.66	11.64	-19.75	22.21	-13.85	-22.39
	3	3.66	-21.54	13.24	5.89	-17.78	16.09	6.84
	4	9.57	-19.61	-22.27	-11.16	-12.19	-18.46	-4.51
1965	1	-12.81	22.40	-29.54	24.56	25.12	16.28	-23.00
	2	-16.03	-23.97	-7.39	-19.45	2.93	-11.03	1.10
	3	28.09	-13.92	12.10	-9.87	6.09	14.42	-23.32
	4	-10.42	20.99	27.57	29.48	-18.90	17.82	-18.83
1966	1	-2.72	-22.51	-8.68	9.75	-10.62	-6.62	29.58
	2	29.77	26.12	4.26	21.28	15.28	27.61	-22.03
	3	7.34	25.04	9.12	25.58	1.93	10.76	4.92
	4	8.34	0.12	-20.67	1.80	-28.17	3.39	26.99
1967	1	-22.45	14.34	-25.09	18.95	0.56	10.45	-20.67
	2	-8.36	-8.88	-1.88	-22.37	-24.08	16.60	21.27
	3	-29.90	-2.24	18.31	26.99	-16.01	9.55	-12.24
	4	-19.32	23.12	7.45	-6.85	-19.61	7.19	8.69
1968	1	-9.18	-10.65	23.64	-20.93	11.66	-20.43	-24.40
	2	-14.96	-23.53	13.31	-14.18	7.83	27.96	20.00
	3	-0.07	-22.76	5.17	5.55	-23.92	-10.81	-2.39
	4	27.65	-15.84	3.23	-21.90	-10.88	-24.36	6.48
1969	1	13.34	14.46	6.54	-17.39	7.54	6.63	-7.29
	2	8.02	-23.81	-20.76	26.56	-28.81	29.11	27.75
	3	22.54	11.28	11.96	25.05	-8.17	12.69	28.92
	4	-24.65	-0.88	-15.33	-12.66	-11.06	-13.66	4.95
MEAN	1	-14.97	-10.80	-22.92	-20.97	16.82	20.22	20.72
	2	9.25	11.74	20.56	15.71	28.30	-19.45	-9.57
	3	5.57	22.63	15.62	14.65	25.53	22.30	-25.12
	4	-4.77	-2.64	-27.75	-26.43	20.79	-26.95	10.88

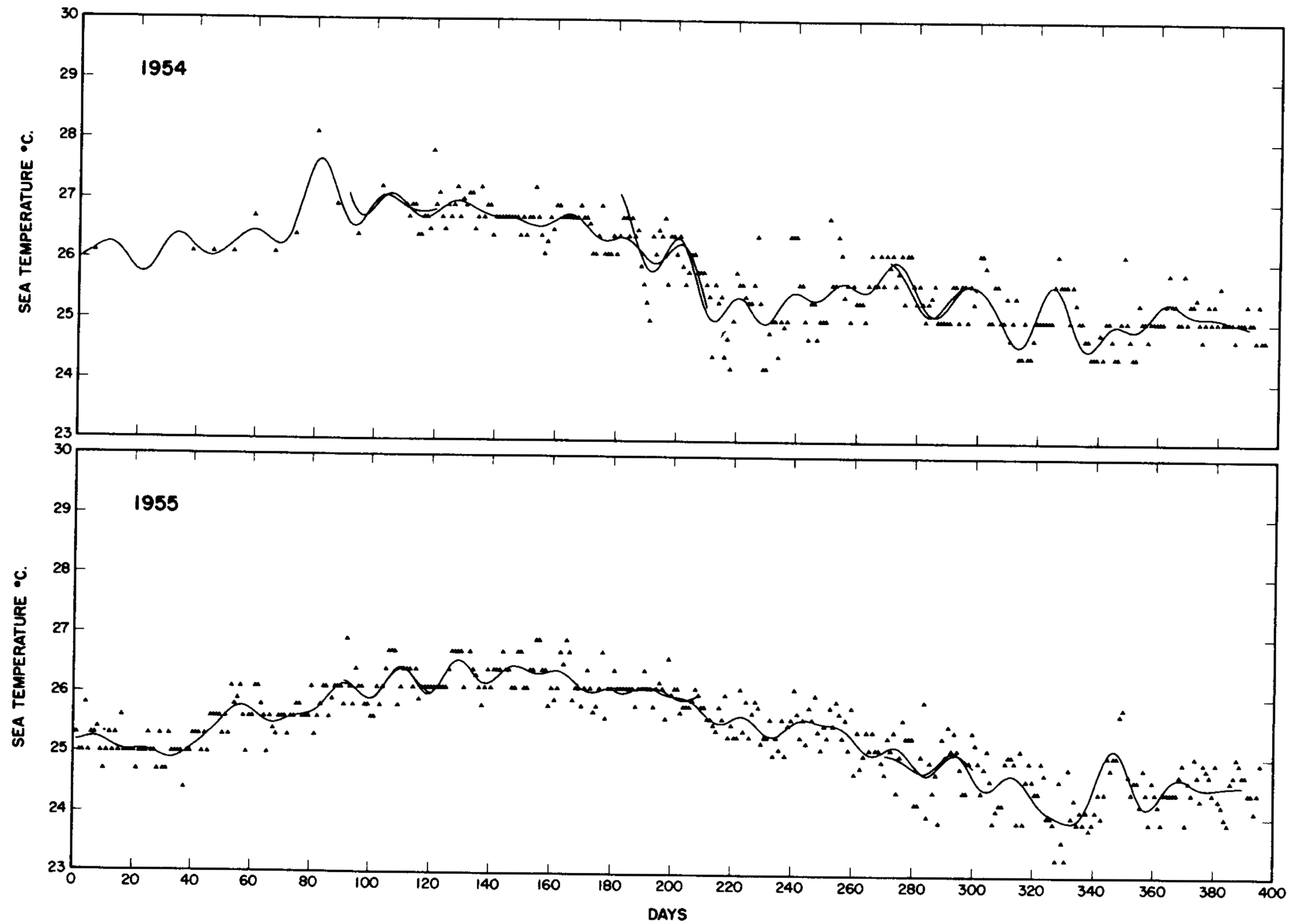
APPENDIX C TABLE 1.—Phase angles and coefficients for sea-surface temperatures, Christmas Island, 1954-69—*Continued.*

				AMPLITUDES						
				N-VALUES						
YEAR	QU.	K	b	1	2	3	4	5	6	7
1954	1	26.0740	0.0070	-0.2543	-0.1310	0.1609	-0.0772	-0.2829	0.0637	0.0743
	2	27.5957	-0.0169	-0.3583	-0.1443	0.1307	-0.1171	-0.1275	-0.1517	-0.0342
	3	26.4268	-0.0139	0.6131	0.0815	-0.0171	0.1382	0.2404	0.1808	-0.1111
	4	25.6919	-0.0090	0.2918	-0.1571	-0.0880	0.2517	0.1746	-0.1001	-0.0949
1955	1	25.1633	0.0067	0.2697	0.1614	-0.0873	0.1008	-0.0360	-0.0632	-0.0698
	2	26.2916	-0.0017	-0.1687	0.0292	-0.0565	-0.0347	0.0635	0.1000	0.1044
	3	26.1027	-0.0107	-0.0046	0.1350	-0.0375	-0.0218	0.0858	-0.0284	-0.0917
	4	24.8187	-0.0041	0.2259	-0.1449	0.1578	-0.1789	0.1836	0.0816	-0.0946
1956	1	24.3949	0.0183	0.4678	-0.1493	-0.0999	0.1311	-0.1107	-0.0674	0.0277
	2	27.7890	-0.0174	-0.4323	-0.2228	0.1648	-0.1209	0.1979	0.0861	0.0232
	3	27.0093	-0.0139	0.5433	-0.1689	0.1687	-0.1784	-0.1535	0.1829	0.0787
	4	26.1278	-0.0049	0.1022	0.0890	-0.1479	-0.0249	-0.0544	0.2481	-0.0168
1957	1	25.4577	0.0176	0.3612	0.1892	0.1458	0.1574	0.1045	0.0540	0.0693
	2	27.7558	0.0025	-0.2169	-0.4098	0.1621	-0.0802	-0.1359	0.2288	-0.1088
	3	27.6226	0.0008	0.4254	0.0970	0.0939	-0.1184	-0.2340	-0.1508	-0.1202
	4	28.0288	0.0066	-0.5153	0.0870	-0.2655	-0.1550	0.0453	0.2115	0.1495
1958	1	28.6567	-0.0050	0.1989	0.1052	0.0983	0.0407	0.0531	-0.0732	-0.0469
	2	28.2124	-0.0091	-0.2768	-0.0700	-0.0795	-0.0835	-0.1346	-0.0783	0.1074
	3	26.6129	0.0	0.3456	-0.1571	0.2536	0.0694	-0.1207	0.0260	0.0474
	4	26.2684	0.0066	-0.2862	0.0679	0.1955	0.1226	0.0867	-0.0855	0.0184
1959	1	26.8512	0.0025	0.3797	-0.3106	-0.2379	-0.1429	0.0562	-0.0545	0.0838
	2	27.8849	-0.0132	-0.2731	-0.1508	-0.1839	-0.1429	-0.0674	-0.0819	0.0641
	3	26.3895	-0.0025	0.5706	0.4088	-0.2084	-0.1502	-0.1315	-0.0472	-0.0216
	4	26.7237	-0.0091	-0.1536	-0.2531	-0.2372	-0.2151	0.1113	0.1418	-0.0299
1960	1	25.6504	0.0042	0.4722	0.0679	0.1461	0.1082	0.0721	-0.0691	0.0460
	2	26.9858	-0.0025	-0.3299	0.1787	-0.1260	-0.0797	0.0321	-0.0404	-0.0377
	3	26.5916	-0.0033	0.3107	0.1345	0.1480	-0.1676	-0.0941	0.0284	-0.0720
	4	26.0923	-0.0049	0.1670	-0.3080	0.1905	-0.1680	-0.0272	-0.1538	-0.1171
1961	1	25.3508	0.0109	0.3816	0.1665	0.0125	-0.1322	-0.1105	0.1068	0.1995
	2	26.7608	0.0025	-0.1900	-0.0611	-0.1699	-0.2098	0.1176	0.0254	0.0986
	3	26.5390	-0.0066	-0.4129	-0.2944	0.0796	0.0568	-0.1021	-0.1658	-0.0319
	4	25.3383	0.0016	-0.2387	0.1069	0.1592	-0.0197	0.1122	0.0896	0.0918
1962	1	24.9107	0.0168	0.1042	-0.2920	0.2226	0.0317	0.0329	0.1702	-0.0358
	2	26.5159	0.0033	-0.2921	-0.2701	-0.1616	0.1077	-0.0996	-0.1532	0.1131
	3	26.9093	-0.0139	-0.2714	0.1472	-0.1661	-0.0770	0.0532	0.0728	-0.0359
	4	25.6078	-0.0090	-0.2539	0.2136	0.0877	0.0963	-0.0275	-0.0509	-0.0641
1963	1	24.5106	0.0202	-0.1137	-0.0707	0.1670	0.0588	0.1332	-0.0370	-0.2257
	2	27.3700	-0.0033	0.5749	0.3627	0.3065	0.1391	0.1594	0.0779	0.2440
	3	27.2705	0.0041	0.3865	-0.2983	0.0802	-0.1328	0.1014	0.1157	-0.0641
	4	27.6762	-0.0066	-0.0697	0.1280	0.0933	0.1408	0.0886	0.0388	0.0603
1964	1	26.7596	-0.0025	0.4138	-0.0692	0.0988	0.1046	-0.0882	-0.0860	-0.0658
	2	26.2958	-0.0017	0.2579	-0.2425	-0.1394	0.0522	0.1147	0.0422	-0.0869
	3	26.0159	-0.0092	0.4285	-0.1814	0.1304	-0.0741	0.1106	0.0733	-0.1184
	4	24.8755	0.0033	0.1803	-0.1368	0.0999	0.1655	-0.0848	0.0989	-0.0299
1965	1	25.0849	0.0151	0.2248	-0.1215	0.0998	-0.0676	-0.1001	0.0939	-0.0430
	2	26.4776	0.0149	-0.2682	-0.2288	-0.1801	-0.0793	-0.0111	0.0295	-0.0471
	3	27.2809	0.0033	0.6393	0.1140	0.3702	0.2312	-0.2351	0.1376	-0.0732
	4	27.3759	0.0098	-0.7293	-0.2572	-0.0559	-0.1109	0.0830	-0.0606	-0.0654
1966	1	28.3584	-0.0076	-0.5145	0.1954	-0.2585	0.0741	-0.1413	0.1728	0.1227
	2	28.5148	-0.0198	-0.8788	-0.3261	0.0899	-0.0254	-0.0921	-0.1870	0.0975
	3	26.6145	-0.0041	0.4308	0.4345	0.2067	0.2310	0.2313	0.0932	0.1898
	4	27.2510	-0.0164	-0.8011	-0.0462	0.6174	0.1390	-0.1069	-0.0289	-0.2482
1967	1	25.3932	0.0076	0.3944	0.1627	0.2864	-0.0779	0.0240	-0.1438	0.0926
	2	26.4576	0.0066	-0.5433	-0.1618	-0.1932	-0.0712	-0.0620	0.0473	0.0553
	3	26.8336	-0.0044	0.1341	0.1093	0.0717	0.0649	-0.0896	-0.0822	0.0505
	4	27.8062	-0.0233	-0.0832	0.0851	0.0784	0.0593	0.0301	0.0046	0.0312
1968	1	25.1250	0.0033	0.5597	0.2691	-0.1413	0.2029	0.0708	0.0239	0.0730
	2	25.9567	0.0066	-0.1308	-0.1705	-0.2038	0.0706	0.0276	-0.1003	0.0172
	3	26.2971	0.0057	0.3013	0.0743	-0.1219	0.2030	0.1886	-0.2381	0.2042
	4	26.9967	0.0	0.1068	-0.0967	-0.1344	0.1499	-0.1181	-0.1165	0.0213
1969	1	26.4649	0.0168	0.1895	0.1321	-0.0869	-0.1854	0.0663	-0.0638	0.0926
	2	28.5357	-0.0165	-0.2815	0.3460	0.1314	-0.1603	0.1393	-0.1347	-0.1080
	3	27.0045	-0.0025	-0.0625	0.1304	-0.0853	-0.1166	-0.0388	0.1949	0.0671
	4	26.4789	0.0074	-0.2169	-0.0886	-0.1190	-0.1058	-0.1537	-0.0858	-0.0390
MEAN	1	25.8878	0.0082	0.2151	0.0484	0.0522	0.0285	-0.0074	-0.0107	0.0132
	2	27.2124	-0.0041	-0.1744	-0.0779	-0.0930	-0.0509	-0.0233	0.0283	0.0229
	3	26.7124	-0.0044	0.1770	0.0789	0.0498	-0.0071	-0.0629	0.0384	-0.0094
	4	26.3568	-0.0019	-0.0997	-0.0317	0.0509	0.0459	0.0341	0.0573	-0.0145

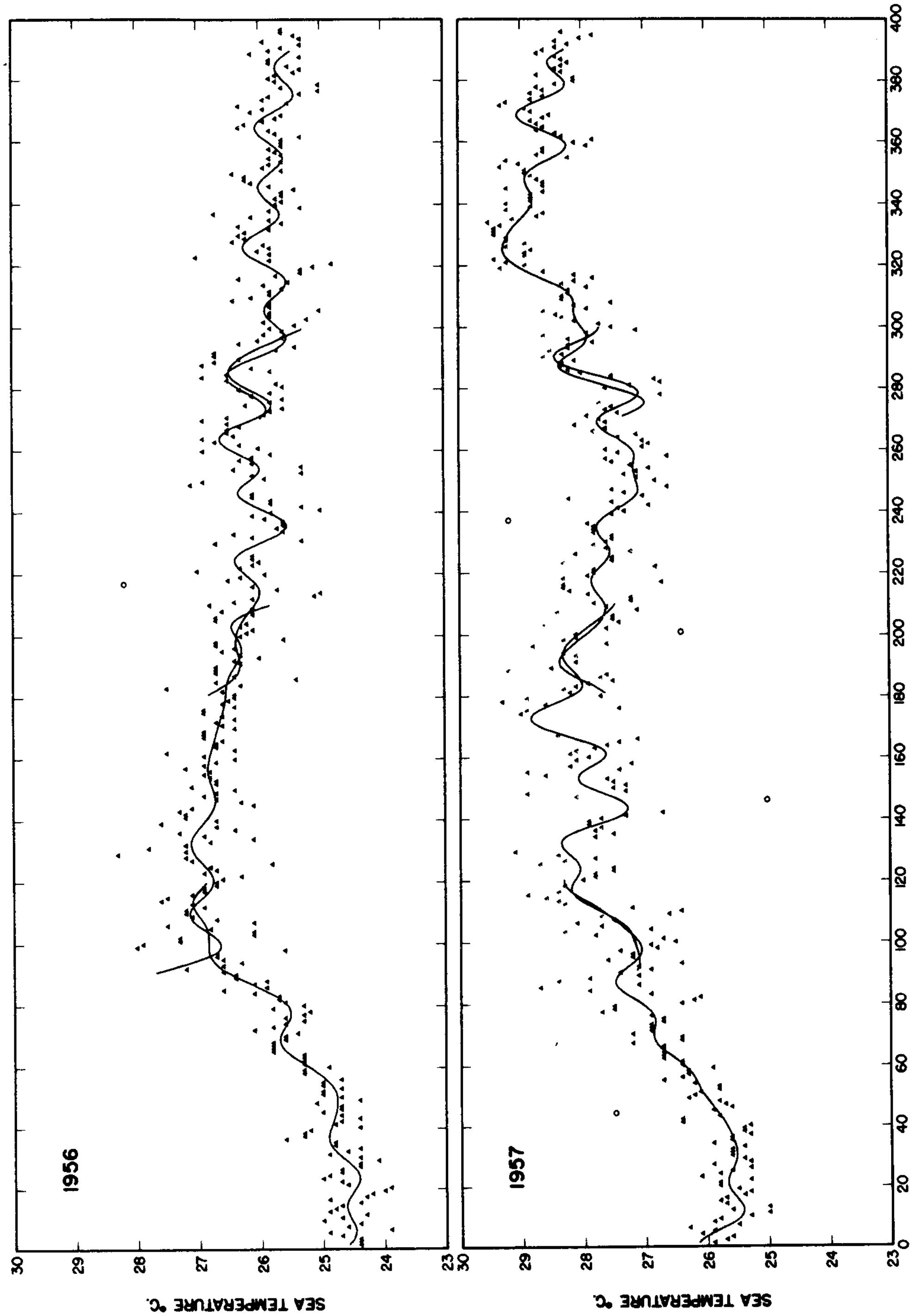
APPENDIX D

Sea-surface temperatures, Christmas Island, 1954-69: Fitted curves with observed values for each year.

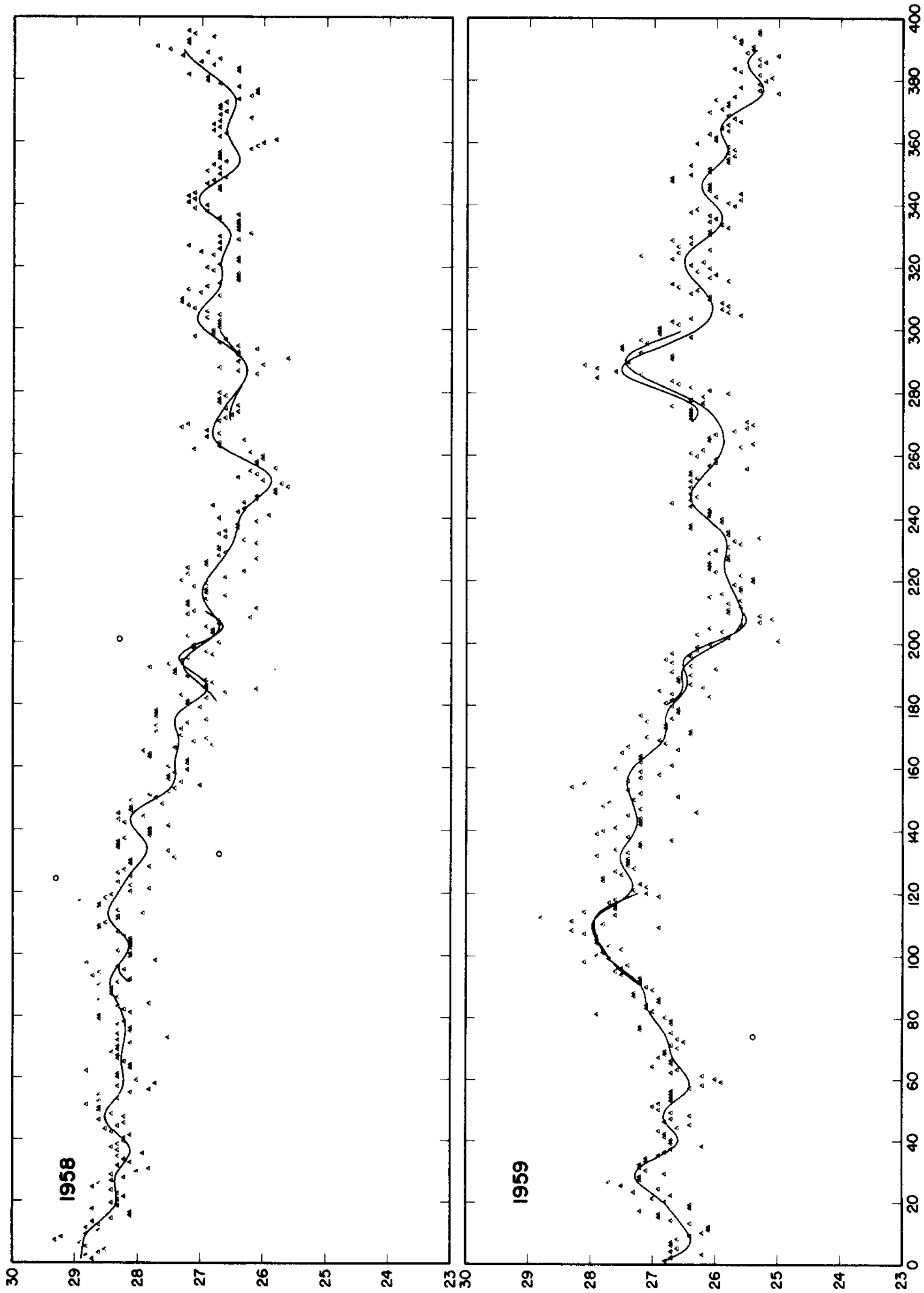
Note: Circled observations have not been used in the harmonic analysis.



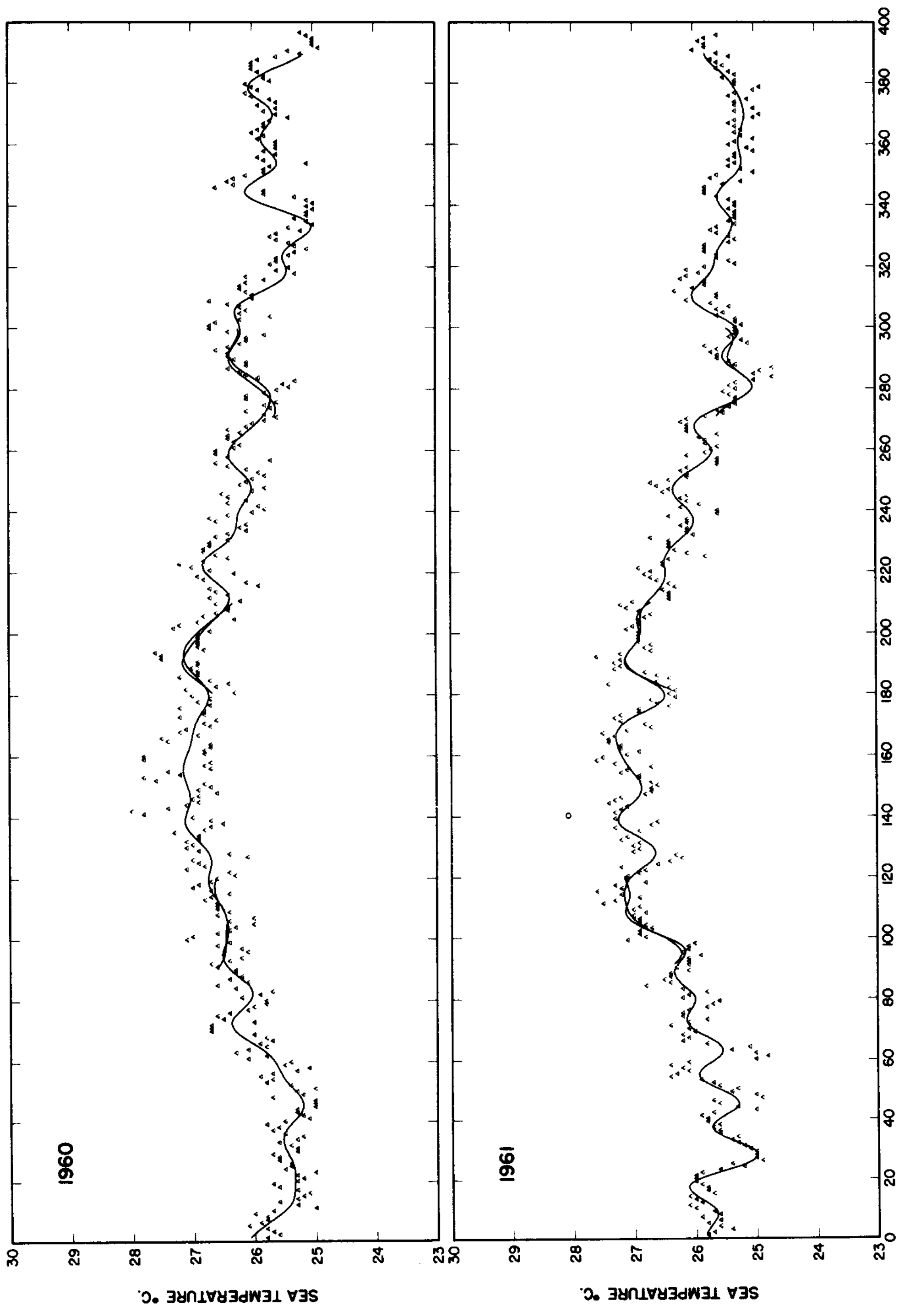
APPENDIX D FIGURE 1.—Sea-surface temperatures, Christmas Island, 1954-69.



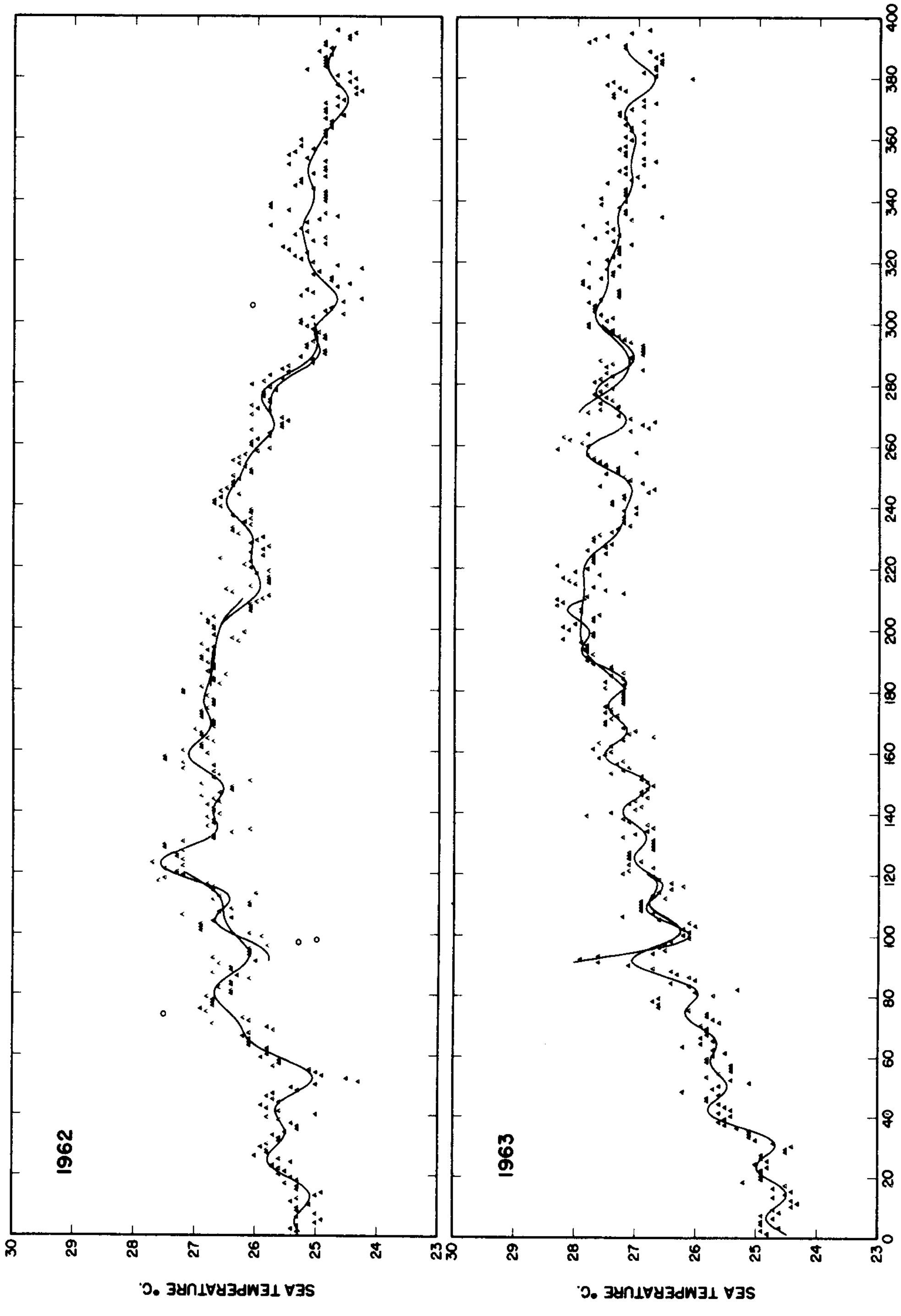
APPENDIX D FIGURE 1.—Sea-surface temperatures, Christmas Island, 1954-69.—*Continued.*



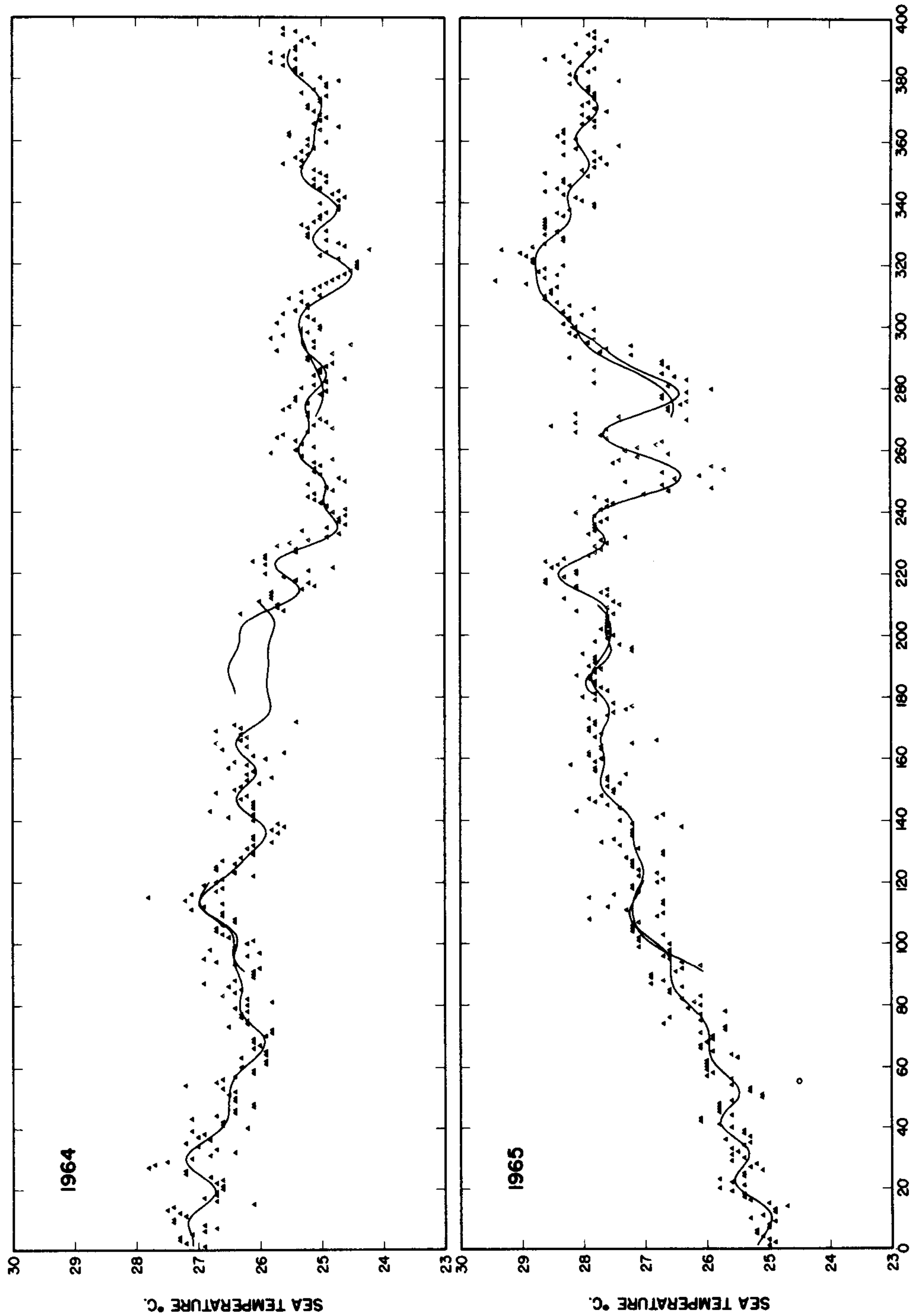
Appendix D Figure 1.—Sea-surface temperatures, Christmas Island, 1954-69.—*Continued.*



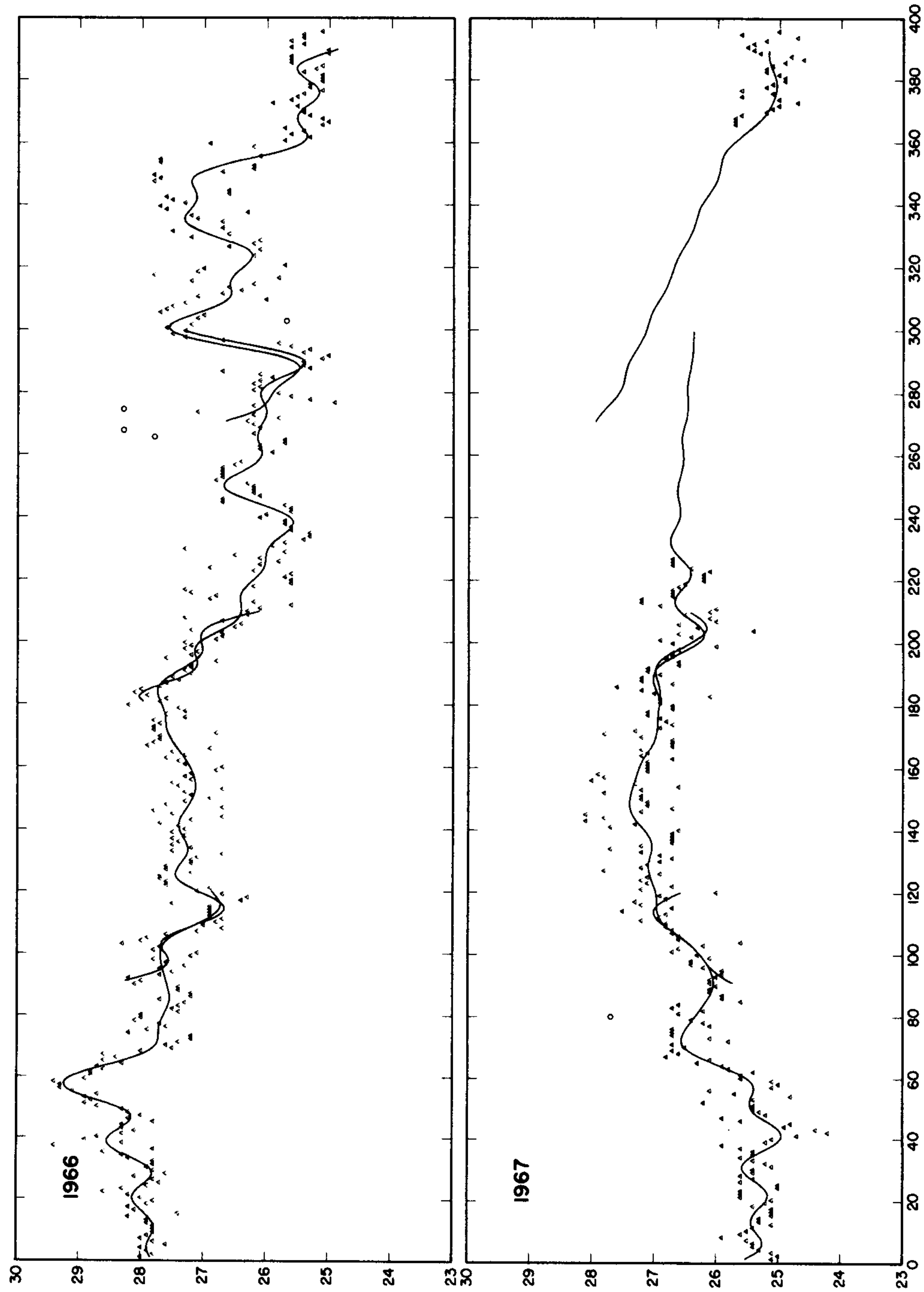
APPENDIX D FIGURE 1.—Sea-surface temperatures, Christmas Island, 1954-69.—*Continued.*



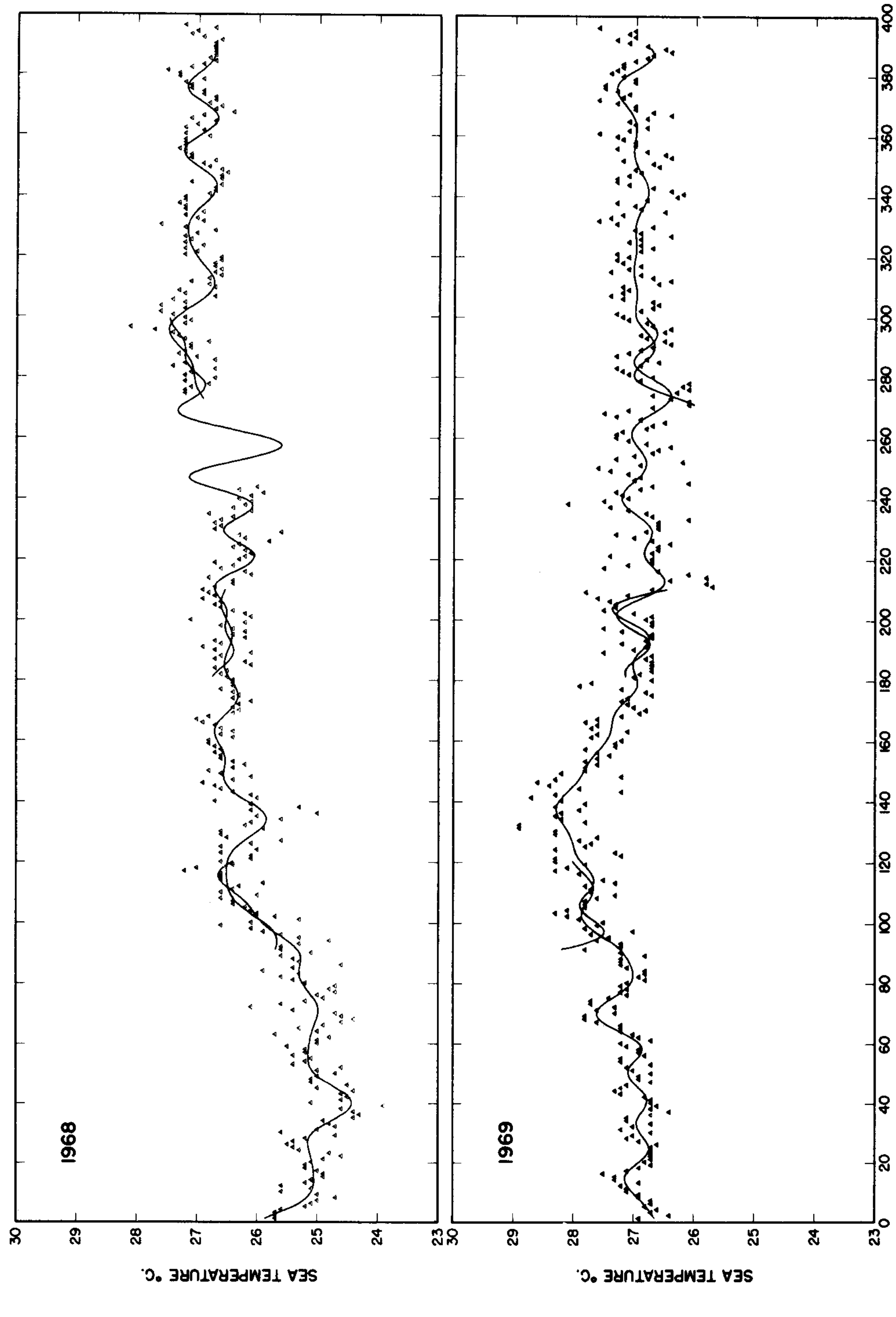
APPENDIX D FIGURE 1.—Sea-surface temperatures, Christmas Island, 1954-69.—*Continued.*



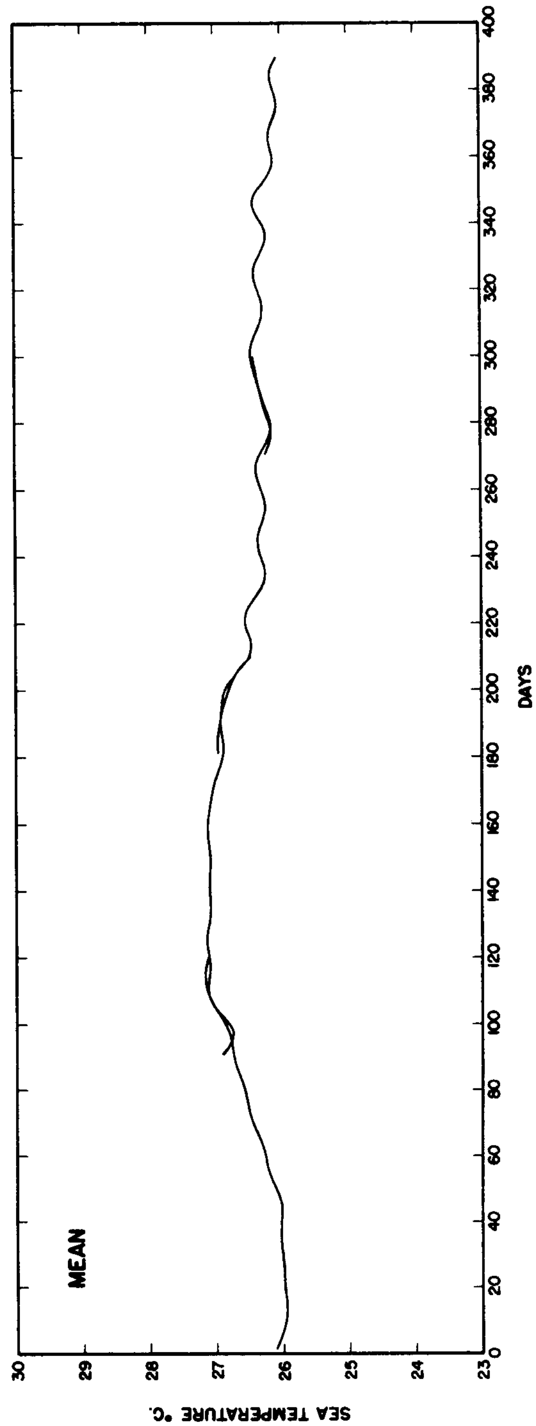
APPENDIX D FIGURE 1.—Sea-surface temperatures, Christmas Island, 1954-69.—*Continued.*



APPENDIX D FIGURE 1.—Sea-surface temperatures, Christmas Island, 1954-69.—*Continued.*



APPENDIX D FIGURE 1.—Sea-surface temperatures, Christmas Island, 1954-69.—*Continued.*



APPENDIX D FIGURE 1.—Sea-surface temperatures, Christmas Island, 1954-69.—*Continued.*