

Long-Run Profit Functions for Multiproduct Firms

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A long-run specification of the multiproduct profit function is developed from the restricted profit function. The multiproduct restricted profit function and the envelope condition are used to estimate the optimal, long-run levels of the quasi-fixed factor. Formulas for long-run Marshallian elasticities of substitution and transformation, economies of scope, product-specific and overall economies of scale, and economic measures of capacity utilization are developed for the translog functional form. The methodology is illustrated by a case study of the New England otter trawl industry.

Key words: economic capacity utilization, fisheries, long-run, profit functions, multiproduct costs and technology.

This article extends the full static equilibrium and restricted (or variable) multiproduct profit functions (Shumway; McKay, Lawrence, and Vlastuin; Lopez 1984, 1985a, b; Weaver) to a long-run specification of technology in which all quasi-fixed inputs are at their optimal, long-run equilibrium levels. Many studies of long-run technology specify a cost function with a single, exogenously determined and constant output (Kulatilaka, Brown and Christensen). In contrast, the long-run multiproduct profit function allows all products (as well as quasi-fixed inputs) to be endogenous without the simultaneous equation econometric problems or assumptions of homothetic separability otherwise required for long-run cost functions. This cost function procedure also ignores the adjustment of output levels to changes in product and factor prices and technology over the long run.

Primary emphasis in this article is upon a comprehensive and unified presentation of the long-run structure of technology and costs when products are multiple and decision vari-

ables to firms. The presentation develops Marshallian elasticities of substitution and transformation. The multiproduct profit function is also extended to consider the structure of long-run multiproduct costs, including long-run economies of scope and product-specific and multiproduct returns to scale. The profit function has previously been used only to examine the structure of production, while cost function studies of multiproduct costs have assumed full static equilibrium and constant outputs. The long-run multiproduct profit function is further developed to provide a measure of economic capacity utilization; previous studies employ single-product cost functions (Schankerman and Nadiri, Morrison 1985, in press). The method is demonstrated with data on the New England otter trawl fleet.

The Long-Run Multiproduct Profit Function

The next two sections consider the limitations to dynamic models, the advantages of long-run models, and develops the long-run profit function.

Long-Run and Dynamic Models

Dynamic specifications of production technology are increasingly advanced in response to the recognized limitations of static models.¹

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¹ Berndt, Morrison, and Watkins; Brown and Christensen; Schankerman and Nadiri; and Kulatilaka provide additional sum-

The most recent approach posits dynamic equilibrium models in which the costs of adjustment are explicitly specified and the firm assumed to be continuously in dynamic rather than static equilibrium. The firm is assumed to face smooth, convex costs of adjusting the stocks of quasi-fixed inputs. An intertemporal cost minimization or profit maximization problem is solved to provide explicit time paths of the quasi-fixed factors. This offers insights into the intertemporal factor substitution possibilities (Lopez 1985b; LeBlanc and Hrubovcak; Berndt, Morrison, and Watkins).

These dynamic equilibrium models have several limitations. They are empirically difficult to apply and have demanding data requirements. Long time series of data are required, and degrees of freedom and powers of tests can be limited. The assumption of smooth, convex costs of adjustment circumscribes asymmetries between investment and disinvestment costs. The internal adjustment costs are the only assumed cause of disequilibrium. Differences between actual and static equilibrium levels of quasi-fixed factors may arise for reasons other than smooth convex internal costs of adjustment, such as institutional rigidities, credit rationing, or physical immobility of input changes (such as lags or irreversibility), or regulatory restrictions. For example, regulatory restrictions hindering capital mobility or credit rationing are commonly found in agriculture and marine fishing industries, so that application of dynamic models requires particular care. Dynamic equilibrium might then be a misspecification.

The restricted (or partial) equilibrium framework developed by Lau, Mork, and later by Brown and Christensen, offers an attractive alternative. Others (Schankerman and Nadiri, Kulatilaka, Morrison 1985, Halvorsen and Smith, Hazilla and Kopp) have subsequently applied the approach in the framework of single-product cost functions. The firm is assumed to be in static equilibrium with respect to a subset of variable inputs conditional upon the observed levels of the quasi-fixed factors. The long-run optimal levels of the quasi-fixed factors are endogenously derived. The restricted and long-run profit functions describe the technology and costs of the

temporary and long-run equilibriums, respectively.

The restricted equilibrium approach includes both dynamic equilibrium and other departures from full static equilibrium, yet the empirical and conceptual complications are obviated.² Unlike the current generation of dynamic equilibrium models, the adjustment process can accommodate any factors causing disequilibrium. Explicit specification of the adjustment process is not required, but the firm's intertemporal behavior (such as the time path of quasi-fixed factors and dynamic substitution and transformation possibilities) is not explained. Nonetheless, an important difference between the short-run and long-run elasticities indicates slow adjustments of the quasi-fixed factors. The model is fundamentally static because future levels of the quasi-fixed factors are not affected by past departure from long-run equilibrium.

This long-run approach is particularly valuable for firm-level studies. Panel data sets typically do not include sufficiently long time series for dynamic models, and both balanced and unbalanced panel data sets and fixed effects are readily accommodated. The long-run procedure does not necessarily require a time series of data and therefore allows analysis from cross-sectional data of the long-run structures of costs and technology (which the dynamic procedure does not offer). The long-run approach is also useful in studies of production technology in natural resource industries because a model dynamic in the production technology should also be dynamic in the resource stock. The restricted equilibrium procedure is thus a practical method for estimating the long-run (multiproduct) technology given firm-level, cross-sectional, or panel data. The restricted equilibrium approach is also conceptually and empirically more general and flexible than the dynamic equilibrium approach.

Long-Run Profit Function

The long-run multiproduct profit function is developed from the restricted profit function. The multiproduct restricted profit function is specified as $HR(P; Z)$, where HR equals re-

maries and discussions in the context of single-product cost function. Appelbaum and Harris provide an alternative intertemporal approach based upon a neo-Austrian framework.

² Multistage optimization of inputs or outputs may also occur under homothetic separability. Full static equilibrium is assumed for these subsets.

stricted profit (total revenue less total variable costs), P is a $V \times 1$ vector of strictly positive prices, and Z represents an $F \times 1$ vector of positive levels of quasi-fixed factors. By Hotellings lemma: $\nabla_P HR = Y(P; Z)$, where ∇ is the vector differential operator, Y is a $V \times 1$ vector of M positive outputs, and N negative variable inputs, and provides conditional output supply and conditional variable input demand equations.

Hotelling's lemma further states (Lau): $\nabla_Z HR = -P'_z$, where P'_z is the vector of shadow prices of the quasi-fixed factors, Z . This term indicates the increase in restricted profits for a unit increase in the level of the quasi-fixed factors. If the firm optimizes with respect to the quasi-fixed factors, the variable inputs and outputs, and is in long-run equilibrium, then by the envelope condition the shadow prices equal the market rental prices of the quasi-fixed factors, P_z (Samuelson):³ $\nabla_Z HR = -P_z$.

The long-run multiproduct profit function H is derived from the restricted profit function HR . For given levels of quasi-fixed factors, the total multiproduct profit function is specified: $HT(P, P_z, Z) = HR(P, P_z; Z) -$

stricted profit-maximizing levels. The first-order condition for maximizing HT with respect to Z is given by the envelope condition. Solving this condition for the optimal, long-run level of Z , $Z^* = Z^*(P, P_z)$ and substituting into total profit, HT , gives the long-run multiproduct function H :

$$(1) \quad H(P, P_z) = HR\{P, Z^*(P, P_z)\} - P'_z Z^*(P, P_z).$$

Long-Run Marshallian Elasticities

The long-run elasticities of substitution and transformation are (Kulatilaka):

$$(2) \quad E_{ij} = \partial \ln Y_i(Z^*) / \partial \ln P_j \quad i, j \in V, F$$

where $Y_i(Z^*)$ is the long-run supply (demand) of product (input) i , $Y_i(Z^*) > 0$ for outputs, and $Y_i(Z^*) < 0$ for inputs. These are Marshallian elasticities since they include both expansion and pure substitution effects. Their derivation follows that presented in Kulatilaka, Halvorsen and Smith, and Brown and Christensen, so that only their final form is presented. All elasticities are evaluated at Z^* .

$$(3) \quad E_{ii} = \frac{(A_{ii} + S_i^2 - S_i)}{S_i} - \frac{(A_{iz} + S_i S_z)^2}{S_i(A_{zz} + S_z^2 - S_z)} \quad i \in V$$

$$(4) \quad E_{ij} = \frac{(A_{ij} + S_i S_j)}{S_i} - \frac{(A_{iz} + S_i S_z)(A_{jz} + S_j S_z)}{S_i(A_{zz} + S_z^2 - S_z)} \quad i, j \in V$$

$$(5) \quad E_{ii} = \frac{S_z}{(A_{zz} + S_z^2 - S_z)} \quad i \in F$$

$$(6) \quad E_{ij} = \frac{S_z(A_{jz} + S_j S_z)}{S_i(A_{zz} + S_z^2 - S_z)} \quad i \in V, j \in F$$

$$(7) \quad E_{ij} = \frac{(A_{jz} + S_j S_z)}{(A_{zz} + S_z^2 - S_z)} \quad i \in F, j \in V.$$

$P'_z Z$, where P'_z is the transpose of P_z and HT is total maximum profit. The long-run profit function H is obtained by first maximizing HR conditional upon the level of Z and then maximizing HT with respect to Z , holding the first V outputs and variable inputs at their re-

Marshallian elasticities derived from the multiproduct restricted profit function estimated with or without the envelope condition are conditional upon the quantity of Z (which is not necessarily optimal). These conditional elasticities do not allow for the effects from induced changes in Z . The formulas for these elasticities for variable inputs and outputs are given by the first terms of equations (3) and (4). The second term in (3) is negative for products and positive for inputs by concavity of the restricted profit function in Z (Lau). The

³ The long run is defined as the state where total profits (restricted profits less the costs of the quasi-fixed inputs) are maximized. The long run so defined is merely a construct to distinguish the observed short-run equilibrium from the desired long-run equilibrium and need never be actually achieved by the observed technology (Kulatilaka).

restricted own-price elasticities are therefore smaller in absolute value than the long-run elasticities, which is consistent with LeChatelier's principle. The long-run cross-price elasticities can differ in absolute magnitude in either direction because the second term in (4) can be either positive or negative for both inputs and outputs.

The Structure of Long-Run Multiproduct Costs

The long-run profit function can also be extended to examine the structure of multiproduct costs. This approach relaxes the maintained hypotheses of the static cost function: exogenously determined products and full static equilibrium of all inputs. Instead, the long-run structure of multiproduct costs is evaluated at the optimal levels of all outputs (and inputs) rather than exogenously determined (not necessarily optimal) levels.

Economies of Scope

An important multiproduct cost measure is economies of scope. Economies of scope measures the effects of joint production upon costs. Production which is joint in inputs has a lower cost than independent production of several products (Baumol, Panzar, and Willig).

Two sources of jointness in inputs, and therefore economies of scope, exist: an interdependent production process and allocatable (quasi-) fixed factors. An interdependent production process leads to economies of scope through local cost complementarity; higher levels of one output reduce the marginal costs of other products. Local cost complementarity can arise from several sources, such as risk minimization, the quasi-public nature and lumpiness of capital, economies of networking, the reuse of an input by more than one product, and the high costs of obtaining information and the organizational and strategic impediments to its market transfer (Sharkey, Bailey and Friedlaender). Allocatable (quasi-) fixed factors yield jointness (Shumway, Pope, and Nash), hence economies of scope, when the marginal allocation of variable inputs depends upon the allocation of the fixed input, and generate product-specific fixed costs.

Gorman extends the work of Baumol, Pan-

zar, and Willig to establish the relationship between cost complementarity and product-specific fixed costs in determining economies of scope. Gorman shows that, even in the absence of cost complementarity, the existence of subadditive fixed costs is a sufficient condition for economies of scope:

$$(8) \quad c_{ij} - [F(S) + F(T) - F(S \cup T)]/Y_i Y_j \leq 0,$$

where c_{ij} denotes the second derivatives of the variable cost function with respect to outputs i and j , $F(S)$ and $F(T)$ are the product-specific fixed costs of the disjoint product sets S and T , and $F(S \cup T)$ is the fixed cost of producing both sets together as a single product set. When there are no product-specific fixed costs, $C_{ij} < 0$ is a sufficient condition for economies of scope, where C represents static equilibrium or long-run multiproduct costs.

Condition (8) implicitly assumes allocatable (quasi-) fixed factors in order to obtain the terms involving product-specific fixed costs. However, in a long-run competitive equilibrium at the firm level, this source of jointness disappears. Shumway, Pope, and Nash (p. 77) state if a competitive equilibrium exists for the (quasi-) fixed factors and the market rental rate equals the shadow price of these factors, then the effects of jointness from allocatable (quasi-) fixed factors vanish. Shumway, Pope, and Nash also note that the dual specification of technology (required to test for economies of scope) cannot yield product-specific demand equations for either the variable inputs or the fixed allocatable inputs, it can only identify total demand equations for each input.⁴ As a consequence, the long-run equilibrium levels of the (quasi-) fixed allocatable inputs, and therefore product-specific fixed costs, cannot be determined by a long-run cost or profit function. Thus, condition (8) should be considered as a short- or intermediate-term test for economies of scope. Also, the condition (8) for economies of scope in the long run becomes a test for local cost complementarity ($C_{ij} < 0$), and the long-run source of jointness

⁴ Since Shumway, Pope, and Nash note that the dual approach is limited when allocatable (quasi-) fixed factors exist, the condition (8) provides a difficult test for a cost function even in the short run. Moreover, use of the static cost function to directly measure overall economies of scope captures only cost complementarities if product-specific fixed costs do not exist, that is, fixed factors are not allocable.

in inputs for the firm is an interdependent production process.

The condition for long-run cost complementarity based on the long-run multiproduct profit function is developed using the results of Lau and Lopez (1984). Lopez indicates the following relationship between the Hessian matrix of the cost function C and the Hessian of the profit function: $H = C_{ij} = H_{ij}^{-1}$, $i \neq j$, i, j are outputs, where C_{ij} is the second partial derivative of C with respect to products i and j , and H_{ij}^{-1} represents the inverse of the second partial derivative of H with respect to the product prices of i and j . Lau relates the derivatives of the long-run (H) and restricted (HR) profit functions, allowing all forms entering H_{ij}^{-1} to be derived from the restricted profit function: $H_{ij} = HR_{ij} - (HR_{ZZ})^{-1}HR_{iZ}HR_{Zj}$, where all terms are defined as before. The derivatives required to evaluate the inverse of this expression with the translog form are derived in the same manner as the Marshallian elasticities. The condition for local cost complementarity with the long-run translog multiproduct profit function is

$$(9) \quad H_{ij}^{-1} = \left[\frac{HR}{P_i P_j} (A_{ij} - S_i S_j) - (A_{ik} + S_i S_k)(A_{jk} + S_j S_k) / (A_{kk} + S_k^2 - S_k) \right]^{-1}.$$

Only the internal brackets require evaluation since $HR, P_i, P_j > 0$. Predicted shares (S_i) are evaluated at Z^* . The test is applied equation-by-equation rather than simultaneously for the system, and it is not a statistical test.

Product-Specific Economies of Scale

Product-specific economies of scale measure the change in costs through variations in the output of one product while holding the quantities of other products constant. Although product-specific economies of scale cannot be directly measured by the translog profit function, a sufficient condition is obtained by examining incremental marginal costs (Baumol, Panzar, and Willig). C_{ii} less (greater) than zero implies decreasing (increasing) long-run product-specific economies of scale for product i . Since $C_{ii} = H_{ii}^{-1}$, the following sufficient condition with the translog form provides the basis for this nonstatistical test:

$$(10) \quad H_{ii}^{-1} = \left[\frac{HR}{P_i^2} (A_{ii} + S_i^2 - S_i) - (A_{ik} - S_i S_k)^2 / (A_{kk} + S_k^2 - S_k) \right]^{-1}.$$

Only the terms in the internal brackets require evaluation since $HR, P_i > 0$. $C_{ii} < 0$ implies decreasing marginal and average incremental cost curves for product i . Under marginal cost pricing, the revenues collected from the sale of product i fall short of the incremental costs of their production (Baumol, Panzer, and Willig), but MacDonald and Slivinski note that overall efficiency may imply what appears to be inefficiencies within the diversified firm. This test can also be applied to single-product profit functions.

Cost Convexity

The diagonal elements of the Hessian submatrix, $C_{ii} = \partial^2 C / \partial Y_i^2$, all $i \in M$, represent product-specific marginal cost curves, while the off-diagonal elements C_{ij} indicate cost complementarities among product pairs. Convexity of the long-run cost function (inherent in the long-run profit function) in outputs can be tested by examining this Hessian submatrix for outputs.

Long-Run Multiproduct Returns to Scale

Long-run multiproduct or ray returns to scale for the profit function measure the behavior of costs for proportional changes in total firm output and all variable and quasi-fixed inputs.⁵ This is a straightforward extension of the concept of single-product scale economies, where the output composition remains fixed while its scale can vary. The degree of long-run ray scale economies equals the ratio between long-run production costs and the revenues that occur with marginal cost pricing. The rev-

⁵ The single-product profit function is not well defined for constant or increasing returns to scale. However, the multiproduct profit function does not suffer from this limitation because the structures of multiproduct costs and industry are dependent upon both the scale and composition of outputs. Even if ray returns to scale are increasing for the existing division of products, the multiproduct cost structure may be such that some other division of outputs among smaller firms may provide an even more efficient form of multiproduct industry organization. In fact, cost subadditivity is required for a single multiproduct firm (natural monopoly), and increasing ray returns to scale provides only a sufficient condition for ray subadditivity. Shaffer shows that multiproduct functions should in both theory and practice cause downward bias in estimated ray scale economies.

enues exceed, are less than, or equal long-run costs as there are decreasing, increasing, or locally constant long-run ray returns to scale.

Long-run multiproduct economies of scale are a weighted sum of long-run economies of scale and long-run product-specific economies of scale (Baumol, Panzer, and Willig; Bailey and Friedlaender):

$$(11) \quad S_M = \frac{w_T S_T + (1 - w_T) S_{M-T}}{1 - SC_T},$$

where the product set M is partitioned into two disjoint subsets T and $M - T$, SC_T is the measure of long-run economies of scope, S_T and S_{M-T} are measures of long-run product-specific economies of scale, and $W_T = (\sum_T Y_i C_i / \sum_M Y_i C_i)$. The degree of long-run overall scale economies for both product sets is thus a weighted average of long-run product-specific scale economies magnified by long-run economies of scope through the denominator. Long-run ray returns to scale can still be captured even if decreasing long-run product-specific returns to scale exist throughout the product set. Long-run overall returns to scale with the long-run translog multiproduct profit function are measured by $(\sum_N S_i + \sum_F S_F) / (\sum_M S_j)$, where all shares are predictions evaluated at Z^* . Because measurement is taken along the expansion path, this is also a measure of long-run overall size economies.

Capacity Utilization Measurement

The long-run profit function can also be applied to studies of economic capacity utilization (CU) when product levels and mixes are decision variables to firms. Recent applications of the long-run single-product cost function to CU measurement can be extended to the long-run multiproduct profit function (Morrison 1985, Schankerman and Nadiri). The effects of changes in product and factor prices on CU measurement and temporary equilibria when outputs are endogenous are directly captured by the multiproduct profit function. In contrast, biased measures of economic CU are likely from single-product cost functions for multiproduct firms after the exogenous shocks inducing the new, temporary equilibrium, unless outputs are separable because CU measures will be on a new product ray.

Noncompetitive product markets are readily accommodated by the approach of Die-

wert. Although not developed here, CU measures based on the long-run multiproduct profit function can be used to adjust productivity measurements for departures from full equilibrium (Morrison 1985, Schankerman and Nadiri).

Capacity utilization measures represent the proportion of available productive capacity currently utilized. Economic measures of CU based on the primal depict the divergence between short-run temporary equilibrium (Y) and long-run full equilibrium (Y^*) levels of outputs, so that $CU = Y/Y^*$ in product space. In the general case of long-run nonconstant returns to scale, the capacity level of outputs Y^* for some product combination corresponds to the tangency of the short-run ray (SRAC) and long-run ray (LRAC) average cost curves. These capacity output levels are in steady state in that the firm has no incentive to change product levels and combinations from Y^* . Since the stocks of quasi-fixed factors position and influence the shape of SRAC, Y^* and CU explicitly reflect short-run constraints (Morrison 1985).

Morrison, and Schankerman and Nadiri provide a dual interpretation of economic CU measures using the cost function. Dual CU measures contain information on the difference between the long-run competitive equilibrium and temporary equilibrium in terms of implicit costs of being away from long-run equilibrium. These disequilibrium costs with the profit function are opportunity costs in terms of restricted or variable profit foregone with the divergence from long-run equilibrium. This measure not only accounts for cyclical changes in the economy, but in natural resource industries, changes in the level and mix of resource availability.

The implicit costs of disequilibria are represented by the difference between the shadow price of the quasi-fixed factor (say capital), P_z^* , and the market rental price, P_z . When the capital stock is inadequate relative to demand, $P_z^* > P_z$; that is, the valuation of an incremental unit of capital stock is high, or conversely, the opportunity cost in terms of restricted profit foregone of having too low a capital stock is high. Alternatively, when $Z^* > Z$ and $P_z^* < P_z$, the marginal unit of capital has a low valuation relative to its market value. This is an opportunity cost of too high a level of capital. The economic CU measure will exceed one when $Y > Y^*$ (and thus $P_z^* > P_z$) and will fall short when the reverse holds (Morrison 1985).

Morrison's (1985) specification of the dual shadow-value measure of CU adapted for the long-run profit function becomes⁶

$$(12) \quad 1 + \frac{Z[P_z^* - P_z]}{HR}$$

With a homothetic technology (so that the scale impact of all inputs is the same) and nonconstant returns to scale, the measure $(1/S_M)(1 + Z[P_z^* - P_z]/HR)$ can also be calculated, where S_M is the measure of ray scale economies calculated at Z . This measure provides the savings at Y from increasing Z to a steady-state level.

The multiproduct profit function measure of economic capacity utilization allows product levels to be endogenous. The output levels determined by the restricted profit function are endogenous and conditional upon the existing stock of capital, Z , and are thus not the output levels to which Z will adjust in reaching a steady state. Morrison notes, however, that this is not a problem for CU calculation, since CU is a short-run notion, so that the existing output levels remain the valid short-run levels for comparative purposes.

Empirical Analysis

This section specifies a translog profit function for estimation, discusses the data, and reports the long-run structures of multiproduct costs and technology.

Empirical Specification

Otter trawlers are fishing vessels which drag a net at the stern or side of the vessel. Otter trawlers often harvest multiple species with the levels and mix of catch as decision variables of the firms. Outputs are endogenous because vessels choose species and locations to target. These multiple products are produced by organizing fuel, labor, and capital (vessel, engine, gear, and equipment). Fishing firms are therefore multiproduct firms producing a vector of endogenous products from a vector of endogenous inputs. This endogeneity suggests that a multiproduct profit function rather than a cost or revenue function is the appro-

⁶ Alternatively, the CU measure (12) can be interpreted as a ratio of short- and long-run ray returns to scale (Morrison, in press).

prate dual representation of the firm's production technology. Moreover, the large number of vessels in the industry, presence of important auction markets, homogenous products, and minimal vertical and horizontal integration among firms assure exogenously determined product prices. Input prices are exogenous because inputs are traded on regional or even national markets.

The translog multiproduct restricted profit function is specified as a second-order Taylor's series approximation by:⁷

$$(13) \quad \ln HR = A_0 + A_T T + \sum_i A_i \ln P_i \\ + \sum_k A_k \ln Z_k + \sum_i \sum_j A_{ij} \ln P_i \ln P_j \\ + \sum_k \sum_l A_{kl} \ln Z_k \ln Z_l \\ + \sum_i \sum_k A_{ik} \ln P_i \ln Z_k + \sum_i A_{iT} \ln P_i T,$$

where T is an index of time, and without loss of generality, symmetry is imposed by $A_{ij} = A_{ji}$ for $i \neq j$ and $A_{kl} = A_{lk}$ for $k \neq l$, and $A_{ik} = A_{ki}$ for $i \neq k$.

The conditional revenue and cost-share equations obtained by Hotelling's lemma are

$$(14) \quad \frac{\partial \ln HR}{\partial \ln P_i} = \frac{P_i Y_i}{HR} = A_i + A_{iT} \\ + \sum_j A_{ij} \ln P_j + \sum_k A_{ik} \ln Z_k, \quad \forall i \in M, N,$$

which are positive for outputs and negative for inputs. Linear homogeneity in prices is imposed on the multiproduct restricted profit function by the restrictions: $\sum_i A_i = 1$, $\sum_i A_{ij} = \sum_j A_{ij} = \sum_i A_{ik} = \sum_k A_{ik} = \sum_i A_{iT} = 0$. All *ex ante* expectations are assumed realized *ex post*.

This study specifies three species groups

⁷ All functional forms for multiproduct functions are limited. The global approximation capacity of the Fourier form is attractive (especially for tests of cost subadditivity), but typically requires a prohibitively large number of estimated parameters. The large regular region and ability to impose many nonnegative sign restrictions before loss of flexibility make the minflex and full Laurent forms attractive but similarly require an extensive number of observations. The generalized McFadden allows easy imposition of convexity, but is not easy to estimate. Lopez (1985a) demonstrates that flexible forms linear in profits a priori impose quasi-homotheticity, input-output separability in multiproduct technologies and additive separability in inputs. The forms using a Box-Cox transformation allow for linear profits and assume a particular form of non-normal disturbances prior to transformation.

($M = 3$), roundfish (cod and haddock), flatfish (yellowtail and other flounders), and a residual, all others; roundfish and flatfish are formed by division indices. Two variable inputs are specified ($N = 2$), energy (fuel and oil) consumption and labor (including captain). One quasi-fixed factor ($F = 1$) is specified, capital, represented by the vessel's gross registered tonnage (GRT). Short-run economic profit is therefore total revenue less the opportunity cost of labor and energy costs; T represents a dummy variable for 1981, where the base period is 1980. Resource abundance is specified as a technological constraint because it is beyond the control of any individual firm but nevertheless affects the production environment within which the firm operates. That is, capital, labor, and energy are organized by firms and applied to the natural resources. Changes in resource abundance may then be viewed as shifts in the production technology that relates the generation of outputs from inputs, i.e., changes in an efficiency parameter (McFadden). These changes are represented by the 1981 dummy variable.

The restricted profit function (13) is jointly estimated with the restricted revenue and cost share equations (14), and all equations have additive disturbances due to errors in optimization. The restrictions for symmetry and linear homogeneity in prices are directly imposed. Because the restricted share equations sum to unity, the energy consumption equation is dropped and its parameters identified through the linear homogeneity and symmetry restrictions. The system of equations is estimated by the iterative seemingly unrelated regression technique. The estimates are consistent, asymptotically normal, equal to maximum likelihood estimates, and are invariant to choice of deleted equation.⁸

The balanced panel data set consists of annual observations for two years, 1980 and 1981, on forty-two full-time otter trawl vessels with at least 85 days absent from port in each year. Home ports are in all of the major and most minor New England ports. Home ports are assigned by a plurality of days absent. The range of sample space almost completely en-

compasses the population of full-time vessels in this sector. The mean sample vessel is 120 gross registered tons (GRT), has a crew size of five, and was built in 1972. The mean of the sample vessel's days absent from port is 167, with a range of 85 to 249. The average vessel makes an average of fifty-seven trips per year of three days' duration (with a range of 1 to 13 days).

The revenue, landing, vessel, and trip characteristics data are from the National Marine Fisheries Service (NMFS) Weighout File. Fuel and oil costs are from federal income tax returns. Most vessel acquisition prices (hull, engine, gear, and equipment) are without measurement error because they are compiled from bills-of-sale; the remainder are from federal income tax returns. Both new and used vessels are included, but only those purchased between late 1976 and 1979 are used in order to eliminate effects of vintage and structural changes in the fishery from the Fishery Conservation and Management Act of 1976. All data are confidential. Because the fuel cost and vessel acquisition price data are from NMFS capital construction fund or guaranteed loan program participants, the sample is likely to reflect some of the more successful and newer vessels.

Returns to captain and crew are determined after each fishing trip by the vessel lay system, which normally yields payments as a percentage of gross or net trip revenue. By this system, net trip returns are apportioned by formulas which can vary by port and sometimes vessel. In this study, returns to labor are assigned an exogenous economic valuation through use of the opportunity cost of labor. This provides an exogenous representation of returns to labor and food costs per person.⁹ The opportunity cost of ordinary crew members is the mean annual wage of total manufacturing, the opportunity cost of a vessel engineer is the mean annual wage of a maintenance mechanic, machinery, and the opportunity cost of the captain is a mean annual wage rate 20% higher than an ordinary seaman's to reflect the captain's entrepreneurial and managerial skills. Crew members are assigned to one of five New England coastal manufacturing cities (Portland, Gloucester, Boston, New Bedford, and Provi-

⁸ The approach illustrated by McKay, Lawrence, and Vlastuin estimates the variable and fixed cost share equations by the Zellner technique, which requires all the regressors to be exogenous. Since the dependent variables of the fixed input share equations include the fixed inputs, (iterative) three-stages least squares may be more appropriate.

⁹ The opportunity cost approach to valuing labor has been used by Clark and Munro for marine fishing industries and Lopez (1984) for family farm labor.

Table 1. Parameter Estimates of Translog Restricted Profit Function

Exogenous Variables	Product Shares			Factor Shares		Profit Function
	Roundfish	Flatfish	All Others	Labor	Fuel ^a	
Share intercept	-4.486 ^b (1.637) ^c	2.512 (1.491)	-0.208 (0.639)	0.434 (0.623)	2.749 ^b (1.337)	10.7375 ^b (3.346)
1981 share dummy	-0.071 (0.084)	-0.002 (0.069)	0.184 ^b (0.080)	-0.064 (0.061)	-0.047 (0.039)	0.617 (0.608)
Roundfish	-0.252 (0.207)	-0.155 (0.105)	-0.115 ^b (0.058)	0.275 (0.145)	0.247 ^b (0.124)	
Flatfish		0.233 (0.207)	-0.051 (0.045)	-0.052 (0.128)	0.025 (0.102)	
All others			0.097 ^b (0.035)	0.036 (0.042)	0.033 (0.026)	
Labor	(symmetric)			-0.048 (0.049)	-0.211 (0.310)	
Fuel					-0.093 (0.332)	
Gross registered tons	0.426 ^b (0.075)	-0.319 ^b (0.057)	0.010 (0.086)	0.016 (0.053)	-0.134 ^b (0.033)	0.296 (0.781)
Gross registered tons squared						0.035 (0.063)

^a Parameter estimates derived from the constraints implied by linear homogeneity and symmetry. Standard errors are computed as first-order Taylor's series approximations.

^b Statistically significant at 5%.

^c Standard errors are in parentheses.

dence) by geographical proximity to their home ports. The opportunity cost of labor is a divisia index of the separate opportunity costs for crew, engineer, and captain which varies by port and crew size. Labor cost data are from the U.S. Department of Labor, Bureau of Labor Statistics, and from comparable state agencies. All profits are therefore economic profits. Energy costs include all sales taxes, while energy prices are port-specific. The capital services price is comprised of the opportunity cost of capital and depreciation. Because all restrictions are directly imposed upon the profit function, prices are deflated by the gross national product (GNP) implicit price index.

Empirical Results

The estimated parameters of the translog multiproduct restricted profit function are presented in table 1. The systems R^2 (Baxter and Cragg) is 0.99, while the OLS R^2 s for the share equations are 0.66 for roundfish, 0.51 for flatfish, 0.30 for all others, and 0.08 for labor. The predicted share equations are consistent with monotonicity over all sample values. The restricted profit function is not convex, although this is not a statistical test. A test of convexity cannot be interpreted as strictly a test of profit maximization because convexity can be violated for a number of other reasons.

For example, Wales shows that the estimates of a flexible functional form may violate convexity even if the data come from a perfectly well-behaved technology. Linear aggregation of the other species assemblage can also cause the apparent failure of convexity.

The optimal capital stock, Z^* , represented by the vessel's gross registered tons (GRT), can be solved from the equation $\nabla_Z HR = P_Z$ using the 1980 arithmetic sample mean real service price per ton (GRT) and evaluated at the point of approximation. Because a closed form analytical solution is not possible with the translog function, a numerical solution is required. The solved value for mean Z^* in the open-access fishery is 99.96, while the observed sample mean value is 120.38.

The divergence between observed Z and estimated Z^* might simply reflect sampling error in the estimated Z^* . A number of hypothesis testing procedures for static equilibrium are possible. Bootstrap and jackknife procedures can be used. Schankerman and Nadiri apply Hausman's test for specification error in a system of simultaneous equations. Kulatilaka applies the delta method to obtain a first-order Taylor's series approximation for the variance of Z^* to form a t -test. Kulatilaka's test in quantity space is

$$(15) \quad t = (Z^* - Z)/[V(Z^*)]^{1/2},$$

Table 2. 1980 Long-Run Marshallian Elasticities

Price	Roundfish	Flatfish	All Others	Labor	Fuel	Capital
Roundfish	3.36	0.06	2.36	1.77	3.87	3.88
Flatfish	0.03	0.01	0.13	0.54	0.09	-0.32
All others	0.61	0.05	-0.06	0.35	0.22	0.54
Labor	-0.75	-0.37	-0.58	-1.56	-0.64	-0.73
Fuel	-1.06	-0.04	-0.68	-0.14	-1.18	-1.14
Capital	-2.19	0.29	-1.17	-0.97	-2.36	-2.23

which is t -distributed, $V(Z^*) = Z^*_A V(A) Z^*_A'$, A is the vector of estimated parameters and therefore a random variable, Z^*_A is a vector of partial derivatives of Z^* with respect to A , and $Z^*_A = -[HR_{ZZ}]^{-1} HR_{ZA}$ evaluated at Z^* .

The estimated t is -0.80 , implying no statistically significant difference between Z and Z^* . This test result may reflect a robust level of Z^* in an industry in which fishermen make long-run investment decisions with expectations of important cyclical and stochastic temporal and spatial fluctuations in resource abundance (Gold). Vessels of a certain size and design are also required to harvest different fishing grounds in different seasons and in the stormy northwest Atlantic waters. The lumpiness and long life of capital in marine fishing industries may further contribute toward the robust Z^* . Moreover, important used and new vessel markets, vessel leaseings, and deliberate vessel sinkings for insurance exist, and otter trawlers are mobile and can easily switch to a gear, location, or targeted species other than those of the owner's original intentions.¹⁰

Table 2 reports that long-run Marshallian elasticities evaluated at the 1980 arithmetic sample mean and $Z^* = Z = 120.38$ GRT.¹¹ The own-price supply elasticities all have the expected algebraic signs except for all others.¹² The own-price supply elasticities for

flatfish and all others are both inelastic, while that for roundfish is quite elastic. All cross-price supply elasticities are positive, indicating complementarity, and are inelastic except in one instance. The inelasticities and complementarities reflect the mixed species nature of the groundfishery, the somewhat limited capability of the otter trawl gear and electronics to target different species, and the distinct spatial distribution of many bottom-dwelling finfish species. Search costs in the form of energy consumption, risk, quality deterioration for some species, and opportunity costs of catch foregone and labor also limit harvesting responsiveness to changes in species prices. The marginal search costs quickly outweigh the incremental revenues obtained.

The long-run factor demands are all elastic and negative as expected. The inputs are all Marshallian complements, most with inelastic cross-price elasticities, perhaps caused by expansion effects outweighing pure substitution effects. Sakai indicates that all inputs can be Marshallian complements because of expansion effects. The elastic long-run factor demands suggest that fishermen are very responsive to fuel price and interest rate shocks such as those of the preceding decade.

The long-run elasticities between inputs and outputs display the expected signs, with increases in product prices leading to increased factor demand and increases in factor prices inducing decreased product supply. These elasticities for flatfish and all others are usually inelastic. They are elastic for roundfish, reflecting the pivotal importance of the ubiquitous cod and haddock as the traditional mainstays of the fishery. Moreover, the generally elastic output responses for capital suggest that the species composition may be particularly impacted by changes in interest rates.

¹⁰ Although the 20% difference in Z^* and observed Z is not statistically significant, it may nonetheless indicate a 20% overcapitalization of the average firm. The interpretation is consistent with the overcapitalization expected to be found in an open-access fishing industry.

¹¹ The long-run Marshallian elasticities change only minimally when evaluated at $K^* = 99.96$ GRT. Because the observed levels of the quasi-fixed input coincide with the desired levels, the restricted and full static equilibrium models are equivalent and there is thus no need to estimate a full static equilibrium model.

¹² The negative sign for all others may be due to aggregation bias, since more than 50 different minor species are linearly aggregated over all trips in each year for each vessel. Linear aggregation technologies imply perfect transformation among outputs. This may be the factor which leads to the absence of convexity. Alternatively, since all measures of incremental marginal costs and product-specific economies of scale are evaluated at the long-

run, optimal levels of outputs and inputs, the increasing product-specific returns to scale for all others and declining average incremental costs imply a declining supply curve for the other species assemblage. This may be caused by a mining down of the resource stock, which includes species that are at high abundance.

The long-run measures of local cost-complementarity evaluated at the 1980 arithmetic sample mean for the observed capital stock are 33.78 for roundfish-flatfish, 1.54 for roundfish-all others, and 21.23 for flatfish-all others. Economies of scope do not exist between any of the product pairs. This absence may be due to the spatial distribution of different fish stocks, the importance of fishing skill in harvesting, and the quasi-public but lumpy nature of fishing boats. Long-run incremental marginal costs are 0.35 for roundfish, 204.08 for flatfish, and -100.00 for all others. These results suggest the presence of decreasing product-specific returns to scale for roundfish and flatfish and increasing product-specific returns to scale for all others. Cost advantages exist to harvesting additional species in this latter category, and firms specializing in harvesting these species are likely (vessels with the Point Judith, Rhode Island cooperative provide an example). Evaluation of the hessian matrix for outputs of the long-run cost function does not indicate convexity.

The 1980 estimate of long-run overall returns to size is 0.65, indicating decreasing ray returns to scale. Cost or profit advantages do not exist to expanding production if product proportions are held constant when all prices and resource abundance are fixed. As the scale of production expands for a given level and mix of the resource stock, vessels harvest in increasingly marginal fishing grounds, further deplete the fish stocks in existing grounds, harvest in more adverse weather, and so on. The absence of economies of scope throughout the product set contributes toward the decreasing ray returns. Moreover, Gold states that "materials dominated" processes are those in which the controlling constraint on output capabilities is the richness of the natural resources utilized or processed, such as the population density of fishing grounds. It may well then be that the measure of long-run overall (and product-specific) returns to scale is altered by changes in the composition and level of resource abundance and density; Kirkley provides empirical support for this thesis with a multiproduct revenue function.

Since the existing capital stock Z is found to be at the optimal long-run level Z^* , the economic CU measure is one. Partitioning observed productivity changes into potential or long-run productivity growth and the effects of temporary equilibrium would not be necessary for any productivity measures. The im-

portance of testing divergences from full static equilibrium are also apparent.

Concluding Comments

The restricted multiproduct profit function in conjunction with the envelope condition allows a long-run analysis of the structures of production and costs without prior assumptions of full static equilibrium and exogenous products. A recent test for economies of scope is also shown to hold only in the short term and may be difficult to implement even with restricted cost functions. Long-run models are also shown to have certain advantages over dynamic models in many potential applications. Measures of economic captivity utilization in multiproduct industries are also shown to be generally more suitable from profit functions and likely to be biased if determined from single-product cost functions. It is also important to test for economic CU measures diverging from one.

The framework is developed for vessel-level data from the New England otter trawl fleet. Product supply elasticities are typically inelastic, and most species are complements in harvesting. Factor demand elasticities are both elastic and inelastic, and all inputs are Marshallian complements. Economies of scope and product-specific economies of scale are generally absent and overall returns to scale are decreasing. Not surprisingly, the industry is primarily composed of single-vessel firms with minimal horizontal and vertical integration. The long-run cost function is also not convex with respect to products.

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