Some Effects of Stock Mixing on Management Decisions

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INTRODUCTION

One of the major concerns of the IWC Scientific Committee on the status and management of sperm whales has been delineating and setting quotas by unit stocks. Examining some aspects of this concern with a simple mathematical model helps to illustrate the need for careful attention to stock delineation and the degree to which two stocks are indeed independent and can be managed as such.

Lord (1971) formulated a general multiple species production model with inter-specific interaction terms and performed some mathematical analysis of the logistic form. However, there has been no attempt to apply the model to an actual fishery. Similarly, Larkin (1963) mathematically examined the competition equations of Lotka and Volterra, the logistic form, for two species in detail. The mixing of multiple stocks, however, is of specific interest to the management of sperm whales. The problem considered here is one of stocks of the same species occupying generally different, though not necessarily distinct, spaces with some degree of mixing among them. For simplicity, I will consider only two stocks and the logistic form of population production.

THE MIXING MODEL FORMULATION

Consider two logistic stocks of the same species which occupy generally different spaces, such that fishing mortality (or fishing effort) can be applied to each stock separately (F_1 and F_2), but between which there is some mixing at intrinsic rates T_1 and T_2. Consider also that these stocks are not different, other than in their origin, such that once a part of stock 1 transfers to the area of stock 2 it is indistinguishable in all aspects from stock 2 (and vice versa). The mixing model is, therefore, the set of equations:

$$\frac{dP_1}{dt} = (K_1P_1 + T_2P_2)\left(\frac{P_{max(1)} - P_1}{P_{max(1)}}\right) - T_1P_1\left(\frac{P_{max(2)} - P_2}{P_{max(2)}}\right) - F_1P_1 \qquad 1(a)$$

$$\frac{dP_2}{dt} = (K_2P_2 + T_1P_1)\left(\frac{P_{max(2)} - P_2}{P_{max(2)}}\right) - T_2P_2\left(\frac{P_{max(1)} - P_1}{P_{max(1)}}\right) - F_2P_2 \qquad 1(b)$$

where 1 and 2 are the designations for stocks 1 and 2.

It can be seen that the model assumes that the rates of mixing are determined by the population size in the area of origin and the amount of space available in the area of transfer – when both populations are at their carrying capacities, P_{max}, (with $F_1 = F_2 = 0$) there is no mixing. Many additional mixing models could be formulated under alternate hypotheses, but I will restrict my analysis in this document to the above equation set.

MIXING MODEL ANALYSIS

Rigorous mathematical analysis of the mixing equations could be presented (cf. Larkin, 1963; Pielou, 1969); however, it will suffice to illustrate simply a few of the implications of the model.

Fig. 1 provides the locus of equilibrium points ($dP_1/dt = 0$, $dP_2/dt = 0$) for equation set (1) – the concave upward curves for equation (1a) and concave downward curves for equation (1b) – with an arbitrary set of parameters. Additionally,

$$P_{max(1)} = 0.5\ P_{max(2)},$$

$$K_1 = K_2 = K$$

and $T_1 = T_2 = T$.

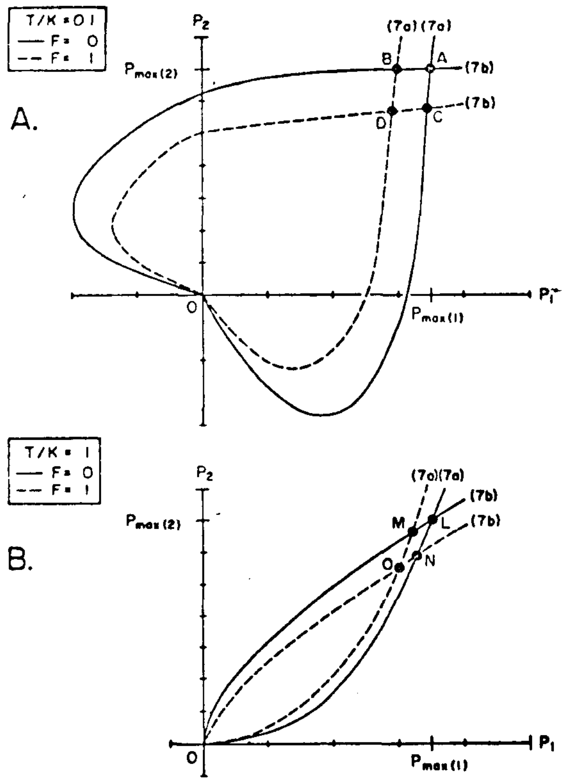

Fig. 1. Locus of equilibrium points for the production model with mixing of two species (equations 1a and 1b). (A) Low relative mixing rate; (B) High relative mixing rate.

Four combinations of fishing mortality at 0 and 1 are plotted for two cases of the mixing coefficient relative to the intrinsic rate of increase coefficient (T/K). Points of stability occur at the intersections of the concave upward and concave downward curves (heavy dots).

Fig. 1A illustrates the relationships when mixing is slow relative to population productivity, $T/K = 0.1$. At $F_1 = F_2 = 0$ both populations achieve their respective P_{max} (point labelled A). Applying fishing mortality to either stock has little affect on the stock not being exploited (point B or C). However, applying fishing mortality simultaneously to both stocks results in slightly lower stock sizes (point D) than exploiting just one or the other (points B and C) due to an increase in export and a loss of supporting import.

Fig. 1B illustrates the relationships when mixing is relatively high. With no fishing mortality, point L is the same as point A (Fig. 1A). An obvious difference between the low and high relative mixing rates is that the curves become more coincident as the relative rate of mixing increases, approaching a common curve as the mixing rate becomes infinite. Furthermore, the degree of synergistic lowering of both stock sizes when both are exploited (point O), as opposed to each being exploited alone (points M and N), is much greater than when the mixing rate is low (Fig. 1A).

MIXING AND THE YIELD RELATIONSHIP

The major question is: what impact does the mixing of stocks have on the shape of the yield curve? Fig. 2 illustrates the combined equilibrium yield and combined fishing effort curves (assuming catchability = 1) for four rates of mixing and three ratios of effort applications to the two

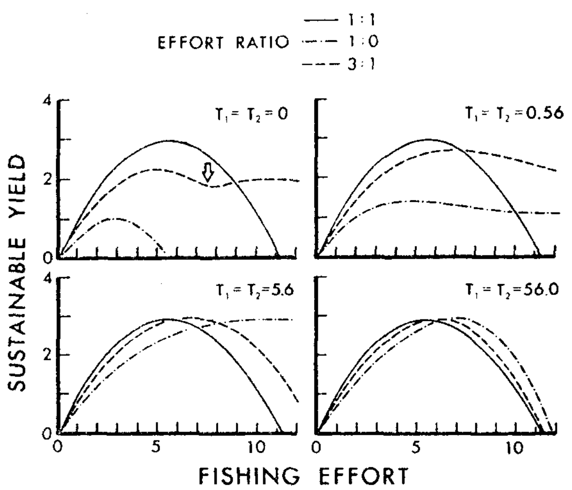

Fig. 2. Total equilibrium yield-total fishing effort curves for the production model with mixing of two species (equations 1a and 1b) at four rates of mixing.

stocks. All parameters are the same as in the previous section. On the vertical axes is the combined equilibrium yield from the two stocks ($Y_1 + Y_2$) and the combined fishing effort ($F_1 + F_2$) is on the horizontal axes. The dash-dot lines represent all the effort being exerted on P_1 alone, the dashed lines represent 75% of the total effort being distributed on P_1 and 25% being distributed on P_2, and the heavy lines represent an equal amount of effort being distributed on both P_1 and P_2.

When there is no mixing between the stocks, $T_1 = T_2 = 0$ (Fig. 2, upper left panel), obviously the total maximum sustainable yield (TMSY) increases to the sum of the MSYs of the two stocks when fished separately. Because of the nature of the parameters selected, the MSY of each stock occurs at the same F, but the MSY of P_2 is twice as large as that of P_1. Any other effort ratio produces a lower TMSY.

When the mixing rate is very large, $T_1 = T_2 = 56$ (Fig. 2, lower right panel), the model behaves as if it were one large stock, $P_1 + P_2$, as expected. The total sustainable yield is nearly the same regardless of the effort ratio.

In the case where mixing is intermediate between the two extremes, $T_1 = T_2 = 0.56$ (Fig. 2, upper right panel) and $T_1 = T_2 = 5.6$ (Fig. 2, lower left panel) some very interesting relationships are implied from which several important scenarios can be derived. Depending on the effort ratio: (1) yield curves can be very flat-topped or descend rapidly with overfishing, (2) TMSY can vary widely at about the same level of total effort, and (3) the TMSY can be the same but at widely varying values of total effort.

Let us consider two scenarios where effort develops first on P_1 and then transfers in part to P_2.

Scenario 1

When the mixing rate is relatively low (Fig. 2, upper right panel) a fishery developing on P_1 would follow the dash-dot curve out to a maximum of 1.5 units of yield at 4–5 units of fishing effort. If 1 to 1.25 units of the 4–5 effort units were shifted to P_2, the equilibrium yield would increase markedly, and if an additional 1 to 1.25 units were transferred to P_2 the TMSY would be double that originally observed and for exactly the same amount of total effort. However, since the equilibrium catch rates differ between P_1 and P_2 at the same fishing effort (that of P_1 is one-half that of P_2) an unequal effort ratio would be tended towards.

When the mixing rate is moderately high (Fig. 2, lower left panel), the same manner of fishery development would have the opposite consequences. A fishery developing on P_1 would follow the dash-dot curve out to an TMSY of 3 units at 10 units of fishing effort. Any redistribution of fishing effort to P_2 would cause the total equilibrium yield to plummet.

Scenario 2

Development of a fishery on P_2 may occur through the addition of effort rather than through its re-distribution. When the mixing rate is relatively low (Fig. 2, upper right panel) development of a fishery on P_1 would follow the dash-dot curve. Additional effort being placed on P_2 could initially cause the overall fishery to begin following the dashed curve with increased equilibrium yield. Trouble would arise if total effort exceeds 8 units and greater effort were added to P_2 such that the effort ratio were even – total equilibrium yield would decrease. With a higher mixing rate (Fig. 2, lower left panel) the same trouble could occur, but it would occur at lesser effort being placed on P_2 than for the lower mixing rate situation.

SUMMARY

The formulation of the mixing model has considerable impact on the yield relationship. The model formulation presented here, along with the assumed parameter values, indicates that knowledge of the mixing relationship can be critical to good management, which can be highly influenced by the distribution of fishing effort (hence quota levels) among stocks. It remains to be seen, however, how well such a model can describe an actual fishery situation.

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