Discount Factors and Risk Aversion in Managing Random Fish Populations

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MENDELSSOHN, R. 1982. Discount factors and risk aversion in managing random fish populations. Can. J. Fish. Aquat. Sci. 39: 1252–1257.

The effects of changes in the discount factor as compared with the effects of changes in risk aversion are examined for a simple stochastic model of fish population dynamics. Numerical results suggest that both optimal harvesting strategies and the resulting population dynamics are insensitive to changes in the discount factor, whereas changes in the degree of risk aversion in the utility function do cause significant changes in both. When risk averse utility functions are viewed as total revenue curves with marginal prices that decrease with supply, the results suggest that the more sensitive price is to supply, then the resulting optimal harvesting policy is smoother. Theoretical results are presented which suggest these results are robust beyond the specific model.

Key words: discount factors, risk aversion, population dynamics, randomness

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Nous analysons, sur un modèle stochastique simple de dynamique des populations, les effets de changements du facteur escompte comparativement aux effets de changements du facteur aversion contre le risque. Les résultats donnent à penser que les stratégies de récolte optimale, ainsi que la dynamique des populations qui en résulte, ne sont pas influencées par les changements du facteur escompte. Par contre, le degré d'aversion contre le risque dans la fonction utilité entraîne en fait des changements significatifs dans les deux. Quand on considère les fonctions utilité opposées au risque en termes de courbes de revenus totaux, avec prix marginaux qui diminuent en même temps que l'offre, les résultats suggèrent que plus le prix est sensible à l'offre et plus douce est la politique de récolte optimale qui en découle. Nous présentons des résultats théoriques vraisemblablement applicables au-delà de ce modèle particulier.

Received January 19, 1981
Accepted May 28, 1982

Reçu le 19 janvier 1981
Accepté le 28 mai 1982

THE standard literature (see Clark 1976, 1980; Anderson 1977; and references therein) on dynamic, deterministic fishery optimization models has stressed the importance of the discount factor ($1/(1 + i)$, where i is the interest rate) in determining optimal harvesting strategies. The discount factor has been emphasized because an optimal harvesting strategy depends continuously on the discount factor, and a stock will be optimally harvested to extinction if the rate of growth of the population is less than the rate of discount (Reed 1974; Clark 1976). Related results have been established for stochastic harvesting models (Reed 1974; Mendelssohn 1980b; Mendelssohn and Sobel 1980), though in a stochastic model the population dynamics may drive the population to extinction even though the harvest levels have not left the remaining population at or near zero.

The presumed sensitivity of an optimal harvesting strategy to changes in the discount factor is important, as a standard procedure for representing increased risk to a manager of a fishery in face of uncertainty is to decrease the discount factor. Even at first glance this is a naive approach, as it assumes a constant attitude towards risk regardless of the initial population size or harvest decision.

What would appear to be a more sophisticated approach towards risk is to assess the manager's utility function and use this as the objective function. This approach to managing fisheries and other natural resources is given in Hilborn and Walters (1977), Keeney (1977), and Holling (1978); a general discussion can be found in Keeney and Raiffa (1976).

One way to compare these differing approaches is to see how changes in the discount factor and changes in the degree of risk aversion affect the policy that should be followed. In this paper this is done for one empirical stochastic model of a fishery. In particular, the actual sensitivity of an optimal harvesting strategy to changes in the discount factor is compared with its sensitivity to changes in the degree of risk aversion in the utility function. Further changes in the population dynamics of a stock being managed under an optimal

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Printed in Canada (J6360)
Imprimé au Canada (J6360)

harvesting policy are examined as both the discount factor and the degree of risk aversion vary. Also, the appendix presents theoretical results which suggest the numerical results presented are robust over a wide class of models.

The Model

The model to be analyzed is a stochastic version of a Ricker spawner–recruit curve put forward by Mathews (1967) for salmon runs in Bristol Bay. Let x_t be the recruits in period t, and y_t the spawners. If x is observed and a decision y is taken, a one-period utility $g(x, y)$ is received. Without loss of generality, $g(x, y)$ can be the expected value (over the random variable θ) of some other utility $h(x, y, \theta)$. Hence the model indirectly includes economic uncertainty.

The utility in period t is discounted by a factor β, $0 \leq \beta < 1$. The recruits in period $t + 1$ are a random function of the spawners in period t and a random variable d,

(1) $x_{t+1} = d4.084y_t \exp\{-0.8y_t\}$; $\ln d \sim N(0, 0.2098)$

where $N(0, \sigma^2)$ denotes a random variable that follows a normal distribution with mean zero and variance σ^2.

The problem is to maximize the expected discounted utility.

(2) $\text{maximize} \sum_{t=1}^{\infty} \beta^{t-1} g(x_t, y_t)$

such that $x_t \geq y_t \geq 0$; Eq. (1).

Computational techniques for solving Eq. (2) and related policy questions are discussed in Mendelssohn (1980b). A 100-point grid is used with no absorbing (zero) population size.

Two different utility functions are examined for $g(x, y)$. The first is $v_1 = 0.5 \ln(x_t - y_t)$ while the second is $v_2 = (x_t - y_t)^\lambda$, for $\lambda = 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 1.0$. These utility functions are plotted in Fig. 1. Utility functions are said to be absolute risk averse (Merton 1969, 1971; Keeney and Raiffa 1976) if $-u''(z)/u'(z) > 0$ for a function $u(z)$. The Pratt measure of relative risk aversion is $-u''(z)z/u'(z)$. Basically, a utility function is risk averse if a certain return is preferred to a lottery with equal or greater value in expectation.

The two utility functions v_1 and v_2 are members of the HARA (hyperbolic absolute risk averse) family of utility functions (Merton 1969, 1971). In particular, if R_1 is the measure of absolute risk aversion, and R_2 is the measure of relative risk aversion, then for v_1, $R_1 = 1/z$ and $R_2 = 1$; for v_2, $R_1 = (1 - \lambda)/z$ and $R_2 = 1 - \lambda$, where $z = x - y$ is the amount harvested. Both v_1 and v_2 have constant relative risk with respect to the amount harvested. However, it is clear that the relative risk aversion for v_2 changes linearly with λ. Hence λ is a measure of the relative degree of risk aversion. For λ approaching zero, the manager is totally risk averse, whereas for $\lambda = 1$ the manager is risk neutral. The utility function v_1 is the limiting case of v_2 as λ approaches zero.

The utility functions v_1 and v_2 are not as rich as they might be given that they only depend on (x_t, y_t) through $z_t = x_t - y_t$. However, there exists little empirical basis for choosing more complicated utility functions that can be given meaningful, practical interpretations. Both v_1 and v_2 can be viewed as total revenue curves. Taking derivatives $v_{1'} = 0.5/z$ ($v_{2'} = \lambda z^{\lambda-1}$) and $v_{1''} = -0.5/z^2$ (respectively, $(1 - \lambda)z^{\lambda-2}$). The first derivative is the marginal price for a supply of z units of fish, with the marginal price decreasing with increasing supply. Therefore, using v_1 and v_2 as the one-period utility functions can be viewed as examining the effect of dropping the usual economic assumption of perfect competition. This is treated analytically in some detail in Mendelssohn and Sobel (1980).

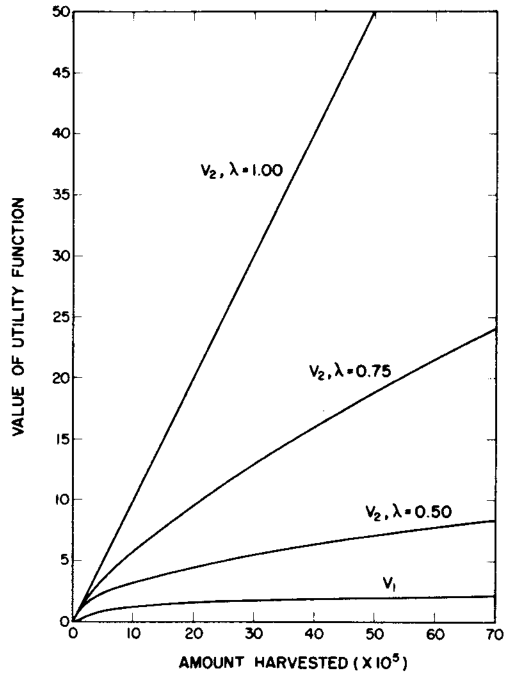

FIG. 1. Graph of the different utilities function.

It is worth emphasizing that "risk" is used here in a very specific way. In some papers, risk is used to denote the randomness in the population dynamics. Here, it is assumed that all random variation in the population is contained in the random variable d. No effort is made to discuss the actual merits or accuracy of Mathew's model. Instead the model is taken as given.

Also, sometimes risk is used to denote undesirable events that could happen to the population that are not directly reflected in the utility function. The risk of extinction or of very small population sizes is one such example. This problem is treated in Mendelssohn (1978).

Risk in this paper denotes the gamble the decision maker faces in the certain reward of the given return function (in this case harvest) this period versus the uncertain return next period. The return function is assumed given. For many problems, the choice of the appropriate return function is crucial (Mendelssohn 1980b).

Results

Optimal harvesting policies and the resulting population

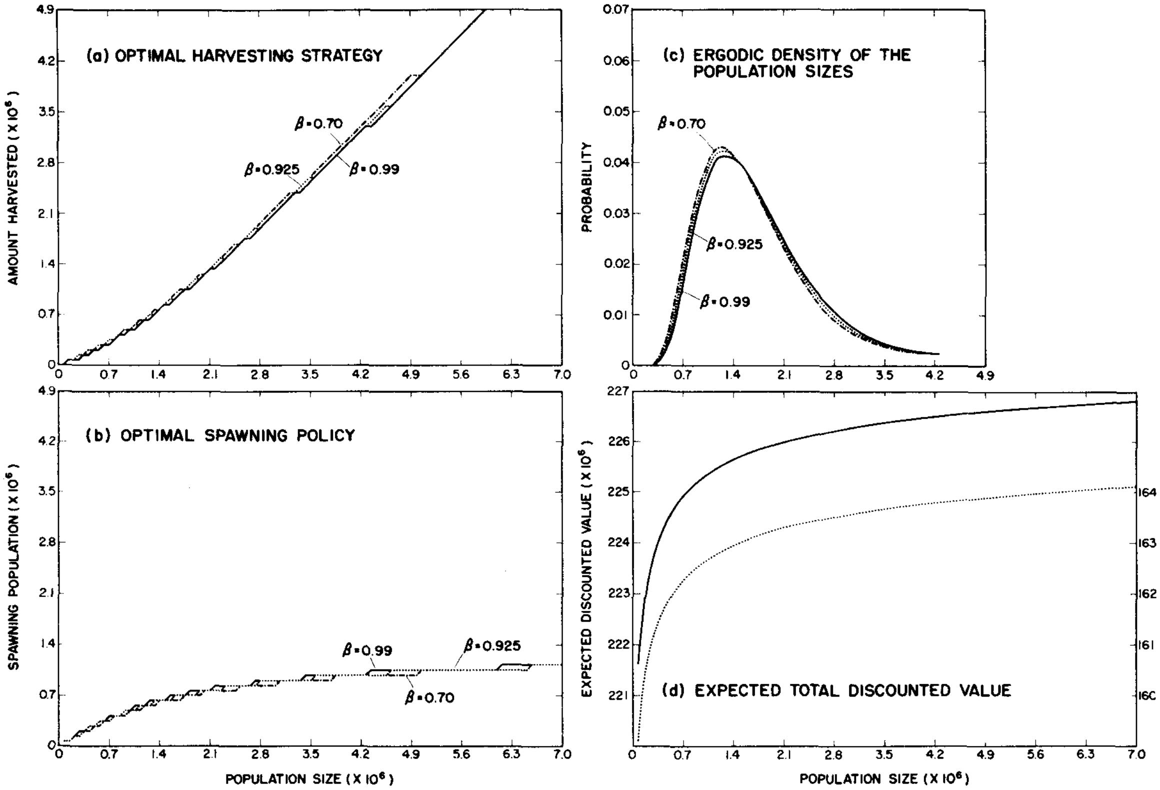

FIG. 2. Effects on an optimal policy and optimal value from varying the discount factor.

dynamics were calculated for both classes of utility functions for discount factors of 0.99, 0.95, 0.925, 0.9, 0.85, 0.80, and 0.70. These discount factors are equivalent to interest rates of 1, 5.23, 8.1, 11.1, 17.65, 25, and 42.86%. Results are shown only for the utility function $v_1 = 0.5 \ln(x - y)$ and several of the extreme values of the discount factor. The results using v_2 with any value of λ are nearly identical to those presented for v_1, and the graphs change uniformly with β, so the graphs in Fig. 2 represent the extreme changes.

Visually it is clear that the curves in Fig. 2 are extremely similar. For the discount factors $\beta = 0.99$ and $\beta = 0.70$ the optimal number of spawners to leave after harvesting agree at 50 out of 100 states and differ by only one grid point at the rest of the states. Moreover, it can be shown that this result remains valid for much finer grids; hence the two policies can be made to be extremely close together.

The long-run (ergodic) population sizes show a slight shift to smaller population sizes as β decreases, but the cumulative ergodic harvest distributions are virtually indistinguishable over the range of discount factors examined.

Figure 2(d) shows an optimal value function, $\sum_{t=1}^{t} \beta^{t-1} v_1(t)$ when starting in each state, for $\beta = 0.99$ and $\beta = 0.925$. Despite the fact that almost identical sample paths are generated by the two optimal harvesting strategies, the change in the valuation of the sample paths is dramatic.

There has been much discussion in the fisheries literature on the appropriate value of the discount factor (Walters 1975; Clark 1976; Walters and Hilborn 1976; Holling 1978). This discussion can be divided into two components—the discount factor that should be used to evaluate the fishery by an external agency such as a government agency (say to compare the benefits derived from a stream enhancement project) or a private individual (when making an entry decision, for example) and that used by the fishery manager to set policy, bearing in mind the effects harvest policy will have on the former decisions. If the previous empirical results are robust, they suggest that the fishery manager need not worry greatly about the exact choice of the discount factor used for determining policy, and that this choice will have little bearing on entry decisions or enhancement projects. This is because a nearly optimal expected present value will be found over a broad range of discount factors that might be used by the external agents.

Figure 3(a–e) shows similar results for the v_2 with $\beta = 0.95$ and $\lambda = 1.00$ and $\lambda = 0.05$. Runs were performed for $\lambda = 1, 0.95, 0.9, 0.8, 0.5, 0.25$, and 0.05 and for different values of β. The results were not sensitive to the particular value of β used. The results changed uniformly with λ; hence the curves for $\lambda = 1.00$ and $\lambda = 0.05$ represent the extreme curves found.

For $\lambda = 0.05$, more is harvested at lower population sizes, while less is harvested at larger population sizes.

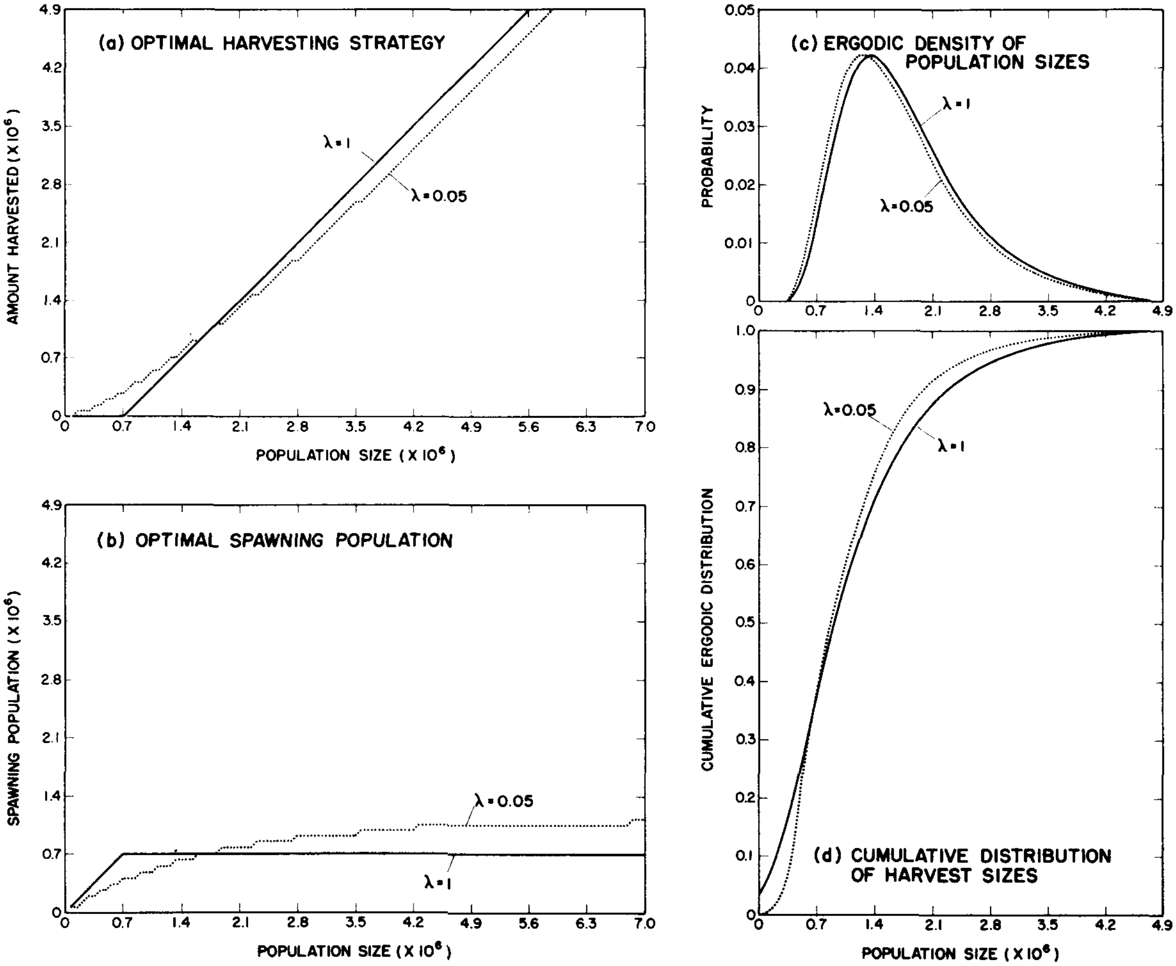

FIG. 3. Effects on an optimal policy and optimal value from varying the degree of risk aversion.

The crossover point is at a population size of 1.89×10^6, which is where (letting $\bar{d}$ denote the mean of d) the equation $\bar{d}4.082y \exp\{-0.8y\}$ has a value equal to $y = 1.89 \times 10^6$. Thus at the crossover point an individual just replaces itself in expectation.

Unlike the case for changes in β, the cumulative harvest distributions when λ varies differ by as much as 8%, and at the most probable harvest sizes, by about 5–6%. These differences are small but significant; the smaller values of λ can be seen to produce a "smoother" harvest in the long run.

These results can best be explained by considering the economic interpretation of the utility functions. At $\lambda = 1$, the price does not vary with supply (in fact it is $p = 1$, which can be viewed as the normalized price); therefore a harvest policy is based entirely on being as close as possible to the point of largest overall expected growth. However, at $\lambda = 0.05$, at low populations, the higher price (i.e. the higher risk) makes it desirable to harvest some now (i.e. the risk makes it desirable to obtain some amount of certain return now). At higher population levels, a balance must be maintained between a desirable price and future growth. For $y \geq 1.89 \times 10^6$, the expected value of x_{t+1} is less than y_t. Harvesting more now lowers the present marginal price per unit of fish, but increases the probability (i.e. decreases the risk) of larger population sizes next period (i.e. of small, unprofitable harvests next period).

The problem of unsmooth harvests from random populations is discussed in Mendelssohn (1976, 1980a, 1980b), Beddington and May (1977), and May et al. (1978). These empirical results suggest that unsmooth harvests have been found in the past because no effort has been made to include in management policies either attitudes towards risk, or marginal prices that are sensitive to supply. A reasonable conjecture is that all other things being equal, for a properly managed random fishery, wider fluctuations in catch will be expected either if outside supplies affect prices or if the entire quota cannot be taken by the fleet. Conversely, smaller fluctuations will be expected when the marginal price is very sensitive to the supply of the managed stock of fish only. Inventories can be viewed as an outside supply that affects

TABLE 1. Changes in relative risk aversion with changes in planning horizon length when following an optimal policy.

n	$\lambda = 1$	$\lambda = 0.5$	$\lambda = 0.05$
2	0.1430	0.0518	0.045
3	0.1684	0.303	0.423
4	0.1684	0.325	0.423
5	0.1684	0.331	0.423
6	0.1684	0.331	0.423
7	0.1684	0.331	0.423
8	0.1684	0.331	0.423
9	0.1684	0.331	0.423
10	0.1684	0.331	0.423

price, and they should have a very destablilizing effect if not taken into account in setting management policy for biologically random fisheries. This is precisely what has been seen in the salmon fishery in Alaska.

The solution of Eq. (2) offers some insights into relative risk aversion and planning horizons when optimal policies are followed. This is because Eq. (2) can be solved by solving the following sequence of recursive equations:

$$f_n(x) = \max_{0 \le y \le x} \; g(x, y) + \beta E f_{n-1}(s[y, d])$$

where n denotes the number of periods remaining in the planning horizon. For Eq. (2), each $f_n(\bullet)$ is concave and continuous (Mendelssohn and Sobel 1980), and hence each is a risk averse utility. Each $f_n(x)$ is the utility of having x units of fish when there are n periods left when following *an optimal policy*. Hence $f_n'(x)$ is the marginal value of one additional unit of fish with n periods remaining, and the two risk averse measures R_1 and R_2 can be calculated as n changes (derivatives are approximated by finite differences) (Table 1). The measure R_2 reflects the relative risk aversion faced by the decision maker given the actual sample paths that will occur when following an optimal policy. What is noticeable is that the relative risk for f_n is much less than that for v_2, that it increases with n, and that it quickly becomes stationary after three to five periods. An optimal harvesting policy reduces the relative risk of the decision maker. Moreover, the effective planning horizon for these problems appears to be 3–5 yr, after which both policy, risk, and preferences remain stationary.

The measure of relative risk R_2 can be seen to be almost a measure of elasticity of the present marginal value of a unit of fish. As an optimal policy "smooths out" the supply of fish, the elasticity of the value per unit of fish decreases.

Conclusions

Any conclusions to be drawn from these results must be tempered by the realization that the results are derived from only limited numerical experimentation. However, the appendix presents theoretical results which suggest that over a broad class of stochastic harvesting models large changes in the discount factor produce only very small changes in an optimal harvesting policy. Hence, there is reason to believe the numerical results are in fact robust.

The numerical results suggest that policy often is insensitive to changes in the discount factor over a range of values likely to be found in practice. Discount factors may adequately represent intertemporal preferences for a dollar now versus an expected dollar in the future, but they do not seem to capture the attitude towards the degree of gamble in obtaining that expected dollar in the future.

Moreover, when the utility functions are viewed as total revenue curves with marginal price sensitive to supply, the results suggest that for optimally managed fisheries highly supply-sensitive prices will lead to stabilized harvests, whether the fishery is economically or biologically stochastic. As most if not all fisheries are stochastic, the usual assumption of perfect competition in fishery economic models and the lack of attention to risk preferences limit the usefulness of the results derived from these models.

External decisions for a fishery, such as entry decisions and enhancement projects, depend on the valuation of the fishery. The valuation of the fishery depends on the utility function used, the discount factor used, and the sample paths of the random harvest stream. Usually, the fishery manager can only control this last aspect through harvesting policy. The results of this paper suggest that under optimal management, misspecification of the discount factor in determining the optimal harvesting policy will have little effect on the external decisions of other parties.

Finally, as the degree of risk aversion increases, the amount harvested is not strictly nondecreasing for all population sizes. This seemingly is in contradiction with the results of Cropper (1976). The numerical results yield an intuitive explanation of why Cropper's result is not correct for this problem.

Acknowledgments

I am grateful to Laurie Tsuji who performed most of the computations used in this paper. This paper has benefited from several careful readings by Suzanne Holt. The work was completed while I was a Visiting Research Associate at the Center for Coastal Marine Studies, University of California at Santa Cruz. I am grateful to the CCMS for its support during this period.

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Appendix

This appendix presents a rationale, but not a total proof, of why in a general setting policy, will change only slightly with changes in the discount factor. The notation and discussion follow Mendelssohn and Sobel (1980). The dynamic programming formulation of the optimal harvesting problem is

$$f(x) = \max_{y \in Y(x)} \{G(x, A(x)) + \beta E f(s[A(x), d])\}$$

For convenience, assume that an optimum always occurs at an interior point, which is true for one-period reward functions like $\ln (x - y)$. Then the optimality conditions are

$$G^{[2]}(x, A(x)) + \beta E f'(s[A(x), d])s^{[1]}[A(x), d] = 0$$

By substitution, this yields

$$G^{[2]}(x, A(x)) + \beta E G^{[1]}(s[A(x), d], A(s[A(x), d]))s^{[1]}(A(x), d) = 0$$

For the specific case of $G(x, y) = (x - y)^{\lambda}$, this reduces to

$$\lambda(x - A(x))^{\lambda - 1} + \beta E\{\lambda(s[A(x), d] - A(s[A(x), d]))^{\lambda - 1}s^{[1]}[A(x), d]\} = 0$$

or that

$$E\{\lambda(s[A(x), d] - A(s[A(x), d]))^{\lambda - 1}s^{[1]}[A(x), d]\} = \frac{\lambda(x - A(x))^{\lambda - 1}}{\beta}$$

Suppose the discount factor is perturbed to $\beta + \delta$, such that $0 \leq \beta + \delta < 1$. Then at the value $A(x)$, the derivative has changed value by an amount

$$\frac{\delta\lambda(x - A(x))^{\lambda - 1}}{\beta}$$

Let $\gamma = |\lambda - 1|$, then this is equivalent to

$$\frac{\delta\lambda}{\beta(x - A(x))^{\gamma}}$$

For example, for $\beta = 0.9$, $\lambda = 0.5$, and $\delta = -0.2$ (so that the new discount factor is 0.7), the resulting term is

$$\frac{-0.0111}{(x - A(x))^{1/2}}$$

so that the derivative is still almost zero at $A(x)$. Moreover, $s[\bullet, d]$ is concave, and an optimum is most often at values of y where $s^{[1]}[\bullet, d]$ changes rapidly. Hence, it requires very small changes in an optimal policy to account for even relatively large changes in the discount factor.