# METHODOLOGICAL PROBLEMS IN SAMPLING COMMERCIAL ROCKFISH LANDINGS 

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#### Abstract

The present sample survey plan, for the estimation of age and species composition of California rockfish landings, which is stratified two-stage with port-month group as a stratum, poses serious operational problems in data collection. A revised plan is suggested which is workable. Formulas have been developed for estimating total catch and its error by species-sex-age groups; optimum sampling and subsampling fractions have been obtained for a given cost function and the precision of the estimator is compared with two other estimators. The method developed has been extended to cover situations other than rockfish.

The paper also deals with double-sampling for specified cost for the estimation of age composition of a species, which is important to predict the status of a stock in future years, the inherent problems in data collection in commercial fisheries, and the measurement errors involved in the survey.


Estimates of the total catch (in terms of number) by species-sex-age and by area of landing and during a given time for commercial rockfish caught in California north of point Arguello are currently based on a probability sample of landings. The commercially important species of rockfish taken by California's fishery with mixed species are widow rockfish, Sebastes entomelas; bocaccio, Sebastes paucispinis; and chilipepper, Sebastes goodei.

A study was undertaken during 1983 under agreement between the present author, the Humboldt State University Foundation, and the Tiburon Laboratory of the National Marine Fisheries Service, NOAA, to determine if the present sampling plan for the estimation of species and age-composition of California rockfish landings is workable. The study revealed that the current plan is not operationally feasible. A revised plan is proposed which is workable and would provide efficient estimates of the parameters based on existing catch data within the usual limitations of budget and personnel and under the assumptions made in the plan. Formulas have been developed for the ratio estimators of mean and total catch and their errors. Optimum sampling and subsampling fractions have been obtained for a given cost function and the precision of the estimator is compared with two other estimators.
For most theoretical population work and for management purposes, the knowledge of the age

[^0]composition is important to predict the status of the stock in future years. Fridricksson (1934) developed the age-length key method for determing age composition from a large number of length measurements. Fridricksson's approach was improved by Ketchen (1950) who provided more accurate results for age groups at the extremities of the distribution. Kutkuhn (1963) mentioned the limitations of the agelength key approach except in situations where price differentials may demand sorting of landings by size criterion. Westrheim and Ricker (1978) pointed out that the age-length key approach will almost always give biased estimates. Clark (1981) and more recently Bartoo and Parker (1983) dealt with methods for control or elimination of bias. Following the method of Tanaka (1953) in which stratification occurs after subsampling for age, Kutkuhn (1963) estimated absolute age composition of California salmon landings by port-month groups. He showed that the sampling procedure is not effective unless the age sample is at least five times costlier than the length sample.

Mackett (1963) found double sampling more efficient than simple random sampling with fixed sampling costs for estimating relative age composition of Pacific albacore landings.

Southward (1976) found that a sample of otoliths proportional to the length frequency of sampled fish from each port was preferable to fixed sample size procedure for estimating age composition of Pacific halibut. Kimura (1977) arrived at the same conclusion as Southward by following a somewhat different approach.

We will present some of the important considera-
tions in sampling for estimating age composition of rockfish landings based on recent widow rockfish data from the California coast. Finally, we will describe some of the measurement errors, which would normally occur in simple random sample of individual fish and which are taken care of in cluster sampling adopted in our approach.
The sampling plan arrived at may produce usable results under the assumptions stated, though some of the assumptions have been under attack during recent years.

## DESIGN OF THE SURVEY

Rockfish are being landed at 14 points on the California coast. Of these, three cater only to commercial fishing, four to sport fishing, and seven to both sport and commercial fishing. The 10 commercial ports are grouped into 6 port groups with a sampler (six in all) assigned to each of the 6 portsEureka, Fort Bragg, Bodega Bay, San Francisco, Monterey, and Morro Bay.
The commercial trawlers make trips varying in length from 1 to 8 d . These vessels maintain log books to keep records of area fished and appropriate catch for each tow. Sampling by tow is generally not feasible because it is not possible for the sampler to be on board during haul time. For the same reasons no estimates of fish being rejected and returned to the sea are obtained because this would involve collection of discarded fish from randomly selected tows within sampled trips.

## Selection Procedure

A two-stage stratified random sampling plan was adopted with port-month group as a stratum and boat trips within a stratum as first-stage sampling units. Fish are sorted at sea into market categories. The first stage sampling units are poststratified into categories and at least one cluster of a given weight is subsampled within each sort-type from a firststage sampling unit. Categories are based upon species composition, size, and quality, but in other contexts they could be strictly size or species categories. Cluster (box) of 25 lb is taken when sampling small fish, or any time small rockíish are landed such that there would be more than 20 fish in the $50-\mathrm{lb}$ cluster. In all other cases $50-\mathrm{lb}$ standard cluster size is selected. A cluster is next separated by number of each species and its weight, which are recorded along with sex, total length, and otolith of each member of a species in the cluster.
The instructions are to "sample all market
categories (sorts) from a boat, and from as many boats as possible and select:
"(i) 1 cluster per $20,000 \mathrm{lb}$ of widow rockfish landed by each boat, up to 4 clusters,
"(ii) 1 cluster for all other species, if less than $5,000 \mathrm{lb}$ landed, and
"(iii) 2 clusters for all species if more than 5,000 lb are landed.
"The second cluster should not be taken if this precludes sampling another boat."

## Estimation with Poststratification of Sample Trips by Categories

Consider the problem of estimation of total catch of a given species for a port-month stratum. Equations for estimation of other characteristics for fisheries with mixed species are straightforward and can be obtained by substituting the value of the characteristic for the catch of the species. Totals across strata are formed by simple additon.

## Notation

For a given species, let
$N=$ total number of trips,
$n=$ number of randomly sampled trips,
$W=$ total weight of fish caught from all trips,
$W_{i}=$ weight of fish caught on trip $i$,
$W_{i j}=$ weight of fish for sort $j$ caught in trip $i$,
$m_{i j}=$ number of clusters sampled from sort $j$ on trip $i$,
$m_{i}=$ number of clusters sampled on trip $i$,
$m=$ number of clusters sampled over $n$ trips,
$W_{i}=\sum_{j}^{L_{i}} W_{i j}$ where $L_{i}$ is the number of sorts in trip $i$,
$y_{i j k}=$ number of fish of the species in cluster $k$ from sort $j$ of trip $i$,
$Y_{i j}=$ total number of the species caught from sort $j$ of trip $i$,
$Y=$ total number of species caught from all trips,
$\overline{\bar{Y}}=$ mean catch per cluster for the species,
$\bar{y}_{i j}=\sum_{k} y_{i j k} / m_{i j}=$ unbiased estimate of $\bar{Y}_{i j}$,
$w_{i j k}=$ weight of the $k$ th cluster from the $j$ th sort of the $i$ th trip,
$\hat{M}_{i}=\frac{W_{i}}{\bar{w}_{i}^{-}}$where $\bar{w}_{i}=\sum_{j} \sum_{k} w_{i j k} / \sum_{j} m_{i j}=$
average weight of sampled clusters in the $i$ th trip.

If $\bar{W}_{i}$ is a constant, its estimate $\bar{w}$ will be given by $\bar{w}=\sum_{i} \sum_{j} \sum_{k} w_{i j k} / \sum_{i} \sum_{j} m_{i j .}$. In practice, $N$ and $M_{i}$ will not be known and will be estimated by $\hat{N}=$ $\frac{n W}{\sum W_{i}}=W / \hat{\bar{W}} ; \hat{M}_{i}=\frac{W_{i}}{\bar{w}}$ respectively, if $\bar{W}_{i}$ is a constant $=\bar{w}$ (say).

## Ratio Estimates of Mean and Total

The ratio estimate of mean catch $(\overline{\bar{Y}})$ per cluster is

$$
\begin{equation*}
\hat{\overline{\bar{Y}}}_{R}=\frac{\sum_{i}^{n} M_{i} \bar{y}_{i}}{\sum_{i}^{n} M_{i}} \doteq \frac{\sum_{i}^{n} W_{i} \bar{y}_{i}}{\sum_{i}^{n} W_{i}} \tag{1}
\end{equation*}
$$

where $\bar{y}_{i}=\sum_{j} M_{i j} \bar{y}_{i j} / \sum_{j} M_{i j} \cong \sum_{j} W_{i j} \bar{y}_{i j} / \sum_{j} W_{i j}$.
The ratio estimate of total catch $Y$ is

$$
\begin{equation*}
\hat{Y}_{R}=\frac{W}{\bar{w}} \hat{\overline{\bar{Y}}}_{R} \tag{2}
\end{equation*}
$$

The above estimators recommended for use are not workable in rockfish sampling because the sampler failed in almost all cases to subsample from more than one category in a sampled trip as would be seen from a sample of basic data for 1982 (Table 1) available for Eureka from the Department of

TABLE 1.-Distribution of landing weights (lb) from all categories and from the sampled category for Eureka for 1982.

|  | Number of <br> clusters <br> sampled <br> $\left(m_{i}\right)$ | Market <br> category <br> sampled | Weight of <br> all fish <br> $\left(W_{i}\right)$ in a <br> given trip | Weight of all <br> fish for the <br> category in <br> a given trip |
| :---: | :---: | :---: | :---: | :---: |
| 1528 | 1 | 269 | 26,550 | 24,176 |
| 1529 | 1 | 250 | 4,133 | 445 |
| 1530 | 2 | 269 | 59,218 | 58,239 |
| 1531 | 1 | 269 | 20,511 | 15,987 |
| 1533 | 1 | 269 | 35,022 | 14,661 |
| 1534 | 1 | 269 | 20,757 | 20,705 |
| 1535 | 1 | 269 | 15,812 | 8,436 |
| 1536 | 1 | 250 | 1,975 | 1,010 |
| 1537 | 1 | 250 | 16,055 | 1,075 |
| 1541 | 3 | 269 | 65,837 | 65,837 |

'Shows the code number of categories which are based on species, size, and quality.
Note: In all cases, only one of the categories could be sampled from a given trip. In boat 1541 there was only one category (269) of fish.

California Fish and Game. The reasons for failure to collect the data are discussed in the section on Collection of Representative Data-Measurement Errors. The above estimators are, however, recommended for use in situations where the problem does not exist and, in particular, for single species where the categories are based on size. The estimates of error are given in Equations (4) and (5).

## Estimation Ignoring Category Variation Within Sampled Trips

Assume that a cluster is selected at random from all possible clusters in a sampled trip. In other words, we ignore categories altogether both in sample selection as well as in estimation. Valid ratio estimates $\hat{\overline{\bar{Y}}}_{1 R}$ of $\overline{\bar{Y}}$ and $\hat{Y}_{1 R}$ of $Y$ are respectively given by

$$
\begin{equation*}
\hat{\overline{\bar{Y}}}_{1 R}=\frac{\sum_{i}^{n} W_{i} \bar{y}_{i}}{\sum_{i}^{n} W_{i}} ; \hat{Y}_{1 R}=\frac{W}{\bar{w}} \hat{\bar{Y}}_{1 R} \tag{3}
\end{equation*}
$$

Note these equations are essentially the same as Equations (1) and (2) except that we now assume that a cluster is randomly selected from all possible clusters in a sampled trip where $W_{i}$ is the total landing weight from all categories for the $i$ th boat trip in the sample ( $W=\sum_{i} W_{i}$ ). In practice, the sampler would tend to subsample from a category which is accessible and is preponderant. This may lead to some bias in the estimate though its contribution to the total error will be negligible, since this would occur at the second stage of sampling.

The estimates of variance of estimated total and mean are approximately given by

$$
\begin{gather*}
v\left(\hat{Y}_{1 R}\right) \doteq\left[\frac{1}{n}\left(1-f_{1}\right) s_{b}^{2}+\frac{f_{1}\left(1-\bar{f}_{2}\right) s_{w}^{2}}{n \bar{m}}\right]\left(\frac{W}{\bar{w}}\right)^{2}  \tag{4}\\
v\left(\hat{\bar{Y}}_{1 R}\right) \doteq\left(\frac{\bar{w}}{W}\right)^{2} v\left(\hat{Y}_{1 R}\right)  \tag{5}\\
\text { where } s_{b}^{2}=\sum_{i}^{n}\left(\frac{W_{i}}{\hat{\bar{W}}}\right)^{2} \frac{\left(\bar{y}_{i}-\hat{\overline{\bar{Y}}}_{1 R}\right)^{2}}{n-1} \\
s_{w}^{2}=\sum_{i}^{n} \frac{\bar{m}}{n}\left(\frac{W_{i}}{\hat{\bar{W}}}\right)^{2} \frac{s_{2 i}^{2}}{m_{i}} \tag{6}
\end{gather*}
$$

and $s_{2 i}^{2}=\sum_{k}^{m_{i}}\left(y_{i k}-\bar{y}_{i}\right)^{2} /\left(m_{i}-1\right) ; \hat{\bar{W}}=\sum^{n} W_{i} / n$;

$$
\begin{equation*}
f_{1}=\sum^{n} W_{i} / W ; \bar{f}_{2}=\frac{\bar{w} \sum_{i}^{n} \frac{m_{i}}{W_{i}}}{n} \tag{7}
\end{equation*}
$$

We will consider an operationally feasible plan in which sample trips at a port during a month are poststratified into categories and clusters are subsampled from each category; where one or more categories are missed due to inadequate field staff and/or management problems, clusters should be selected from other boat trips containing the missed categories.

Assuming that the cluster weight of the unequal cluster size varies over trips, i.e., $\bar{w}_{i}=\sum_{j} \sum_{k} w_{i j k} /$ $\sum_{j} m_{i j}$ estimates of mean and total are
$\hat{\overline{\hat{Y}}}_{2 R}=\frac{\sum_{i}^{n} W_{i} \hat{R}_{i}}{\sum_{i}^{n} W_{i} / \bar{w}_{i}} ; \hat{Y}_{2 R}=W \frac{\sum_{i}^{n} W_{i} \hat{R}_{i}}{\sum_{i}^{n} W_{i}}$
where $\hat{R}_{i}=\frac{\bar{y}_{i}}{\bar{w}_{i}} ; v\left(\hat{Y}_{2 R}\right)$ and $v\left(\hat{\overline{\bar{Y}}}_{2 R}\right)$ can be obtained similar to Equations (4) and (5).

## Estimation Based on Categories as Domains of Study

This method is almost as precise as proportional stratified sampling if within each port-month stratum (a) a minimum of four landings or boat trips ( $n_{j} \geqslant 4$ ) is selected for each category and (b) the landing weights are available by categories after the season to serve as weights at the estimation stage. The minimum number in (a) is mainly based on limitations of field staff and budget restrictions. The ratio estimates of mean catch per cluster, total catch, and their errors, assuming clusters of equal size and using categories as domains of study are given by

$$
\begin{equation*}
\hat{\overline{\bar{Y}}}_{3 F} \doteq \sum_{j} W_{j} \overline{\bar{y}}_{j} / \sum_{j} W_{j} ; \hat{Y}_{3 R} \doteq \sum_{j} \hat{Y}_{j} \tag{9}
\end{equation*}
$$

where $\overline{\bar{y}}_{j}=\sum_{i}^{n_{j}} W_{i j} \bar{y}_{i j} / \sum_{i}^{n_{j}} \mathrm{~W}_{i j}$

$$
\begin{align*}
\hat{Y}_{j}= & \overline{\bar{y}}_{j} \frac{W_{j}}{\bar{w}}  \tag{11}\\
v\left(\hat{\overline{\bar{Y}}}_{3 R}\right)= & \sum_{j} \frac{W_{j}^{2}}{W^{2}} v\left(\overline{\bar{y}}_{j}\right) \\
& +2 \sum_{j} \sum_{<k} \frac{W_{j} W_{k}}{W^{2}} \operatorname{cov}\left(\overline{\bar{y}}_{j}, \overline{\bar{y}}_{k}\right) \tag{12}
\end{align*}
$$

and

$$
\begin{equation*}
v\left(\hat{Y}_{3 R}\right) \doteq v\left(\hat{Y}_{j}\right)+2 \sum_{j} \sum_{<k} \operatorname{cov}\left(\hat{Y}_{j}, \hat{Y}_{k}\right) . \tag{13}
\end{equation*}
$$

Both $v\left(\overline{\bar{y}}_{j}\right)$ and $v\left(\hat{\bar{Y}}_{3 R}\right)$ are of standard forms and can be obtained as in Equation (4). Similarly, $v\left(\hat{Y}_{j}\right)$ and $v\left(\hat{Y}_{3 R}\right)$ can be obtained. The covariance terms in Equations (12) and (13) are ignored when the subsamples from different categories are from different boat trips and are, therefore, independent. In rockfish sampling this was found true, because the sampler failed in almost all cases to subsample from more than one category. In general, for all fish where sampling from more than one category per boat trip is feasible, e.g., with few species-sizequalities, Equation (13) should be used.
Assume that the clusters vary in size over trips. For any sort (say $j$ )

$$
\begin{equation*}
\hat{Y}_{j_{1}}=\left[\sum_{i} W_{i j} \hat{R}_{i j} / \sum_{i} W_{i j}\right] W_{j}=\hat{R}_{j} W_{j} \tag{14}
\end{equation*}
$$

and

$$
\begin{equation*}
\hat{\bar{y}}_{j_{1}}=\sum_{i} W_{i j} \hat{R}_{i j} / \sum_{i} W_{i j} / \bar{w}_{i j} \tag{15}
\end{equation*}
$$

where $\hat{R}_{i j}=\frac{\bar{y}_{i j}}{\bar{w}_{i j}} ; \hat{R}_{j}=\sum \hat{R}_{i j} W_{i j} / \sum_{i} W_{i j}$.
If $n_{j}$ is small compared to $N_{j}$ and if the same subsampling strategy is applied to each of the sample landings, we have, ignoring contribution due to second-stage sampling units,
$v_{1}\left(\hat{R}_{j}\right)=\frac{1}{n_{j}\left(n_{j}-1\right)} \sum_{i}\left[\frac{W_{i j}}{\hat{W}_{i j}}\right]^{2}\left(\hat{R}_{i j}-\hat{R}_{j}\right)^{2}$.
Another estimator $v_{2}\left(\hat{R}_{j}\right)$ is the jackknife

$$
\begin{equation*}
v_{2}\left(\hat{R}_{j}\right) \doteq \frac{\left(n_{j}-1\right)}{n_{j}} \sum_{i}\left(\hat{R}_{i j}^{\prime}-\hat{R}_{j}^{\prime}\right)^{2} \tag{18}
\end{equation*}
$$

where $\hat{R}_{i j}^{\prime}=\frac{\hat{R}_{1 j} W_{1 j}+\ldots+\hat{R}_{(i-1)} W_{(i-1) j}+\hat{R}_{(i+1)} W_{(i+1)}+\ldots+\hat{R}_{n j} W_{n j}}{W_{1 j}+\ldots+W_{(i-1) m_{j}}+W_{\left(i+1 m_{j}\right.}+\ldots+W_{n_{j}}}$
and

$$
\hat{R}_{j}^{\prime}=\frac{1}{n_{j}} \sum_{i} R_{i j}^{\prime} .
$$

Thus $\hat{R}_{i j}^{\prime}$ is obtained by omitting trip $i$ from the sample for sort $j$ and calculating $\hat{R}_{i j}$ instead of $\hat{R}_{i j}$ as in Equation (16).
Hence, for category $j$ of a species
or

$$
\begin{align*}
v\left(\hat{Y}_{j_{1}}\right) & =W_{j}^{2} v_{1}\left(\hat{R}_{j}\right) \\
& =W_{j}^{2} v_{2}\left(\hat{R}_{j}\right) \tag{20}
\end{align*}
$$

where $v_{1}\left(\hat{R}_{j}\right)$ and $v_{2}\left(\hat{R}_{j}\right)$ are given by Equations (17) and (18).
For estimate of total over all sort groups for a species

$$
\begin{gather*}
\hat{Y}_{4 R}=\sum_{j} \hat{Y}_{j_{1}}  \tag{21}\\
v\left(\hat{Y}_{4 R}\right)=\sum_{j} v\left(\hat{Y}_{j_{1}}\right)+2 \sum_{j} \sum_{<k} \operatorname{cov}\left(\hat{Y}_{j_{1}}, \hat{Y}_{k_{1}}\right) \tag{22}
\end{gather*}
$$

A simpler formula $v\left(\hat{Y}_{4 R}\right) \doteq \sum_{j} v\left(\hat{Y}_{j_{1}}\right)$ can be used where subsamples from different categories are from different boat trips and are, therefore, independent.
It is, however, more reasonable to assume that the frequency distribution of fish caught is more uniform within a category so that cluster weight would be approximately a constant within a category. If so, the estimates of mean and total are given by

$$
\begin{align*}
\hat{\bar{Y}}_{5 R} & =\sum_{j} W_{j} \overline{\bar{y}}_{j} / \sum_{j} W_{j} ; \hat{Y}_{5 R}=\sum_{j} \hat{Y}_{j}  \tag{23}\\
\text { where } \overline{\bar{y}}_{j} & =\sum_{i} W_{i j} \bar{y}_{i j} / \sum_{i} W_{i j} ; \\
\hat{Y}_{j} & =\frac{\sum_{i} W_{i j} \bar{y}_{i j}}{\sum_{i} W_{i j}} \frac{W_{j}}{\bar{w}_{j}} \tag{24}
\end{align*}
$$

and $\bar{w}_{j}$ is the simple mean weight of clusters in the $j$ th group. Where the assumption of constant cluster weight within a category is not valid, the more general results given in Equations (14) and (15) should be used.

## Comparison of Methods: Ignoring Category Variation Versus Poststratification by Categories

We will compare the efficiency of the estimators (3), ignoring variation due to categories, with the estimators (9), based on poststratification of landings by categories at a port during a month. The analyses were based on Eureka and Monterey data for 1982. The coefficients of variation (c.v.) of mean catch per cluster for a species based on categories as domains of study (method 2 ) were in almost all cases lower (Table 2) than ignoring category variation (method 1). Since method 1 results in underestimation of c.v.'s because sampling is actually based on a stratified random sample instead of a simple random sample, the increased precision of method 2 is all the more striking.
The c.v. of the estimated mean catch by sex-age groups for a species for which the number of sample landings were $\geqslant 10$ (Table 3 ) were in all cases less for method 2 than for method 1. It may, however, be pointed out the c.v.'s are likely to be affected by factors such as growth, maximum age, and maximum size of fish. These have not been considered in this study. Thus, estimates based on categories as domains of study proved more efficient than ignoring categories altogether. Besides, method 2 has the added advantage of providing estimates by

TABLE 2.-Coefficient of variation (c.v., in percent) of mean catch by species at Eureka and Monterey based on the two methods during 1982.

|  | Sample size <br> Location <br> and <br> species <br> (number of <br> boat trips <br> sampled) |  | c.v. (\%) |  |
| :--- | :---: | :--- | ---: | :---: |
|  |  | Method 11 | Method 222 |  |
| Eureka | 88 | 11.48 |  |  |
| $\quad$ Widow rockfish | 88 | 30.83 | 32.33 |  |
| Chilipepper | 88 | 26.01 | 24.40 |  |
| Bocaccio |  |  |  |  |
| Monterey | 54 | 18.31 | 6.62 |  |
| $\quad$ Widow rockfish | 54 | 15.68 | 13.92 |  |
| Chilipepper | 54 | 12.57 | 10.32 |  |
| Bocaccio |  |  |  |  |

'Method 1, based on random categories (i.e., ignoring stratification by categories).
${ }^{2}$ Method 2, based on categories as domains of study.

| Eureka |  |  |  |  | Monterey |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of boat trips sampled | Sex | Age <br> (yr) | c.v. (\%) |  | Number of boat trips sampled | Sex | Age <br> (yr) | c.v. (\%) |  |
|  |  |  | Method <br> 1 | Method $2$ |  |  |  | Method 1 | Method 2 |
| Widow rockfish |  |  |  |  |  |  |  |  |  |
| 17 | M | 7 | 19.71 | 18.83 | 10 | F | 13 | 39.98 | 24.29 |
| 18 | F | 7 | 13.50 | 10.94 | 10 | F | 12 | 35.16 | 20.49 |
| Chilipepper |  |  |  |  |  |  |  |  |  |
| 11 | F | 13 | 39.98 | 24.89 | 24 | F | 9 | 18.48 | 7.63 |
| 11 | F | 12 | 34.77 | 31.21 | 21 | F | 7 | 22.09 | 9.81 |
| Bocaccio |  |  |  |  |  |  |  |  |  |
| 15 | M | 6 | 30.10 | 19.82 | 14 | M | 7 | 27.46 | 12.45 |
| 19 | F | 6 | 35.87 | 32.45 | 20 | F | 7 | 24.34 | 10.06 |

${ }^{1}$ 'Age-sex groups for which primary sampling units (landings) are $\geqslant 10$.
market categories which is of considerable economic importance.

## COST FUNCTION

Consider the cost function

$$
\begin{equation*}
C=c_{1} n+c_{2} n \bar{m} \tag{25}
\end{equation*}
$$

where $c_{1}$ is the average cost (in minutes) per boat trip due to transport, contact, and delay in making a contact, $c_{2}$ the average cost in data collection (identification of species, sex, length, otoliths, etc.) per cluster within clusters per boat trip and $C$ is the total cost involved in visiting the primary sampling units (boat trips) and collecting data from the $n$ boats with an average of $\bar{m}$ clusters per boat sampled. Data collected at Tiburon by the California Department of Fish and Game and the National Marine Fisheries Service show that $c=111.80 \mathrm{~min}, c_{2}=$ 58.3 min so that $\frac{c_{1}}{c_{2}}=2$ apply. However, from more recent studies conducted $\frac{c_{1}}{c_{2}}=3$.

The components $c_{1}$ and $c_{2}$ were estimated at
Activity Percent Mean (in minutes)

| Transport | 50.0 | 81.7 |
| :--- | :---: | :---: |
| Contact | 5.0 | 8.7 |
| Delay (off loading, etc.) | $\underline{13.0}$ | $\underline{21.4}$ |
|  | 68.0 | 111.8 |
| Data collection | Percent | Mean (in minutes) |
| Species ${ }^{1}$ | 7.7 | 14.0 |
| Sex, length | 5.8 | 10.6 |
| Otolith | 10.8 | 19.7 |
| Preparation time | $\underline{7.7}$ | $\underline{14.0}$ |
|  | 32.0 | 58.3 |

${ }^{1}$ Excluding samples dominated by single species.
Minimizing Equation (4) subject to Equation (25) for the optimum allocation we have

$$
\begin{equation*}
\bar{m}_{\mathrm{opt}}=\frac{s_{w}}{\sqrt{s_{b}^{2}-\frac{s_{w}^{2}}{\bar{m}}}} \sqrt{\frac{c_{1}}{c_{2}}} \tag{26}
\end{equation*}
$$

TABLE 4.-Optimum values of $m$ for estimating species catch per cluster by categories for different variance and cost ratios, 1978.

| Species | Category' | $n$ | $s_{b}^{2}$ | $s_{w}^{2}$ | $\bar{m}$ | $c_{1} / c_{2}=\frac{m_{\text {op1 }}}{2}$ | $c_{1} / c_{2}=3$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Eureka |  |  |  |  |  |  |  |
| $\quad$ Bocaccio | 250 | 25 | 1.80 | 3.01 | 2.16 | 3.86 | 4.73 |
| Chilipepper | 250 | 13 | 24.45 | 3.13 | 1.92 | 0.52 | 0.64 |
| $\quad$ Widow rockfish | 250 | 11 | 59.49 | 8.71 | 2.46 | 0.56 | 0.68 |
| Monterey |  |  |  |  |  |  |  |
| $\quad$ Bocaccio | 253 | 31 | 95.15 | 4.20 | 1.97 | 0.63 | 0.77 |
| Chilipepper | 253 | 33 | 43.71 | 4.16 | 1.94 | 0.45 | 0.55 |
| Widow rockfish | 253 | 12 | 22.38 | 4.66 | 2.00 | 0.68 | 0.84 |

${ }^{1}$ Code numbers of categories which are based on size, species and quality.

The variation among clusters ( $s_{b}^{2}$ ) in different landings at Eureka and Monterey for 1978 was in almost all cases greater than between clusters within the same landings (Table 4); also the optimum number of clusters per boat for estimating species number was mostly unity. Data from other ports follow the same pattern. Since a minimum of two clusters is needed to provide an estimate of between cluster within trip variation, a subsample of two clusters per category per trip is recommended. In practice, it is preferable to select a systematic sample of clusters separated in time.

## VARIANCE COMPONENTS: SPECIES-AGE AND LENGTH GROUPS

A two-level nested analysis of variance for length and age with unequal sample size for species based on sample landings at ports during 1979 (Table 5) shows that both the variation, because of length and age, was generally high among sample landings compared with clusters within landings. Also, variation between clusters was generally of the same order as within clusters, and the optimum number of clusters was $\leqslant 2$. Data for other ports and years (not shown in the table) mostly supported the findings.
On the whole, both the variation in species number (Table 4) as well as in length and age (Table 5) was consistently high among sample landings relative to between clusters within landings; also, variation among clusters was not significant compared with variation within clusters. Hence, for precise estima-
tion of species number, length, and age composition for a category at a port during a season, data should be collected from a large number of landings and from few clusters (two) from a category within a sample landing.

## RELATIVE EFFICIENCY OF ESTIMATORS USING POSTSTRATIFICATION

Consider the three estimators of total catch for a sort of a species at a port during a year. We will use the same selection procedure with poststratification by sorts but different estimation procedures.

$$
\begin{gather*}
\hat{Y}_{j}^{\prime}=\frac{N_{j}}{n_{j}} \sum_{i=1}^{n_{j}} \bar{y}_{i j}  \tag{27}\\
\hat{Y}_{j}=\frac{\sum_{i}^{n_{j}} W_{i j} \bar{y}_{i j}}{\sum_{i}^{n_{j}} W_{i j}} \frac{W_{j}}{\bar{w}_{j}}  \tag{28}\\
\hat{Y}_{j_{1}}=\hat{R}_{j} W_{j} \tag{29}
\end{gather*}
$$

where $\hat{R}_{j}$ is given by Equation (16), $\bar{y}_{i j}$ is the simple mean of species number per cluster for sort $j$ from the $i$ th sample, $\hat{Y}_{j}$ is the same as Equation (24) with a constant cluster weight within a sort group, and $\hat{Y}_{j_{1}}$ is a more general estimator based on the assumption that cluster weight varies among trips. For $v\left(\hat{Y}_{j_{1}}\right)$ use $W_{j}^{2} v_{2}\left(\hat{R}_{j}\right)$ where $v_{2}\left(\hat{R}_{j}\right)$ is the jack-

TABLE 5.-Two-level nested ANOVA of length and age of species with unequal sample sizes by ports during 1979. $\mathrm{MS}=$ mean square; $F=F-$ RATIO, Statistic; $P=$ observed probablity level.

| Source | Age |  |  |  | Length |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | df | MS | F | $P$ | df | MS | F | $P$ |
| Widow rockfish at Eureka |  |  |  |  |  |  |  |  |
| Samples | 15 | 34.45 | 4.75 | $<0.005$ |  | 37.86 | 3.09 | $<0.025$ |
| Clusters (within samples) | 13 | 7.25 | 1.19 | 0.35 |  | 12.27 | 1.43 | $\sim 0.18$ |
| Within clusters | 320 | 6.09 |  |  |  | 8.58 |  |  |
| Chilipepper at Monterey |  |  |  |  |  |  |  |  |
| Samples | 43 | 31.74 | 4.05 | <0.001 | 48 | 145.20 | 4.02 | <0.001 |
| Clusters | 39 | 7.84 | 1.80 | $\sim 0.001$ | 44 | 36.10 | 1.43 | $\sim 0.035$ |
| Within clusters | 320 | 4.35 |  |  | 971 | 25.25 |  |  |
| Bocaccio at San Francisco |  |  |  |  |  |  |  |  |
| Samples | 10 | 84.97 | 6.95 | $<0.001$ | 10 | 317.88 | 6.98 | <0.001 |
| Clusters | 15 | 12.23 | 1.20 | $\sim 0.30$ | 16 | 45.55 | 0.80 | $\sim 0.75$ |
| Within clusters | 225 | 10.20 |  |  | 227 | 57.11 |  |  |

knife estimator of Equation (18) and for $v\left(\hat{Y}_{j}^{\prime}\right)$ see Sukhatme (1954). $\hat{Y}_{j}^{\prime}$ is generally subject to considerable bias.

The c.v. of total catch of bocaccio, chilipepper, and widow rockfish for different categories by port-year groups (Table 6) show that the estimators $\hat{Y}_{j}$ and $\hat{Y}_{j_{1}}$ are highly efficient compared with $\hat{Y}_{j}$; also, $\hat{Y}_{j}$ turns out to be slightly superior to $\hat{Y}_{j_{1}}$ since the jackknife estimator $v_{2}\left(\hat{Y}_{j_{1}}\right)$ is an underestimate and does not take into account the contribution of the within component of variance. Thus, the empirical evidence supports strongly the use of the estimator $\hat{Y}_{j}$.

TABLE 6.-Coefficient of variation (in percent) of estimates of total catch of bocaccio, chilipepper, and widow rockfish per cluster by ports during 1978 and for different categories for the three estimators $\hat{\gamma}_{j}^{\prime}, \hat{Y}_{j}$, and $\hat{\gamma}_{i_{i}}$

| Port | Category | Number of boat trips sampled | $\hat{\gamma}_{j}^{\prime}$ | $\hat{Y}_{j}$ | $\hat{Y}_{j_{1}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Bocaccio |  |  |  |  |  |
| San Francisco | 253 | 20 | 13.51 | 10.24 | 11.64 |
| Fort Bragg | 250 | 86 | 16.21 | 7.36 | 8.14 |
| Monterey | 253 | 31 | 12.07 | 17.93 | 19.51 |
| Eureka | 250 | 25 | 40.11 | 26.00 | 29.84 |
| Chilipepper |  |  |  |  |  |
| Eureka | 250 | 13 | 37.66 | 34.52 | 42.33 |
| Widow rockfish |  |  |  |  |  |
| Monteray | 250 | 12 | 111.20 | 43.47 | 68.29 |
| Eureka | 250 | 11 | 72.69 | 27.81 | 33.90 |

## AGE-COMPOSITION: DOUBLE SAMPLING

Studies mentioned in the Introduction section have shown that since aging from otoliths of each individual fish in a sample is more expensive than an easily measured quantity such as length, it may pay 1) to choose a random subsample from the whole sample of length measurements for age determination or 2) stratify the sample according to length classes and choose a subsample from each class for age determination. The technique is profitable only if the correlation between length and age is fairly high.

It may be recalled that considerable bias is introduced by applying age-length keys developed during a year to subsequent years. Both Kimura (1977) and Westrheim and Ricker (1978) showed that agelength keys can yield most inefficient estimates of numbers-at-age with substantial overlap of lengths between ages. In the latter case the correlation between length and age will be low for the larger and
the very small sizes. Consequently, we will need a higher sampling intensity at the tails to provide reliable estimates of age for such sizes.
In the construction of length strata for selection of the subsample, additional questions arise on 1) number of strata to choose, 2) strata boundaries to decide, and 3 ) the number of sampling units to be allocated to each stratum for deriving maximum gain from double sampling. These are discussed as follows.

## Number of Strata

The values of $V\left(\bar{y}_{s t}\right) / V(\bar{y})$ (Cochran 1977) are given below as a function of $L$, the number of strata using the linear model

$$
\begin{equation*}
y=\alpha+\beta x+\varepsilon \tag{30}
\end{equation*}
$$

where $y$ is the length, $x$ the age of female widow rockfish and

$$
\begin{equation*}
\frac{V\left(\bar{y}_{s t}\right)}{V(\bar{y})}=\frac{\rho^{2}}{L^{2}}+\left(1-\rho^{2}\right) \tag{31}
\end{equation*}
$$

where $\rho$ is the correlation between length and age in the unstratified sample and $L$ the number of strata. It can be shown for this model that when $L$ $\geqslant 6$ and $\rho>0.95$, there is hardly any gain due to stratification (Table 7). The improvement in stratification is highest for data set 1 for which $\rho^{2}$ $=0.7004$ and lowest for set 3 for which $\rho^{2}=$ 0.5278 . The results for the regression model indicate that unless $\rho$ exceedes 0.95 , little reduction in variance is to be expected beyond $L=6$. Data sets 1,2 , and 3 support this conclusion. In fact, there does not seem to be any profit resulting from increase in strata beyond $L=5$.

## Strata Boundaries

For the length-age strata on 239 females (widow rockfish) landed during 1982 at San Francisco and the rule based on the cumulative of $\sqrt{f(y)}$ (Cochran 1977) where $y$ denotes the length in centimeters, the nearest available points for the two strata are

|  | Stratum |  |
| :--- | :---: | :---: |
|  | 1 | 2 |
| Boundaries | $36-47 \mathrm{~cm}$ | $48-55 \mathrm{~cm}$ |
| Intervals on <br> cum $\sqrt{f}$ | 18.70 | 23.72 |



It turns out that the division point is approximately the same for young as well as old widow rockfish. For length-age data (1981) based on 444 females (widow rockfish) landed at Eureka, the boundaries using 2 and 3 strata are

|  | Stratum |  |  |
| :---: | :---: | :---: | :---: |
|  | 1 |  | 2 |
| Boundaries | $31.5-47 \mathrm{~cm}$ |  | $46.5-55 \mathrm{~cm}$ |
| Intervals on cum $\sqrt{f}$ | 17.70 |  | 29.01 |
|  | Stratum |  |  |
|  | 1 | 2 | 3 |
| Boundaries | $31.5-46 \mathrm{~cm}$ | $46.5-49 \mathrm{~cm}$ | $49.5-55 \mathrm{~cm}$ |
| Intervals on $\operatorname{cum} \sqrt{f}$ | 17.70 | 13.12 | 15.89 |

## Optimum Allocation Plan

Double sampling with regression is more efficient than single sampling (when the first sample is measured for age alone) for the same cost if

$$
\begin{equation*}
\rho^{2}>\frac{4 \frac{c}{c^{\prime}}}{\left[1+\frac{c}{c^{\prime}}\right]^{2}} \tag{32}
\end{equation*}
$$

where $\rho$ is the correlation between length and age of fish, $c$ and $c^{\prime}$ are respectively the costs of aging and measuring a fish. Assuming that the average cost of aging a rockfish (including small and large fish) is 6 min and of measuring it is 1.2 min (estimates based on measurements by W. Lenarz of Tiburon Laboratory), we have from Equation (32)
or

$$
\rho>0.7453 .
$$

For the three data sets (Table 7) the values of $\rho^{2}$ are respectively $0.7004,0.6515$, and 0.5278 so that Equation (32) is approximately satisfied. However, neither $\rho$ nor $\frac{\mathrm{c}}{c^{\prime}}$ are large enough to suggest that double sampling will be much more efficient than single sampling.

We will illustrate the use of double sampling for stratification by analyzing 1981 length-age data at Eureka to estimate the proportion of female in age group 11, based on a sample of 444 fish. For the three length strata, $h=1,2,3$ with stratum boundaries based quadratic fit of length on age are 31.5-43, 43.5-49, 49.5-55. (Note this is different than boundaries based on length only.) Also

$$
\begin{aligned}
& c_{0}=1.2 \mathrm{~min}, c_{1}=3.8 \mathrm{~min}, c_{2}=3.8 \mathrm{~min} \text {, and } c_{3}=8 \mathrm{~min} \\
& w_{1}=0.0653, w_{2}=0.5451 \text {, and } w_{3}=0.3896 \\
& s_{1}=0.1825, \quad s_{2}=0.4966, \quad s_{3}=0.1503, \\
& \text { and } s=0.4343
\end{aligned}
$$

where $w_{1}, w_{2}$, and $w_{3}$ are the proportions of fish in the sample, $c_{0}$ is the cost of measuring a fish and $c_{1}, c_{2}, c_{3}$ are respectively the costs of aging them in the three length groups. From Cochran (1977, p. 331) we have
$v_{\min }\left(p_{s t}\right)=\frac{1}{C^{*}}\left[\sum w_{h} s_{h} \sqrt{c_{h}}+\left(S^{2}-\sum w_{h} s_{h}^{2}\right)^{1 / 2} \sqrt{c^{\prime}}\right] 2$
$=0.8915 / C^{*}$
where $p_{s t}$ is the estimated proportion and $C^{*}=$ $E(c)=E\left(c_{0} n+\sum_{n} c_{h} n_{h}\right)$ with $n_{1}=14, n_{2}=120$, $n_{3}=48$ and $n^{\prime}=444$. The efficiency of double sampling with respect to single sampling is given by

$$
v_{s r s}(p) / v_{\min }\left(p_{s t}\right)=1.27
$$

where $v_{\text {srs }}(p)=0.1885 / \frac{C^{*}}{6}$, i.e., double sampling is $27 \%$ more efficient than single sampling. However, as noted by Ricker (1975) the increase in accuracy achieved by combining a length sample with a smaller age sample may not be great unless fish used for age determination is taken from the same stock, during the same season and using gear having the same selective properties as the length-frequency samples. This point will generally be met if fish are subsampled systematically for age from fish arranged in increasing (or decreasing) order of length from a port-month stratum. Our studies have shown that the best length-age fit does not change significantly if age determination is made on every other fish arranged in ascending order of length.
It is difficult to obtain reliable estimates of the numbers at age for the extremely small or larger sizes because lengths cannot be used for estimating age. There is need for search for other auxiliary variables (other than length) associated with age and for increase in sampling rate at the tails. In double sampling where lengths are obtained in the first phase, a number of small clusters may be used separated in space and time to provide a large number of fish at the tails for estimating numbers at age. The extent of bias in estimation of numbers
at age through length-age key approach may be tested by Monte Carlo simulation.

## COLLECTION OF REPRESENTATIVE DATA-MEASUREMENT ERRORS

Owing to uncertainty of arrival times and varying unloading procedures, no objective method is available to ensure random sampling of the trips. When the vessels return to port, they are usually available for sampling except when they are transhipped immediately due to inclement weather, lack of processing facilities, uncooperative buyers, or unscheduled deliveries at short notice. It is, however, not unreasonable to regard a set of sample landings during a week at a port as random and representative of the totality of all landings at the port for the month.
Although rockfish are landed by categories, which are mostly determined by market agreement based on size, composition, and condition of the catch, the number of categories per delivery cannot be predetermined. This number would vary from delivery to delivery and from dealer to dealer. Also, there are no guarantees that a complete boat sample, covering clusters from each category, can be taken on any sampling day and some of the categories are actually missed in sampling. Some of the possible reasons for missing the categories are 1) when landing weight would not occur during regular hours, one of the sorts may have already been shipped before the sample could arrive at the spot; 2) often one of the sorts may be quite small and there may be a buyer at the dock waiting for the fish to be taken away; 3) while the sampler is working on a sort, the other sort(s) will have either been processed or shipped away; and 4) the sampler may
be prevented from taking a sample from another sort by the skipper who may not like some of his fish being cut and otoliths removed for biological studies. This may happen at ports where either processing facilities are inadequate or fish are bought by local merchants immediately after landing. The question arises if failure to sample from all categories of a sample landing as originally planned would cause appreciable bias and loss in efficiency in the estimates of species catch and its distribution and whether a more efficient method could be developed that is operationally feasible. This point has been examined in the present paper.

The present technique of selecting a cluster (box) of fish as second stage sampling unit is preferred to random selection of a specified number of individual fish because in practice the potential of personal bias of the sampler could be considerable. Often fish chosen by the latter technique are ones closest to the sampler or those that fell in a certain position. Tomlinson (1971) felt that in this approach the sampler may tend to choose a fish with certain qualities and thus may introduce procedural bias.

The selection of a representative cluster would depend whether samples after sorting on the vessel come from bins, strap boxes, or off conveyor belts. Buyers from small markets occasionally select fish from the top of bins. Hence, to avoid this bias, it is preferable to select the cluster from the conveyor belt which exposes unsorted fish from the lower portion of the bin. However, where small market buyers do not buy fish, a cluster may be selected from a bin. Where many bins are present a systematic sample of two clusters, preferably from the beginning and end of the trip may be selected. Where fish are graded on a conveyor belt before they enter the plant (e.g., Fieldslanding at Eureka) the sampler should try to intercept the landings prior to secondary sorting or obtain separate weights for each subsort category. In general, selection of a cluster for a market category should be done before any presorting is done at the port.

It has been pointed out earlier that bias may result from personal selection of fish within a cluster. If the sampler were to select a number of clusters with few fish per cluster, a cluster will on the average contain more big fish. This would lead to high nonsampling bias. Sometimes, the top few fish in a bin are selected and put there to impress small buyers. The resulting bias in selection can be avoided by taking all the fish in a cluster (e.g., 50 lb ) from one side of the box.

For obtaining reliable and comprehensive information on population characteristics, it is essential
for the sampler to maintain good relationships with both the skipper and the buyer; this will depend to a large extent on the expertise of the sampler gained in the course of the field work.

## SUMMARY

1. The sampling scheme at a port during a month with poststratification of sampled trips into categories and subsampling of clusters from each category (see sections on Estimation with poststratification and Estimation ignoring category variation) is not workable for estimating rockfish catch since some of the categories may be missed in sampling due to inadequate field staff and/or management problems.
2. For other commercial fish where the above problem does not exist and landing weights by categories are not available at the end of the season, the methods (see sections on Estimation with poststratification and Estimation ignoring category variation) are recommended, e.g., for single species where the categories are based on size.
3. For estimating the catch of rockfish, a twostage sampling plan is recommended with boat trips as first stage units poststratified into categories and clusters subsampled from a category; estimates are based on categories as domains of study with landing weights available for each category. A minimum of four landings or boat trips should be used for each category, to provide efficient estimates. With few categories, this number is likely to be large.

Where only one category is subsampled for each boat in the sample, $v\left(\hat{Y}_{3 R}\right)=\sum_{j} v\left(\hat{Y}_{j}\right)$. In all other cases Equation (13) should be used.
4. The design described in the above paragraph is recommended for use in other fisheries where landing weights are available for each category. Equations (9) and (21) are recommended for the estimation of catch according as the clusters are of equal or unequal size. Equations have been provided for the more practical case when cluster weight can be treated as constant within a category but different among categories.
5. Estimates of species catch by sex and age based on method 1 are less efficient than those based on method 2 which is based on categories as domains of study (Tables 2, 3).
6. Method 2 is preferred to method 1 when there
is variation among categories. This is true for all fish.
7. With few categories (species-size-qualities) the chance of missing a category is reduced. Equations (9) and (13) should be used for clusters of equal size and Equations (21) and (22) for unequal size clusters. This result is, of course, applicable to all commercial fish.
8. As far as practicable, selection of a cluster for a market category should be done before any presorting is done at the port either from bins, strap boxes, or off conveyor belts.
9. Variation (within categories) in length and age for a species was considerably higher among boat trips than among clusters within boat trips. Also, variation among clusters was not significant, compared with variation within clusters (Table 5). Hence, for precise estimation of species number, length, and age composition for a category at a port during a season data should be collected from a large number of landings and from few clusters from a category within a sample landing. This result should hold for all commercial fish.
10. For the cost function $C=c_{1} n+c_{2} n \bar{m}$ where $c_{1}$ is the average cost (in minutes) per boat trip due to transport, contact, and delay in making a contact, $c_{2}$ the average cost of data collection (identification of species, sex, length, otoliths, etc.) per cluster per boat trip and $C$ is the total cost involved in visiting the primary sampling units (boat trips) and collecting data, the optimum number of clusters per sampled trip for a fixed cost for a category is two (Table 4). This should provide valid estimates of error as required in Equations (13) and (22).
11. The principal contribution of the paper is that a minimum of four sample landings be subsampled for each category from a port-month stratum, i.e., about 1 per week and two clusters of 50 lb ( 25 lb for small fish) each should be sampled to provide port-year estimates with a reasonable degree of accuracy.
If a category is infrequently landed, sampling should be directed towards the infrequent category, as long as the number of landings for the category is less than four per month.
12. The efficiency of the ratio estimator (Equation (28)) based on poststratification by categories at port-year level and using constant cluster weight within a category was compared with two other estimators, including the ratio estimator based on jackknife. Empirical evidence indicated that the ratio estimator using constant
cluster weight within a category proved most efficient for estimation of species catch.
13. Age-length keys can yield most inefficient estimates of the numbers at age for extremely small and large fish. It is suggested that cluster sampling for length be based on a number of clusters separated in space and time; also, sampling for age should be intensified for small and large fish. This approach is applicable to all fish.
14. Double-sampling was adopted for estimating proportion of widow rockfish in 11-yr age group. A sample of fish was divided into 3 strata and optimum allocation for age was adopted within strata. The estimated proportion was $27 \%$ more efficient than if single sampling were adopted.

The best length-age did not change significantly if age determination is made on every other fish selected in ascending order of length.
The method is general and is applicable to all fish.

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