

Index Numbers and Productivity Measurement in Multispecies Fisheries: An Application to the Pacific Coast Trawl Fleet

DALE SQUIRES

*Southwest Fisheries Center
National Marine Fisheries Service, NOAA
P.O. Box 271
La Jolla, California 92038*

ABSTRACT

This study is concerned with the measurement of total factor productivity in the marine fishing industries in general and in the Pacific coast trawl fishery in particular. The study is divided into two parts. Part I contains suitable empirical and introductory theoretical material for the examination of productivity in the Pacific coast trawl fleet. It is self-contained, and contains the basic formulae, empirical results, and discussion. Because the economic theory of index numbers and productivity is constantly evolving and is widely scattered throughout the economics literature, Part II draws together the theoretical literature into one place to allow ready access for readers interested in more details.

The major methodological focus of the study is upon the type of economic index number that is most appropriate for use by economists with the National Marine Fisheries Service. This study recommends that the following types of economic index numbers be used: chain rather than fixed base; bilateral rather than multilateral; one of the class of superlative indices, such as the Tornqvist or Fisher Ideal.

PART 1

Introduction

Purpose of study

Measurement of industry productivity is important to planning, public regulation, and monitoring industry performance over time. Yet, with the exceptions of Bell and Kinoshita (1973), Norton et al. (1985), Kirkley (1984), and Duncan (undated), little attention has been given to measuring productivity in marine fishing industries. Moreover, these initial studies can be extended in scope and methodology to draw upon recent advances in the economic theory of index numbers and productivity measurement.

This study addresses these issues and has five explicit purposes. First, the study applies recent advances in the theory of index numbers to the Pacific coast trawl fleet. Second, an informal evaluation is made of the data sources available for productivity measurement of this fleet. Third, many different index-number procedures are available, and this study evaluates the most important and widely used procedures. Fourth, the interpretation of productivity is unclear for marine fishing industries, and this study clarifies this issue. Fifth, the economic theory of index numbers is constantly evolving and is widely scattered throughout the literature. This study draws this literature together to make it more accessible to other applied economists.

The productivity indices are developed for U.S. vessels of the Pacific coast trawl fleet homeported in Washington, Oregon, and northern and central California over the 1981-85 period. These vessels employ bottom, shrimp, and midwater trawl gear to harvest numerous species of groundfish and pink shrimp which are sold to domestic shoreside processors or to foreign processors participating in joint venture operations.

Applications of productivity measures

Productivity measures, used as indicators of relative economic performance in fisheries, portray fishery trends and problems. Productivity measures can be an effective means to monitor the economic performance of a fishery. Only by accurate knowledge of a fleet's performance over time can effective policies be designed. For example, declines in fleet productivity could signal a need for government assistance or regulation. Alternatively, productivity declines after years of government assistance could imply that public resources committed to the fishery have been dissipated through overcapitalization. Government support might be either scaled back or refocused to foster efficiency without encouraging increases in input usage. If public regulation of fisheries is to be concerned with net economic and social benefits, regulators should be aware of changes in productivity and in resource stock levels.

Small or negative productivity gains in fishing industries can be associated with lagging profits, returns to labor, and employment, because fishermen must compete with foreign fishery imports and other protein substitutes, such as meat and poultry, where productivity is a main component of competitive advantage. Rising productivity in the fishery sector can also help mitigate inflationary pressures in fish products, because products can be supplied at declining costs over time. In turn, this helps to maintain the competitive capability of fisheries in relationship with its close substitutes such as meat and poultry (Bell and Kinoshita 1973).

International comparisons of fishing fleet productivity can help clarify differences in international competitiveness of fishing industries. Decline in a fishery's productivity coupled with rising imports of the species being harvested could suggest a need for corrective action by industry bodies or government. Declines in productivity coupled with a structural shift in consumption patterns toward more fish could signal an increase in imports or a decrease in exports in the future.

Regional differences in productivity can be linked to the geographical distribution of productive resources within the fishery. For example, the empirical results presented later in this report show that in 1983 trawl fleet capital declined in Washington while it increased in northern and central California. Moreover, Washington and Oregon experienced positive growth rates in fleet total-factor productivity over the time period 1981-85, while northern and central California experienced declines. Public regulation might become more effective by adopting a more explicitly regional-based approach.

The Pacific trawl fleet

The Pacific trawl industry off California, Oregon, and Washington is composed of several different commercial gear and vessel types harvesting a wide array of species. The most important harvested species include pink shrimp, Dungeness crab, flatfish (Dover, English, petrale, rock, and rex sole), roundfish (sablefish, Pacific cod, ling cod, and Pacific whiting), the *Sebastes* complex (yellowtail, canary, widow, bocaccio, chillipepper, and shortbelly rockfish), and thornyheads.

The contributions of each species and species assemblage to total revenue are provided in Table 16. This table shows the proportions or shares of total regional exvessel revenue received for each species assemblage in each year. Pink shrimp, rockfish, and Dover sole consistently provide the highest proportion or share of total revenue. The contributions of each region to total revenue, or revenue shares, are reported in Table 17. Oregon generally provides about 45% of the total revenue, Washington around 22%, northern California around 20%, and central California the balance. The species' and regions' revenue shares are not static, but change over time.

Three separate trawl fisheries exist for pink shrimp, groundfish, and midwater species. Gear switching occurs, most notably between otter and shrimp trawls and between otter and midwater trawls. Huppert and Korson (1987) note that smaller inshore trawlers in the Crescent City, California, area also shift among target species by harvesting Dungeness crab in the winter and otter trawling for soles and rockfish during the fall and spring. The midwater trawl fishery harvests Pacific whiting in joint ventures and widow rockfish for shoreside landings. For a brief period in 1982, midwater trawlers caught shortbelly rockfish. Many vessels that participate in the pink shrimp and groundfish fisheries have insufficient horsepower to operate the midwater trawls.

Increased profits and the availability of vessel financing led to a significant expansion of the shrimp fishery in the late 1970s (Deweese 1986). Since the early 1980s, however, the shrimp fishery steadily declined in importance, dropping from 19,923 short tons worth \$20 million in 1981 to 4,814 short tons worth \$3.81 million in 1984. A resurgence began in 1985, with landings rising to 12,779 short tons worth \$7.61 million (all dollars are \$1981 values) (Table 1). The contribution of shrimp to regional exvessel revenue declined from 36% and 46% of total 1981 revenue in Washington

and Oregon, respectively, to 13% and 9% in 1984, before rebounding to 29% and 22% in 1985 (Table 16).

During the early 1980s, the groundfish fishery continued the expansion begun following the Magnuson Fisheries Conservation and Management Act of 1976. Table 2 indicates the number of coastwide otter and shrimp trawl vessels in the fishery between 1981 and 1985. Some of the otter trawl vessels were formerly pink shrimp trawlers that switched to otter trawling after the decline in pink shrimp catches and abundance. The decline in the number of otter trawl vessels during 1983 and 1984 can be attributed, in part, to declining rockfish stocks (particularly widow rockfish), continual decline in real groundfish prices, and the effects of high interest loans taken out to finance the fleet's expansion. Some of the larger vessels transferred their operations to Alaska, while other vessels have sunk, burned, been repossessed, transferred to other fisheries, or simply tied up due to financial difficulties.

Otter-trawl groundfish landings in Washington, Oregon, and northern and central California during the 1980s have declined from a 1982 peak of 113,492 short tons valued at \$44.33 million (1981 constant dollars) to 79,938 short tons worth \$32.26 million in 1984, before increasing slightly in 1985 to 82,988 short tons worth \$34.86 million (Table 1). Although the number of vessels, tonnage landed, and total revenue all peaked in 1982, the total frequency of landings (i.e., the number of fish receipts recorded) reached a high one year later, with 15,436 landings (Table 4).

The midwater trawl fishery developed in the late 1970s as joint ventures (JV) with foreign fishing companies. With the exception of limited fishing for shortbelly rockfish in 1982, the joint-venture fishery targets Pacific whiting almost exclusively. This fishery occurs primarily in the summer months. Additional sources of revenue for these vessels include widow rockfish and JV operations in Alaska. Annual landings and revenue by Pacific whiting JV vessels were stable during 1982-85 at around 80,000 short tons valued at around \$10 million (\$1981 dollars) before declining in 1985 to 34,934 short tons worth \$3.2 million (Table 1).

The Pacific trawl fleet has undergone a rapid modernization since 1976. Dewees (1986) examines the rates of adoption of eight technological innovations during this period. The innovations ostensibly contributing most to increases in the technical capabilities (or "fishing power") of trawlers have been the development of midwater trawling, chromoscopes, sonar, and track plotters. Midwater trawling represents a major change in the harvesting process. Midwater trawlers have shown a greater reliance on these electronics since otter trawlers can still function satisfactorily without them.

With other factors affecting productivity held constant, these technological innovations should increase productivity in the Pacific trawl industry. Benefits from technological progress can be realized, however, only if productivity gains are not dissipated through overcapitalization in the fishery. Moreover, the rate of technical progress should be comparatively high in open-access fisheries because of the intense competition among fishermen to harvest limited fish stocks.

In recent years, most of the commercial groundfish stocks have been harvested at or near the maximum sustainable yields (MSY) estimated by the Pacific Fishery Management Council's Groundfish Management Team. Dover sole, Petrale sole, other flatfish, Canary, yellowtail, and widow rockfish, ling cod, Pacific ocean perch, and sablefish in particular are all harvested at levels close to, or surpassing, MSY (PFMC 1985; Huppert and Korson 1987). The Pacific whiting catch remains well below its estimated MSY. The pink shrimp resource is somewhat ephemeral and receives intensive exploitation (Korson 1984).

Changes in resource abundance and composition will affect productivity in marine harvesting industries. With the fishery stocks being harvested at levels close to or at their MSYs in the Pacific coast trawl fisheries, economic index measures of fleet productivity are likely to remain unchanged or exhibit downward movements over 1981-85, reflecting constant or declining levels of stock abundance.

Methodological background

This section provides an introduction to the methodology of productivity measurement and the theory of economic index numbers. Productivity is first defined, then the concept of economic index numbers is discussed, followed by discussions of different index-number formulae, chain and fixed-base indices, and bilateral and multilateral indices. Readers interested in additional methodological issues can refer to Part 2 and its Appendices.

Productivity defined

Productivity is traditionally used to explain the physical output per unit of input. Higher productivity means that more can be produced with the same bundle of inputs or, conversely, that the same output bundle can be produced from fewer inputs.

Historically, productivity measurement focused upon one factor, such as output per unit of capital or output per man-hour (Bell and Kinoshita 1973). These partial productivity measures may provide misleading results, since output increases may arise from the increased use of other inputs or changes in capacity utilization. This limitation to partial productivity has led to emphasis upon total-factor productivity.

Dividing the level of production (total output) by an index of all inputs creates an index of total-factor productivity. Properly constructed, the total-factor productivity index accounts for all changes in the quantities of inputs. Variation in the total-factor productivity index tracks the productivity residual which is not accounted for by changes in the volume of economic inputs. With this introduction, the concept of total-factor productivity is now rigorously developed.

Growth-accounting framework

The standard framework for estimating productivity change is derived from the theory of production. Consider the following one output-two input production function:

$$Y(t) = A(t)f[K(t), L(t)], \quad (1)$$

where $Y(t)$ denotes total landings at time t , $K(t)$ denotes the flow of capital services at time t , $L(t)$ is the flow of labor services used at time t , and $A(t)$ is an efficiency parameter allowing for shifts in the production function. The production function defines the maximum output achievable with the given quantity of inputs, $L(t)$ and $K(t)$, and is determined by the state of technical knowledge and resource abundance, $A(t)$. Total landings can grow from several sources: (1) as existing firms expand their input usage, (2) as new firms enter the industry, and (3) as technology advances and resource abundance increases, causing shifts in the aggregate production function.

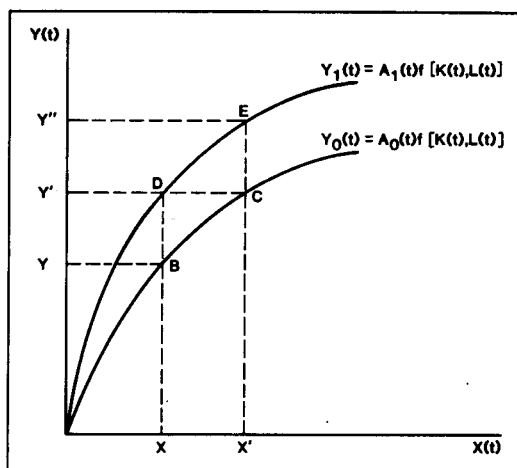


Figure 1
Production function representation of total-factor productivity.

Intuition into the meaning of productivity is provided by Figure 1. Two different levels of the production function in equation (1) are presented: $Y_0(t) = A_0(t)f[K(t), L(t)]$ and $Y_1(t) = A_1(t)f[K(t), L(t)]$, where $Y_1(t) > Y_0(t)$. The vertical axis represents different catch or output levels, where $Y' > Y$. The horizontal axis represents different levels of an index of aggregate input, X , where $X' > X$. When the state of technical knowledge and resource abundance both remain constant but a larger quantity of inputs is used to harvest fish, $X' > X$, and firms move along the existing production function, $Y_0(t) = A_0(t)f[K(t), L(t)]$, from point B to point C. Firms harvest more fish by using more capital and labor, and total catch increases from $Y(t)$ to $Y'(t)$. Total catch can also increase when technological innovations are adopted by the fleet, even if the same amount of inputs is used and resource abundance remains constant. In this case, the state-of-technology index increases from A_0 to A_1 , and the production function shifts from $Y_0(t) = A_0(t)f[K(t), L(t)]$ to $Y_1(t) = A_1(t)f[K(t), L(t)]$. At a constant input bundle X , the shift upwards of the production function is denoted by moving from point B to point D and catch increases from Y to Y' . Fishing firms can now harvest a larger catch, Y' , with the same level of inputs, X . Economists say that fishing firms are now more productive.

The basic problem of productivity analysis is to use data on the prices and quantities of inputs and outputs to allocate the growth of $Y(t)$ among the growth rates of $K(t)$, $L(t)$, and $A(t)$. The growth-accounting framework used in this study proceeds nonparametrically by first taking logarithms of equation (1) and then logarithmically differentiating equation (1) with respect to time. The logarithmic differential of (1) can be written as:

$$\begin{aligned} d \ln Y(t) / dt &= (dY/dt)(1/Y) & (2) \\ &= [\partial \ln Y(t) / \partial \ln K(t)] [d \ln K(t) / dt] \\ &+ [\partial \ln Y(t) / \partial \ln L(t)] [d \ln L(t) / dt] \\ &+ [\partial \ln Y(t) / \partial \ln A(t)] [d \ln A(t) / dt] \end{aligned}$$

where $\partial \ln Y(t)/\partial \ln A(t)$ is set equal to unity because it is a technology-shift parameter, $\partial \ln Y(t)/\partial \ln K(t) = [\partial Y/\partial K] [K/Y]$ is the output elasticity for capital (the percentage increase in output with a 1 percent increase in capital), denoted E_K , and $\partial \ln Y(t)/\partial \ln L(t) = [\partial Y/\partial L] [L/Y]$ is the output elasticity for labor (the percentage increase in output with a 1 percent increase in labor), denoted E_L .

The logarithmic derivatives are interpreted as rates of growth so that the rate of output growth becomes:

$$Y^*/Y = E_K K^*/K + E_L L^*/L + A^*/A, \quad (3)$$

where the asterisk * denotes time derivatives. The rate of output growth is thus allocated among growth in capital and labor, technical progress, and changes in resource abundance.

Because E_K and E_L are unobservable, equation (3) cannot be used for empirical analysis. One further step is required. Assuming that inputs are paid their value of marginal product:

$$\partial Y(t)/\partial K(t) = P^K(t)/P(t), \quad \partial Y(t)/\partial L(t) = P^L(t)/P(t), \quad (4)$$

where $P(t)$, $P^K(t)$, and $P^L(t)$ are the prices of output, capital services (rental price), and labor services (wage rate), respectively. Substituting (4) into (3) gives:

$$Y^*/Y = S_K K^*/K + S_L L^*/L + A^*/A. \quad (5)$$

Because in competitive open-access equilibrium, industry profits are dissipated (Gordon 1954) and firms display locally constant returns to scale (Baumol et al. 1982), total costs equal total revenue, and any input's cost share equals its revenue (or income) share. Therefore, $S_K = [P^K K]/[PY]$ and $S_L = [P^L L]/[PY]$, that is, the cost (equals revenue) shares of capital and labor. Given (3), these shares are equal to the production elasticities E_K and E_L . These shares provide weights to the growth of capital and labor over time.

The final step is to rearrange equation (5) to give:

$$A^*/A = Y^*/Y - S_K K^*/K - S_L L^*/L. \quad (6)$$

Productivity in fisheries, that is, technical progress and change in resource abundance, is therefore measured as the residual of output growth after accounting for the growth of inputs. Intuitively, output grows over time as inputs increase and is reflected in movement along the frontier of the industry production function, while technical progress and changes in resource abundance cause shifts in the production function. The residual (6) thus is a measure of production function shift, and is called the productivity residual.

Tracking the total-factor productivity index for a fishing industry therefore provides information on technical progress and changes in resource abundance of exploited fish stocks. Because the productivity index is measured as a residual in equation (6), changes in productivity might also include changes in the economic efficiency of the individual fishing firms, altered fishing regulations, variations in economic capacity utilization, or variations in exogenous conditions like weather.

The effects of changing resource abundance can be disentangled from the productivity residual. This topic is the subject of current research, and will be discussed in a future report.

After defining and clarifying the issue of productivity in marine fishing industries, attention must be turned to measuring productivity in some way. Economic index numbers have been developed by economists for tasks such as productivity measurement. The next

section provides a brief introduction and survey of this important topic, while readers interested in additional details can refer to Part 2.

Economic index numbers

Productivity, production, and input use are more effectively measured by economic index numbers than by physical measures. Physical measures (e.g., total catch per hour towed) fail to distinguish changes or differences in composition or quality over time and space, or between fishing firms. Simply lumping together total tonnage of catch in one time period and comparing with total tonnage from a subsequent time period neglects the change in catch composition. Different products are then compared, and the assumption is implicitly maintained that a ton of Pacific whiting, for instance, is perfectly substitutable by a ton of pink shrimp.

Economic index numbers deal with situations in which industry outputs and inputs are too diverse to measure simply by weighing or counting. Economic index numbers provide weighted measures of the different kinds of outputs (species) or inputs (capital, labor, fuel). Shares or proportions of total revenue (revenue shares), for example, can be employed to combine the different outputs into a weighted measure of total output, and shares of total costs (cost shares) can similarly be employed to aggregate different inputs into a weighted measure of total input.

The different outputs (inputs) are combined into weighted measures of total output (input) by functions. These functions are called aggregator functions since they aggregate the individual components (e.g., outputs) into the composite (e.g., total output). Different formulae for the aggregator functions have different implications for the properties of the index numbers formed. The economic theory of index numbers is concerned with these relationships between the properties of index numbers and the properties of the underlying aggregator functions they represent.

Both individual quantities and individual prices can be aggregated into a composite quantity or price. Quantity aggregator functions aggregate quantities of individual outputs or inputs into composite measures of total-output quantity or total-input quantity, while price aggregator functions aggregate prices of individual outputs or inputs into composite measures of total output or input prices.

Index-number formulae

A number of different types of economic indices exist. Each type of index offers an approximate scalar measure of a multidimensional change over time in prices, quantities, or productivity. The different indices approximate these intertemporal changes in different ways, according to their theoretical properties (manifested by their formulae). Differences in indices can be viewed as differences in their abilities to provide approximations to the intertemporal changes in prices, quantities, or productivity.

Consider a concrete example of the way in which the different indices provide different approximate scalar measures of intertemporal changes. Suppose the problem is to measure intertemporal changes in an aggregate output bundle, which in fisheries is the change in total catch over time. One of the most important issues in constructing an economic index number for cases such as these is to account for intertemporal changes in the relative composition of this bundle, that is, the changes in species mix. When output (species) prices change relative to one another, fishermen alter the

individual species (output) composition of their catch (aggregate output). An index number that does not properly incorporate these intertemporal changes in output composition into the aggregate measure becomes increasingly biased over time, that is, the errors in approximation increase.

The different index numbers approximate changes such as these in different ways, and thus have different degrees of accuracy. When intertemporal changes (in productivity or output and input prices and quantities) are relatively small, the different indices all provide reasonably accurate and similar approximate scalar measures of these changes. Typically, the larger the changes over time, the more the measures from different indices diverge from one another. This departure occurs because the indices provide approximations in differing ways.

Four economic index numbers are commonly applied: Laspeyres, Paasche, Tornqvist, and Fisher Ideal. These indices correspond to different methods of approximation (reflected in the formulae of their aggregator functions) with correspondingly different properties. The Laspeyres and Paasche indices have traditionally been widely applied, but the Tornqvist and Fisher Ideal are increasingly used.

Laspeyres and Paasche indices—The Laspeyres and Paasche indices are the most widely used. In forming aggregates, these indices weight individual outputs or inputs with prices or quantities. The Laspeyres index for quantities of inputs or outputs may be written:

$$Q_L = \sum P_i^0 X_i^1 / \sum P_i^0 X_i^0,$$

where P_i^1 and X_i^1 represent the price and quantity of good i in time t . Since prices are held fixed at their base time-period levels, the Laspeyres index indicates how much of the change in value of total quantity resulted from pure quantity changes. A Laspeyres price index can similarly be specified in which quantities, used as weights, are held fixed at their base time-period levels.

The Paasche quantity index for inputs or outputs may be written:

$$Q_P = \sum P_i^1 X_i^1 / \sum P_i^1 X_i^0.$$

In contrast to the Laspeyres quantity index, prices (rather than quantities) are held fixed at their new levels. The Paasche price index holds quantities, used as weights, fixed at their current levels.

In summary, the Laspeyres quantity index weights the individual quantities to be aggregated with base time-period prices; the Laspeyres price index weights with base time-period quantities; the Paasche quantity index weights with current prices; and the Paasche price index weights with current quantities.

The Laspeyres and Paasche indices provide approximations to intertemporal changes which capture only the two most extreme classes of changes in the composition of the aggregate: either perfect or no substitution among the individual elements of the aggregate. (This is because the indices correspond to linear or fixed-coefficient/Leontief aggregator functions.) If, over time, substitution among inputs or outputs occurs, the indices can provide biased measures of the true aggregate either because substitution is not allowed or perfect substitution occurs.

Tornqvist and Fisher Ideal indices—The Tornqvist and Fisher Ideal indices provide more accurate approximations to changes than the Laspeyres or Paasche indices because intermediate substitution

possibilities are incorporated. The individual components (e.g., species) of the aggregate (e.g., total catch) do not have to be either perfect or nonsubstitutes; instead, intermediate substitution possibilities are allowed. This occurs because the prices or quantities from both time periods under comparison enter the index to account for the possible changes in the mix of the inputs or outputs of the index.

The Tornqvist quantity index may be specified:

$$Q_T = II_i [X_i^1/X_i^0]^{0.5} (S_{i1} + S_{i0}),$$

where X_i^k is the value of the i th price or quantity in time k , S_i^k is the share of total revenue (cost) in time k of output (input) i , II denotes natural logarithm, and II_i is the product operator. Revenue or cost shares are used as weights.

The Fisher Ideal index is the geometric mean of the Laspeyres and Paasche indices, and may be written as:

$$Q_F = [\sum S_i^1 (X_i^1/X_i^0) / \sum S_i^0 (X_i^1/X_i^0)]^{0.5}.$$

In order to empirically assess the differences among the Laspeyres, Paasche, Tornqvist, and Fisher Ideal index formulae, this study computes all four types of output indices in each of the four regions and for all regions combined. Part 2 provides additional theoretical discussion on this general topic.

Chain and fixed-base indices

Indices may be formed by either the chain or fixed-base methods. The fixed-base procedure directly compares all changes in prices, quantities, or total-factor productivity to some initial base period. The base period may remain constant or may be changed after some period of time.

Fixed-base indices can be expressed in general form as:

$$P_{0,t} = P_t/P_0,$$

where $P_{0,t}$ represents the fixed-base index comparing price in time t with that of base time 0, P_t is price in time t , P_0 is price in the initial time 0, and the prices are calculated by some index-number formula (e.g., Laspeyres or Tornqvist).

Chain indices directly compare adjacent observations in a sequence of index numbers. Nonadjacent observations are compared indirectly by using the intervening observations as intermediaries. This practice results in transitive comparisons. The general form of the chain index can be written:

$$P_{0,t}^{ch} = P_{01} \times P_{12} \times P_{23} \times \dots \times P_{t-1,t},$$

where each individual term, P_{ij} , is computed by the index-number formula used, and represents the change from time period i to time period j [$i < j$]. $P_{0,t}^{ch}$ thus compares output in time t with output in time 0, the base time period. This formula reflects the basic relationship:

$$P_t/P_0 = P_1/P_0 \times P_2/P_1 \times \dots \times P_t/P_{t-1}.$$

Since all values are represented in terms of the reference period 0 (in this study, 1981), comparisons between adjacent time periods, say i and $i-1$, are achieved with the following formula:

$$P_i = P_{0i}/P_{0i-1}$$

where $0 < i-1 < i < t$.

The values used to weight the individual quantities or prices (aggregated into a composite quantity or price) are kept up-to-date in the chain index, while the fixed-base index compares time periods for which the weights can be very different. As producers change their production patterns in response to changes in relative price, fixed-base indices maintain weights which may have changed and are no longer representative of current output or input mixes. Chain indices are generally preferred on *a priori* grounds for these reasons.

This study empirically assesses the fixed-base and chain indices for total output in each region and the entire fleet. Additional methodological discussion is provided in Part 2 for interested readers.

Bilateral and multilateral indices

Two basic types of indices can be used, bilateral and multilateral. Bilateral indices provide intertemporal comparisons of total-factor productivity (TFP) for any region or interspatial comparisons between regions for any given time period. Because of the large number of possible binary combinations which are not necessarily transitive, bilateral indices are inappropriate for comparisons that are not binary (e.g., TFP in region i in time t with TFP of region j in time $t+1$).

The Tornqvist bilateral index of total-factor productivity can be written:

$$\ln TFP_k - \ln TFP_l = \sum_i 0.5 (R_{ik} + R_{il}) [\ln Y_{ik} - \ln Y_{il}] - \sum_i 0.5 (W_{ik} + W_{il}) [\ln X_{ik} - \ln X_{il}]$$

where k and l are adjacent time periods (or regions), the Y_{ij} are output indices for output i of time j , the X_{ij} are input indices for input i of time j , the R_{ij} are product revenue shares, and the W_{ij} are input cost shares.

Multilateral indices have been proposed by Caves et al. (1982a) to provide transitive comparisons in a multilateral setting. Transitive comparisons are achieved by making all possible binary comparisons in terms of the geometric mean of all observations. For example, any two regions in different time periods are compared with each other by comparing both with the geometric mean.

Multilateral indices directly compare adjacent and nonadjacent observations but only by destroying the fixity of historical comparisons. As additional observations are added over time, thereby expanding the set of comparisons, the multilateral index changes because the geometric mean of the observations changes. In contrast, bilateral indices do not directly compare nonadjacent observations and the historical comparisons remain intact.

The Tornqvist multilateral index for total-factor productivity (TFP) may be written (Caves et al. 1982a):

$$\ln TFP_k - \ln TFP_l = \sum_i 0.5 (R_{ik} + R_i^*) [\ln Y_{ik} - \ln Y_i'] - \sum_i 0.5 (R_{il} + R_i^*) [\ln Y_{il} - \ln Y_i'] - \sum_i 0.5 (W_{ik} + W_i^*) [\ln X_{ik} - \ln X_i'] - \sum_i 0.5 (W_{il} + W_i^*) [\ln X_{il} - \ln X_i']$$

where an asterisk associated with a variable indicates the arithmetic mean, and an apostrophe indicates the geometric mean.

This study empirically evaluates the bilateral and multilateral procedures for all of the indices computed. The Tornqvist formula is used because it is the only one for which the theoretical properties of both the bilateral and multilateral indices have been examined. Additional discussion is provided in Part 2.

Economic performance index

Norton et al. (1985) present an index of industry economic performance which attempts to measure economic performance over time or space. The index recognizes that economic performance could change due to changes in prices as well as productivity. The index incorporates the effects upon industry of economic performance of real prices for aggregate output, aggregate input, and total-factor productivity.

The economic performance index in general form may be written:

$$EP_{kl} = TFP (P_k^Y / P_k^X)$$

where TFP again refers to total-factor productivity, P^Y refers to an aggregate-output price index, P^X refers to an aggregate-input price index, and k and l refer to time periods or regions.

This study provides economic performance indices using the Tornqvist direct-chain index of TFP and the Tornqvist implicit real-price chain indices of aggregate output and aggregate input. Both bilateral and multilateral indices are formed.

The economic performance index is developed through an *ad hoc* procedure and the theoretical properties of the index are unknown. For example, when productivity is increasing (decreasing) and the price ratio is widening (narrowing), the results are unambiguous: industry economic performance is improving (declining). However, when productivity is increasing (decreasing) and the price ratio is narrowing (widening), the overall effect is not clear. Instead, the systematic properties of the index require further analysis, and results should therefore be treated as preliminary.

Data and index construction

Introduction

This section provides a description of the sources and methods used in the construction of the panel (pooled cross section and time series) data set used for analysis of the Pacific trawl fleet.

Output indices

The output indices are developed for U.S. vessels with landings in Washington, Oregon, and northern and central California which harvest in the fishery conservation zone for years beginning in 1981. The Washington landings exclude fish harvested in Puget Sound but do include vessels harvesting in the fishery conservation zone and landing in Puget Sound ports. The northern California or Eureka region includes landings in ports in the Crescent City, Eureka, and Fort Bragg areas. The central California or Monterey region includes landings in ports in the Bodega Bay, San Francisco, and Monterey areas. Nine species assemblages are specified: Dover sole, Petrale sole, other flatfish, rockfish, Pacific cod and ling cod, sablefish, pink shrimp, and Pacific whiting.

The output indices require total dollar value and total pounds of landings for each geographical region and each year for each species of concern. The revenue and catch data are from the PacFIN Management Database. All species and market categories within each of the nine species groups are linearly aggregated. The Pacific whiting species category includes fish harvested by both domestic and joint-venture vessels. Joint-venture whiting revenue and landings data are apportioned between Washington and Oregon in the Columbia area according to the home ports of the joint-venture whiting vessels. Revenue shares by region are reported in Table 17.

Input indices

Three major categories of inputs are distinguished: labor, capital, and fuel. The share or proportion of each input in total costs by region is reported in Table 33.

Labor index—The labor input indices are constructed from three categories of labor: ordinary crewmember, engineer, and captain. Total crew size is currently unavailable for the fleet, but since most vessels have a total crew of three (captain, engineer, and ordinary seaman), all vessels are assigned this crew size. Greater refinement will be possible as more information becomes available. As such, the labor indices presented in this study are strictly preliminary and may be subject to revision after refinement of the database.

Crew sizes are stock values and alone do not provide a satisfactory measure of the annual flow of labor services. Crew sizes are converted into annual flows of labor services by multiplying the number of people in each labor category in each region and time period by the corresponding measure of fishing time used in the study, the number of landings (discussed below). The flow of labor services is thus in man-landings per year.

Quality adjustments of effective annual flows of labor services are not possible with the level and extent of data available. For example, Jorgenson and Griliches (1967) adjust for changes in the quality of labor due to changes in the educational composition of the labor force. In fishing industries, years of fishing experience would be desirable. Ideally, the flow of labor services could also be adjusted for changes in labor efficiency that accompany changes in intensity of effort or time per person (Denison 1962).

Each labor category is valued at its opportunity cost.¹ This provides an exogenous representation of both remuneration to labor and food costs. The data sources include:

- (1) *County Business Patterns* for California, Oregon, and Washington, Bureau of the Census, U.S. Department of Commerce, Wash., D.C.
- (2) *Area Wage Survey of Grays Harbor and Pacific Counties 1984*, Labor Market and Economic Analysis Branch, Washington State Employment Security Department, Olympia, WA 98504.
- (3) *Oregon Occupations, 1984-85*, Oregon Career Information System, University of Oregon, Eugene, OR 97403.
- (4) *Area Wage Survey*, Bureau of Labor Statistics, U.S. Department of Labor, Wash., D.C.

¹In economics, the term "opportunity cost" describes the cost of undertaking a particular activity in terms of the foregone benefits of the next-best alternative. Thus the opportunity cost to capital owners is usually specified to be the interest they could receive if they placed this money in a financial institution. Similarly, the opportunity cost to a fisherman is the wage foregone (or opportunity lost) to this person by not working in the next-most-rewarding alternative.

(5) *Area Planning Information*, Employment Development Department, State of California, Sacramento, CA 95814.

All mean annual incomes used for opportunity costs in all labor categories are in 1981 dollars after deflation by the GNP implicit price index.

The opportunity cost of crew labor is an economic measure based upon reported earnings of workers in the counties in which the trawlers are homeported. Data on mean annual income for ordinary crewmembers are from source (1) above, where the opportunity cost is assumed to be the mean wage earned in manufacturing, transportation, and retail trade sectors. Captains are assumed to have managerial and entrepreneurial skills which imply a higher opportunity cost than for ordinary crewmembers. For lack of any specific alternatives, captains are given an opportunity cost 20 percent higher than ordinary crewmembers. The same data source is used as for ordinary crewmembers.

Vessel engineers are assumed to have an annual opportunity cost equivalent to the wages of an auto mechanic in their home ports (complete data are not available for the preferred category of diesel mechanic). Hourly wage rates for individual California ports are obtained from source (5). Since these wage rates for experienced journeymen auto mechanics are given only in ranges, the midpoint of each year's range is selected. Because data prior to 1984 are usually absent, these rates are assumed to change year-to-year at the same proportional rate of change as in San Francisco, where more timely data are obtained from source (4). Hourly wage rates for Oregon coastal auto mechanics are obtained for 1984-85 from source (3), and are assumed to change over time at the same rate as Portland auto mechanics, from source (4). Hourly coastal Washington auto mechanic rates are obtained for 1984 from source (2), and are assumed to change at the same rate as Seattle wages, from source (4) for previous years. Auto mechanics are assumed to work 40 hours per week, 50 weeks per year.

The home port of each vessel in every year is not known because home ports are obtained from the PacFIN Research Database, whose timeliness lags the annual vessel inventory used to compile annual numbers of vessels by region. Moss Landing is selected as the representative port for central California, Crescent City for northern California, Newport for Oregon, and Westport (Grays Harbor) for Washington.

In order to compute a single index of real labor input services for each region, a Tornqvist multilateral index is employed:

$$\ln L_k - \ln L_l = \sum_3 0.5 (W_{ik} + W_i^*) [\ln L_{ik} - \ln L_{il}] - \sum_3 0.5 (W_{il} + W_i^*) [\ln L_{il} - \ln L_{il}']$$

where L_{ik} is the quantity of labor services in the i th labor category for the k th (time-differentiated) region, L_k is the aggregate index of labor input for the k th region, W_{ik} is the compensation share for category i in region k , W_i^* is the arithmetic mean over all regions and time periods of compensation shares for category i , and L_{il}' is the geometric mean of the number of labor services in category i over all regions and time periods. A Tornqvist bilateral labor index is also constructed for use in the Tornqvist bilateral TFP and aggregate input indices. Tornqvist multilateral and bilateral chain aggregate labor indices are reported in Table 29 and Table 30, respectively.

Fuel index—Fuel consumption rates are estimated following an economic-engineering procedure. Annual fuel cost data for 120

vessel-years covering the 1981-83 period are divided by port-specific mean annual prices for cash purchases of No. 2 marine diesel fuel for 400-gallons. The estimate of annual fuel consumption for each vessel is then divided by its respective number of landings to give its mean fuel consumption per landing by region. Average fuel consumption per landing is then calculated for all vessels in each region. Each region's total annual fuel consumption is then derived by multiplying the regional mean fuel consumption per landing by the total number of landings in each year for that region. All vessels are assumed to use diesel fuel rather than gasoline. All prices are deflated by the GNP implicit price index to provide constant 1981 dollars. Fuel cost data are from confidential federal financial statements.

Annual diesel fuel cash prices are for 400 gallons of No. 2 marine diesel fuel. The 1981-83 port prices were obtained in the following manner. First, 1985 prices from marine fuel docks in each sample port were obtained by telephone interviews in February, May, and November with operators of marine fuel docks from 31 ports in Washington, Oregon, and California. These 1985 prices are averaged and deflated to 1981 levels by the GNP implicit price index, and are assumed to vary over time at the same rate as diesel fuel prices at petroleum terminals in San Francisco, Portland, or Seattle reported in *Platt's Oilgram Price Report: An International Daily Oil/Gas Price and Marketing Letter* (McGraw-Hill). It is reasonable to assume that individual port prices follow prices at major oil terminals, since marine-fuel dock port prices are essentially established on a formula basis from the terminal prices. Regional Tornqvist bilateral and multilateral chain indices for fuel are reported in Tables 29 and 30, respectively.

The total number of landings for Washington vessels of U.S. ownership fishing in the fishery conservation zone and landing anywhere in Washington is from the Washington Department of Fisheries, Olympia, WA 98504, while the number of landings for the other three regions is from the PacFIN Management Database. Joint-venture vessels' fishing time is calculated in weeks of fishing, where the beginning and ending dates of each vessel's fishing season are obtained from logbooks for the years 1981-84. The total number of days for each vessel is then divided by 7. When more complete information becomes available, the number of actual days fished can be taken from the logbooks rather than the beginning and ending dates of a season.

Capital index—The quantity of capital actually used in production is not the stock of capital (e.g., the number of vessels) but the flow of productive services from this capital stock. Thus more services for production are available from vessels actually fishing than from the same vessels tied up in ports.

The price of these capital services is a rental price for capital services on organized markets (e.g., tool rental). When capital services are not exchanged on markets, costs are imputed to firms to reflect the opportunity cost to capital owners of their money tied up in the capital stock and the depreciation of the capital equipment (Jorgenson 1974).

The capital services price per vessel for any given year (t) and size class (j) is given by:

$$P_{kjt} = r_t P_{Aj} + d_t P_{Aj}$$

where P_{Aj} is the mean vessel acquisition price per vessel in size class j , r_t is the opportunity cost of capital in time t , and d_t is the depreciation rate. Depreciation measures the present value of all future declines in productive capability.

This capital services price is an imputed price, and provides an accurate measure of the economic value of capital services to capital owners when the stock of capital is in full static equilibrium in any given time period. That is, the actual capital services are equal to the optimum flow of services, and firms are capable of making the required adjustments to their stock of capital in order to attain the optimum amount in each year. When, for example, the existing capital stock is inadequate relative to demand, firms face a relative shortage of capital and have incentives to invest. An additional unit of capital then has an economic valuation greater than that measured by the capital services price. Alternatively, when the stock of capital is greater than that required for full equilibrium in any year, firms have a relative surplus of capital stock and incentives to disinvest or even leave the industry. In this case, the economic valuation of capital services is lower than the measured imputed price. This study assumes that capital is in full static equilibrium and that the imputed price of capital services accurately measures the economic value of these services.² The value of all capital services is also assumed equal to the sum of the values of the individual capital services (Christensen and Jorgenson 1969, 1970). Aggregation from the firm to the industry is therefore assumed possible.

P_{Aj} is expressed in 1981 prices, and is the mean vessel acquisition price per vessel from vessels purchased 1976-82. Of the total 106 vessels used to calculate these vessel acquisition prices, 15 are class I (1-49 registered feet), 81 are class II (50-74 registered feet), and 10 are class III (75+ registered feet). The vessel acquisition prices are from confidential financial statements. Stable and consistent functioning of capital markets and industry expectations are assumed after the Magnuson Fishery Conservation and Management Act was announced, but no changes are assumed after this time. Limited data require the assumption of no capital gains or losses. The relatively limited number of years for vessel acquisition prices mitigates the effects of capital vintage. Property taxes are not applied to fishing vessels on the U.S. west coast, and are therefore not included in the capital services price.

The opportunity cost of capital in any year, r_t , is assumed equal to the annual corporate bond rate on seasoned issues rated BAA by Moody's, reported in the *Federal Reserve Bulletin* of San Francisco. The annual depreciation rate is set at 7 percent, as suggested by the Southwest Regional Office of the National Marine Fisheries Service, NOAA, Terminal Island, California, and roughly (but not exactly) corresponds to straight-line depreciation with a 15-year economic life and zero scrap value of the vessel and gear.

The price of capital services is used in constructing the index of capital services for the fishery. Constructing this index requires weights that reflect the annual capital cost. The annual capital cost of vessels in each region and year for each length class is estimated as the product of the number of vessels and the annual price of capital services per vessel. The capital services price is assumed constant across regions due to the general mobility of vessels.

The index of capital services also requires annual quantity flows of capital services. The first step in measuring the flow of capital services is to collect annual vessel counts. These are compiled by the National Marine Fisheries Service at Terminal Island, California (Korson 1981-85). Northern and central California vessels since 1985 are assumed to have the same home ports on a percentage

²Current research is in the process of incorporating variations in capacity utilization of capital following the procedure of Hulten (1986) and Berndt and Fuss (1986). This procedure adjusts the price of capital services rather than the stock of capital. The same research is accounting for changes in population abundance of the resource stock and variations in its catchability.

basis as those of the 1984 vessels in the PacFIN Research Database in La Jolla, CA. Three vessel size-classes based on Coast Guard registered length are distinguished: 1-49 ft; 50-74 ft; 75+ ft. The assignment of home states for a few vessels in the annual vessel inventories is inconsistent with the home port classification of the PacFIN research database, in which case the PACFIN assignment is followed.

The annual vessel counts are stocks of capital potentially available for productive purposes in any given year. This study assumes that all vessels fully utilize the potential productive capacity available in each year, that is, the firms are at full static equilibrium in capital. Capital services are therefore assumed to be proportional to capital stocks. Ongoing research will relax this assumption.

The index of real capital services is aggregated over the three vessel length-classes by the Tornqvist multilateral index formula:

$$\ln K_k - \ln K_l = \sum_1 \sum_3 0.5 (W_{ik} + W_i^*) [\ln K_{ik} - \ln K_i'] \\ - \sum_1 \sum_3 0.5 (W_{il} + W_i^*) [\ln K_{il} - \ln K_i']$$

where K_{ik} is the number of vessels in size class i for the k th (time-differentiated) region, K_k is the aggregate index of annual capital services for the k th region, K_i' is the geometric mean of the number of vessels in category i (over all regions and time periods in i), W_{ik} is the k th region's share of total annual vessel capital cost attributed to vessels of type i , and W_i^* is the arithmetic mean of annual capital costs of vessels in class i (over all regions and time periods in i). Tornqvist bilateral indices are also constructed for use with the Tornqvist bilateral aggregate input and TFP index numbers. Tornqvist multilateral and bilateral capital chain indices are reported in Tables 29 and 30, respectively.

Empirical results

Introduction

This section has three objectives: (1) Review the empirical results and relate them to industry events; (2) evaluate empirically the different types of index procedures; and (3) assess the sources of available data.

The empirical results are reported in Tables 6 through 36. These tables include annual indices by region and fleet of total-factor productivity (TFP), TFP growth rates, aggregate output, each individual output, aggregate input, each individual input, implicit prices, and an index of industry economic performance in the spirit of Norton et al. (1985). Revenue and cost shares are also reported. Tornqvist multilateral and bilateral chain indices are reported for all categories in order to evaluate the multilateral and bilateral indexing approaches. Laspeyres, Paasche, Fisher Ideal, and Tornqvist fixed-base indices for aggregate outputs are reported in Tables 21-24 in order to empirically evaluate fixed-base versus chain indices and to evaluate the four different types of index formulae.

The growth-accounting results presented here assume constant returns to scale, full capacity utilization, technical efficiency, and marginal cost pricing. The empirical results are strictly preliminary and may be subject to revision because crew sizes and detailed vessel counts require additional refinement.

The Pacific trawl fleet

The bilateral total-factor productivity (TFP) indices are reported in Table 7. The empirical results indicate a substantial decline in productivity during 1982 and 1983 compared with 1981. Total-factor productivity then grew in 1984 and 1985. Fleet TFP grew at an average growth rate of nearly 0.38% for the industry over 1981-85 with particularly strong growth for 1984-85 (Table 9).

The 1982-83 multifactor productivity decline may largely be due to the harvesting of widow rockfish at levels beyond estimates of maximum sustainable yield and the decline of the pink shrimp fishery (Table 13). Rockfish and pink shrimp are the two most important species groups by share of total revenue (Table 16). Fleet aggregate output as a whole declined by 1.3% in 1982 and 0.47% in 1983 (Table 15). General increases in aggregate input also contributed to the 1982-83 decline in TFP (Table 26). Industry aggregate input usage grew by 8.73% in 1982 and 1.28% in 1983 (Table 28).

The TFP rise in 1984 and 1985 (Tables 7 and 9) can be attributed to an important decline in aggregate input usage from the 1983 level (Table 28). Industry aggregate output declined by 0.46% in 1984 and grew by 2.65% in 1985 (Table 15). No individual species appears to dominate the general rise in aggregate output, although pink shrimp landings were up (Table 13), and in 1985 pink shrimp constituted 29% and 22% of total revenue in Washington and Oregon, respectively (Table 16), and 16% of fleet revenues (Table 18). Fleet aggregate input declined by 11% in 1984 and by 1.5% in 1985 (Table 28).

Capital is an important component of aggregate input in terms of cost share (Table 33). Table 30 reports individual capital indices. Capital began to decline in Washington in 1983, while northern and central California experienced increases in capital in 1983 before a decline in 1984. Vessels may have transferred from Washington and Oregon to California in 1983. A number of vessels may also have left the active fleet due to sinkings, burnings, or financial difficulties. Other vessels, particularly larger ones, are known to have transferred fishing activities to Alaska.

Additional factors may have contributed to the 1984-85 rise in TFP. It is likely that the vessels leaving the fishery were relatively inefficient harvesters, in which case TFP would increase if many of the remaining vessels were operated by skippers and crews with fishing skills superior to those vessels that left the active fleet. The larger vessels that transferred to Alaska may have been more inefficient than medium and smaller vessels under the reduced level of resource abundance. This could be due to decreasing overall returns to scale as stocks, particularly widow rockfish, declined. As those larger vessels left the fleet, productivity measures of the remaining vessels should have increased. Productivity may also have increased as fishermen became more skilled with the technological innovations previously introduced in the production process (an increase in technical efficiency).

Regional differentials in total-factor productivity growth are demonstrated by the bilateral total-factor productivity indices reported in Table 7. By 1985, total-factor productivity for the entire fleet, and in Washington and Oregon surpassed 1981 levels, but not in northern and central California. Tables 14 and 26 indicate that the 1985 aggregate output level is below the 1981 level in Oregon and northern California regions and for the fleet, but that important regional variations exist in input usage. While input usage in Washington and Oregon remains well below 1981 levels, and in fact continued to decline through 1985, input usage in California is still above the 1981 level.

Fleet economic performance depends upon the real prices of outputs and inputs in addition to total-factor productivity. Tornqvist bilateral implicit chain indices for constant-dollar aggregate output prices and aggregate input prices are reported in Tables 20 and 32, respectively.³ By 1985, aggregate output real prices increased above 1981 levels for the entire fleet and northern and central California while remaining below 1981 levels for Washington and Oregon. By 1985, aggregate input real prices for the fleet had remained below 1981 levels, while the 1985 aggregate input prices rose above the 1981 level for Washington. The general increase in aggregate output price and a general decline in aggregate input price by 1985 suggest an improvement in the ratio of product price to input price for the Pacific trawl fleet which reinforced the recent gains in TFP. On the whole, the economic performance of the fleet should have returned to 1981 levels although economic conditions for individual vessels may differ.

Economic performance

The economic performance index of Norton et al. (1985) attempts to combine indices of prices and productivity into a single measure of fleet economic performance. Tornqvist bilateral chain economic performance indices are reported in Table 36. As discussed above, the results should be interpreted with caution due to the uncertain theoretical basis of these indices.

The indices indicate that overall fleet economic performance in 1985 is above that of the initial time period, 1981. Reinforcing recent productivity gains are the general increase in aggregate output price and general decline in aggregate input price as noted above.

Methodological evaluation

Multilateral vs. bilateral indices—Comparisons of multilateral and bilateral indices are made for the Tornqvist chain indices. These indices are computed for total factor productivity (TFP) in Tables 6 and 7, aggregate outputs in Tables 11 and 14, individual outputs in Tables 10 and 13, aggregate inputs in Tables 25 and 26, and individual inputs in Tables 29 and 30.

All of the multilateral indices are normalized in terms of 1981 Washington by dividing all values by the value of 1981 Washington. Setting 1981 Washington equal to 1.00 provides a more convenient basis for making comparisons. All relative relationships are preserved by normalization. Multilateral indices for the fleet are similarly normalized in terms of the 1981 value.

The computed Tornqvist multilateral and bilateral chain indices generally differ little from one another in tracking turning points (i.e., increase to decrease or decrease to increase) and trends. For example, both sets of TFP indices (Tables 6 and 7) indicate productivity declines in 1982 or 1983 and agree with the occurrences of all increases and decreases. This coincidence of turning points occurs because the growth rates of multilateral and bilateral indices generally differ little in magnitude.

Similarity in growth rates can be demonstrated for the most important index, the total-factor productivity index, by regressing the 1982-85 multilateral growth rates upon the 1982-85 bilateral growth

rates with an intercept term. Using this result, it is possible to (1) test to determine whether the intercept term is significantly different from zero and thus whether one set of growth rates over- or understates growth by a fixed percentage relative to the other; (2) test to determine whether the slope coefficient is significantly different from unity as a check on the proportionality of one set of growth rates to another; and (3) examine the R^2 as a measure of the linear proximity of the two sets of growth rates. Regressing the multilateral TFP growth rates upon the bilateral TFP growth rates produces the expected result: the intercept is not significantly different than zero, the slope is not significantly different from unity, and the R^2 is very high. (Intercept coefficient = 0.005, S.E. 0.005; slope coefficient = 1.054, S.E. 0.031; R^2 = 0.984, and F-statistic for overall regression = 1123.004.)

Comparability between the multilateral and bilateral indices is diminished at the highest levels of aggregation, because relatively minor differences for individual outputs and/or inputs begin to accumulate at higher levels of aggregation. For example, proportionately greater differences are exhibited among TFP indices than among individual outputs or inputs. Thus the regional individual species indices (Tables 10 and 13) track individual outputs by region in a similar manner, but the 1983-84 indices for the fleet (Tables 11 and 14) differ (the multilateral index indicates a small increase, whereas the bilateral index indicates a slight decrease).

Although the multilateral and bilateral indices nearly always agree on turning points and usually agree on trends and growth rates, the magnitudes of the indices relative to the initial year (1981) can nonetheless differ in important ways.

To formally compare the magnitudes of multilateral and bilateral TFP indices (rather than growth rates), the multilateral TFP index is regressed upon the bilateral TFP index for the years 1982-85 with an intercept term. The estimated intercept coefficient is 0.319 (S.E. 0.250), the estimated slope coefficient is 0.577 (S.E. 0.266), the R^2 is 0.208, and the overall F is 4.707. These results suggest that a fixed displacement between indices does not exist but that the multilateral TFP measures tend to be about 40 percent lower than the bilateral TFP measures. The R^2 of 0.21 suggests that an additional 79 percent of the variation in the multilateral index estimates exists after the 40 percent proportionality difference has been accounted for (Hazilla and Kopp 1984a,b).

The principal reason for the difference in magnitude relative to the initial year lies in interpretation of the index. The multilateral index for the initial year represents deviations from the geometric mean (of all regions and years), while the bilateral index for the initial year is the constant value 1. That is, the multilateral and bilateral initial values can differ considerably because of the different initial-period magnitudes. This difference is accentuated if the initial year differs markedly from the geometric mean of all years and regions. Both base year and chain multilateral indices are affected by the difference in interpretation of the initial time period.

The computed multilateral and bilateral indices also differ markedly in the years 1982-85 when an intertemporal change occurs which is substantially different from the geometric mean and the value of the preceding year. Consider the Pacific whiting multilateral and bilateral output indices. The Pacific whiting catch from joint-venture vessels is relatively large in total tonnage and can vary considerably from year-to-year and region-to-region.

The computed multilateral and bilateral indices can also differ in an important way when very small intertemporal changes occur. The bilateral index can track the small change but the multilateral index can fail to pick up a change in trend from increasing to

³Implicit prices are calculated through Fisher's weak-factor reversal test. This relationship states that the product of the price index and the quantity index should equal the expenditure ratio between the two time periods. Part 2 and Appendix 3 provide additional discussion.

decreasing or vice versa, because all of its comparisons are in terms of the geometric mean. Thus the multilateral procedure tracks turning-points most effectively when the values are closer to the geometric mean, but may experience difficulties when all the values are substantially different from the geometric mean.

In summary, the multilateral index has superior theoretical properties to the bilateral index, but can demonstrate empirical limitations in relatively extreme situations. Fundamentally comparable results are demonstrated with the two indices. However, for official reports likely to receive widespread distribution, the bilateral procedure should be used since fixity of historical comparisons remains intact (explanations are not required when different numbers occur in subsequent years) and interpretations are easier.

Chain vs. fixed-base indices—As noted above, fixed-base indices compare all changes to some initial base period, while chain indices make comparisons by a process of chaining binary (period-to-period) comparisons back to the original time period (in this case, 1981).

Table 14 reports Tornqvist bilateral chain indices of total output by region, and Table 24 reports Tornqvist bilateral fixed-base indices of total output by region. Comparison between the two tables indicates that very different results and types of information are provided by the two approaches. For example, the fixed-base indices indicate that 1985 total output is less than 1981 in all regions, while the chain indices indicate that 1985 total output is greater for Washington and central California. This difference may be due to the changing species composition of catch over time (see Tables 16 and 18) with which the fixed-base index procedure has difficulty in dealing. Although the choice between the two procedures is somewhat dependent upon the type of information to be presented, the chain procedure is generally preferred on theoretical grounds.

Laspeyres, Paasche, Fisher Ideal, and Tornqvist indices—As discussed above, a number of different index-number procedures exist, each index corresponding to different functional forms of the aggregator function and consequently each index number having different theoretical properties. The Laspeyres and Paasche indices implicitly assume either no or perfect substitution between individual commodities, while the class of superlative indices does not require commodities to be perfect or zero substitutes. Changes in the composition of an aggregate are therefore correctly captured.

To facilitate an empirical evaluation of these indices, fixed-base aggregate-output bilateral indices are developed for each region. Laspeyres indices are presented in Table 21, Paasche indices in Table 22, Fisher Ideal indices in Table 23, and Tornqvist indices in Table 24. An examination of Tables 21-24 indicates that different results are possible, depending upon the choice of index number. Consider, for example, the years 1983-85 for northern California: The Laspeyres and Fisher Ideal indices report a decline in landings from 1983 to 1984, with an increase from 1984 to 1985. In contrast, the Paasche and Tornqvist indices report continual increases.

The Fisher Ideal index lies between the values of the Paasche and Laspeyres indices as expected, since the Fisher Ideal index is the geometric mean of the Paasche and Laspeyres indices. Values of the Fisher Ideal and Tornqvist indices do differ.

As the degree of data disaggregation increases (so that a quantity may become zero), the Fisher Ideal index number formula remains well defined while the Tornqvist does not (because of the log transformation which is undefined when the untransformed

variable is zero). The consequence of this property is clearly demonstrated in Tables 10 and 13 for Pacific whiting landings in Washington. In 1981, no landings of Pacific whiting were made in Washington by either the coastal fleet or joint-venture vessels. However, large landings in 1984 were reported by joint-venture vessels homeported in Washington (with landings consequently assigned to Washington). To allow a Tornqvist index number to be calculated for 1984, a small value is given to 1981 (an *ad hoc* procedure), but an enormous value is calculated for the Tornqvist index. This result carries over to the 1984 fleet value, where the Fisher Ideal (Table 23) indicates a decrease from 1983 to 1984 and an increase from 1984 to 1985. In contrast, the Tornqvist (Table 24) reports just the opposite result.

Recommended index-number procedure—The recommended index-number procedure for analysis with a widespread and disparate audience is either the Tornqvist or Fisher Ideal bilatera chain index. The major advantage of the Fisher Ideal is that it is well defined when the data are so highly disaggregated that a zero output or input occurs, while the Tornqvist is not. Chain indices are preferred to fixed-base indices. Multilateral indices have greater versatility than bilateral indices, but do not have fixity of historical comparisons. The Tornqvist multilateral chain index can be quite suitable for technically sophisticated audiences.

Data evaluation

The data currently available are sufficient for potentially satisfactory construction of productivity, quantity, and price indices for outputs and inputs. Nevertheless, certain limitations exist, many of which can be corrected or at least mitigated with time.

The PacFIN Management Database provides timely data for revenues and quantities of outputs. Fuel prices and interest rates are readily obtainable on a consistent and timely basis through telephone surveys and various issues of the *Federal Reserve Bulletin*, respectively. Wage and income data used to construct the opportunity cost of labor are available, but there is a 1-2 year lag in the most recent data of the Bureau of the Census. Nevertheless, this data lag does not do great harm to the analysis. Crew-size information for all vessels is currently incomplete, and scope for updating and refinement certainly exists.

Cost data from the NMFS confidential financial-cost database are adequate for the task at hand. The cost sample is not comprehensive for all vessels, nor is it systematically derived on the basis of sampling theory. Yet the rather large sample does provide a reasonable degree of confidence in its adequacy. This confidence should improve as the cost data are updated on a continual basis.

The Pacific whiting joint-venture data are among the least satisfactory of all the data currently available. The fishing-time information needs to be updated from logbooks and from confidential sources.

Refinement of the concept of a fisherman's opportunity-cost and better information about the most likely alternatives would improve the analysis. If the information was available, capital gains or losses, i.e., the revaluation of assets, could be important in computing the real cost of capital services (even though most capital gains are not realized). Little is known about the actual rate of economic depreciation of capital assets.

The use of an engineering approach to measuring fuel consumption may also be subject to limitations since the effects of changing

economic conditions might not be fully incorporated into this procedure. Fuel consumption and more accurate fishing-time data may become available from logbooks. The quality of the analysis might improve in this case, and the engineering approach to measuring fuel consumption can be replaced by direct measurement. Finally, the absence of prices and the spotty and incomplete data on materials, supplies, ice, and other trip costs preclude their use in the analysis. In turn, this may lead to some form of omitted variable bias.

In summary, sufficient data of acceptable quality are available to construct output, input, and productivity indices. These data require additional refinement, upgrading, and updating to improve the quality of the analysis, particularly for crew sizes and vessel homeports. As such, the empirical results presented in this study are strictly preliminary, and may be subject to revision after final refinement of the database is completed.

Concluding remarks

This study is developed to address five issues of productivity measurement in the Pacific trawl fleet: (1) Apply the most recent advances in productivity measurement and the economic theory of index numbers; (2) utilize more extensive data sources than those previously used in productivity and performance studies of marine fishing industries; (3) evaluate the many different index-number procedures that are currently available; (4) clarify the meaning of productivity in marine fishing industries; and (5) collect into one accessible place the recent advances in productivity measurement and the economic theory of index numbers. Part 2 and its Appendices address task (5).

In conclusion, this study recommends the use of chain indices rather than fixed-base indices, either the Fisher Ideal or Tornqvist index, and bilateral indices for recurring publications. The data currently available are adequate for satisfactory construction of productivity, quantity, and price indices. Certain limitations do exist but are potentially correctable if existing efforts at the Southwest Fisheries Center (National Marine Fisheries Service, La Jolla, CA) aimed at expanding and updating economic bases are continued along their current lines. These current efforts include expanding and developing the PacFIN Research Database, logbook data, the cost database, and input prices.

PART 2

Additional topics in productivity measurement

This section considers several additional aspects of productivity measurement to supplement the general framework developed in Part 1. This section proceeds by first considering measurement of productivity by the growth-accounting framework using economic index numbers and by the structural framework using econometric methods. Next, the assumptions behind the growth-accounting framework are reviewed, followed by a discussion of interspatial and intertemporal productivity, and a review of duality-based measures of productivity.

Growth-accounting and structural frameworks

Total-factor productivity can be interpreted and measured by either the growth-accounting framework with economic index numbers or by the structural framework using econometric methods. The structural approach measures productivity change as the rate of technological progress when measured by a production, cost, revenue, or profit function. It also allows detailed examination of the structure of production, including measurement of economies of scale, tests for full static equilibrium (Kulatilaka 1985, Schankerman and Nadiri 1986), technical inefficiency (Nishimizu and Page 1982), and corrections for deviations from competitive markets and marginal cost pricing (Denny et al. 1981). This detailed information requires estimation of an econometric model.

The structural approach is parametric and global, since it can yield information about the full range of the estimated aggregate production or cost function and requires parametric specification of this function. The growth-accounting framework is nonparametric since it is based on Divisia indices of multifactor input and multifactor productivity. It is also local, since the only information about the nature of the production technology is embodied in the marginal productivity conditions. These conditions allow calculation of the slope of the aggregate production function using only relative prices, but only along the observed surface of the aggregate production function. The results are thus local to this observed range (Hulten 1986).

Assumptions of the growth-accounting framework

The growth-accounting framework for measuring total-factor productivity makes several assumptions: Constant returns to scale, technical efficiency, perfect competition in input and output markets, and full static equilibrium of all inputs. When these assumptions are not satisfied, conventional indices of total-factor productivity growth include not only the effect of technical change, but may also include some or all of the effects from nonconstant returns to scale, technical inefficiency, market imperfections, and departures from full static equilibrium.

Consider first the assumption of perfect competition: This assumption is in part acceptable in marine fishing industries. Most importantly, fishermen are generally pricetakers in the product and factor markets, but entry and exit may be difficult. Readers interested in the concept are referred to Kendrick (1973) and Denny et al. (1981). Consider next the returns to scale: Parametric representation of the

production technology is necessary to identify and estimate the separate effects due to scale economies and technical change (Diamond et al. 1978, Chan and Mountain 1983). Once a measure of nonconstant returns to scale is available, the TFP index can be adjusted by dividing by the measure of scale economies (Caves et al. 1982b). Alternatively, James Kirkley (Va. Inst. Mar. Sci., Gloucester Point, VA 23062, pers. commun. Feb. 1987) suggests that the nonparametric method of Diewert and Parkan (1985) can be used to examine the effects of scale economies and technical change (although only the lower bounds can be examined).

Nishimizu and Page (1982) note that a distinction should be drawn between technological change and changes in the efficiency with which known technology is applied to production. Given a level of technology, explicit resource allocation may be required to reach the "best-practice" level of technical efficiency over time. After the adoption of a new technology (e.g., stern trawling), productivity gains are possible as firms master the new technology over time. A parametric approach is required to address this issue. The growth-accounting framework adopted in this study implicitly assumes that all firms are technically efficient, that is, that production is on the frontier of the industry-production function.

Fourth, consider the effects of vessels not in full static equilibrium in all of their inputs: If producers are assumed to be in long-run equilibrium when in fact they may be in short-run temporary equilibrium, the productivity residual may be systematically underestimated. (Berndt and Fuss 1986, Hulten 1986, Winston 1974). The traditional method for TFP measurement is appropriate only if the firm's output is always produced at the long-run equilibrium point, i.e., the point of tangency between the short-run unit or average total cost curve and the long-run unit cost curve. Instead, if there are divergences from static equilibrium, the firm is not operating along the long-run average cost curve, and the conventional measure of TFP includes variations in capacity utilization of the fixed inputs. For example, if a decline in overall resource availability or a change in its composition (species mix) causes a temporary equilibrium due entirely to underutilization of harvesting capacity, then a perfectly competitive fishing firm would not be in long-run equilibrium and measurement of TFP might be biased.

The problem of disequilibrium is corrected by one of two methods. Most frequently, the quantity of the quasi-fixed factor is adjusted to reflect the degree of capacity utilization (Jorgenson and Griliches 1967). Alternatively, the price of the fixed factor is adjusted to reflect its true shadow value, that is, the contributions of quasi-fixed inputs are valued at their shadow prices rather than market prices (Berndt and Fuss 1986). Morrison (1985a,b, 1986) provides further discussion on economic measures of capacity utilization obtained by econometric means, while Hulten (1986) and Berndt and Fuss (1986) discuss this in a growth-accounting framework. Current empirical research is addressing this issue.

Intertemporal and interspatial productivity

Intertemporal total-factor productivity can be interpreted as a rate of shift over time in a production function*. As discussed in Part 1, the mechanism generating these rates of change is usually assumed to be technological progress, so that measurement of technological change is equivalent to the measurement of a change in intertemporal TFP. The input effect is associated with movements along

the aggregate-production function, while the multifactor productivity residual is associated with shifts in the aggregate production function. This interpretation is generally attributed to Solow (1957), Jorgenson and Griliches (1967), and Hulten (1975). Denison (1962) and Star (1974) instead stress a diversity of factors which might be captured in the residual total-factor productivity measure. Changes in total-factor productivity in marine fishing industries might also be due, in part, to changes in resource availability and species composition, since higher levels of resource abundance should allow any given input-bundle to harvest more outputs. Changes in resource availability might also impact upon capacity utilization of a quasi-fixed factor such as capital and upon the rate of fuel utilization.

Interspatial total-factor productivity has a somewhat different interpretation than intertemporal total-factor productivity. Interspatial total-factor productivity can be defined as the proportional differences in an index of outputs between different production regions (or firms) relative to the proportional differences in an index of inputs. Interspatial productivity differences arise not from the dynamic process of technological change, but rather from static differences in technology across producing regions (or firms). Interspatial productivity differences in marine fishing industries are also likely to directly reflect differences in resource availability and composition (and indirectly through economies of scale).

Duality-based measures of productivity

The growth-accounting model developed in Part 1 measures multifactor productivity as the residual output not accounted for by the share-weighted growth rates of the factor inputs. As Ohta (1975) notes, total-factor productivity can also be equivalently measured by the residual diminution in average cost not accounted for by the input prices (and changes in scale economies if constant returns to scale are not assumed). Although this result is not of importance in this study, its general outlines are developed along the lines of Hulten's (1986) discussion for the sake of completeness and because so much of the recent productivity literature is developed in terms of costs.

Under certain regularity conditions (McFadden 1978, Lau 1978), the existence of a cost function dual to the production function $Y = f(K,L)$ is implied:

$$C(t) = C[P^K(t), P^L(t), Y(t), t] \quad (1)$$

$$= P^K(t)K(t) + P^L(t)L(t),$$

where $K(t)$ and $L(t)$ are the cost-minimizing quantities of capital and labor, respectively. Under constant returns to scale, this can be written:

$$C(t) = B(t)c[P^K(t), P^L(t)]Y(t), \quad (2)$$

where $c(t)$ is termed the unit cost function, since $C(t)/Y(t)$ is the average cost of producing $Y(t)$ under cost minimization.

The sources of growth implications of equation (1) are derived from Shephard's Lemma, which implies that $\partial C(t)/\partial P^K(t) = K(t)$ and $\partial C(t)/\partial P^L(t) = L(t)$. This implies that:

$$C^*/C - Y^*/Y = B^*/B + S_K(P^K^*/P^K) \quad (3)$$

$$+ S_L(P^L^*/P^L).$$

*Either full or partial equilibrium econometric models of production can be used.

This expression states that the growth rate of average cost equals the growth rate of the shift parameter, B^*/B , plus a Divisia index of input prices. Under constant returns to scale, $PY = P^K K + P^L L = C$, implying that $A^*/A = -B^*/B$. In other words, real average cost decreases at a rate equal to the growth rate of the Hicksian efficiency parameter.

This result means that the total-factor productivity residual can be measured as the residual growth rate of output not explained by the Divisia index of inputs, or as the residual diminution rate of average cost not explained by the Divisia index of input prices.

Economic theory of index numbers⁵

Introduction

The economic theory of index numbers is concerned with the relationships between the properties of index numbers and the properties of the underlying aggregator functions they represent. The demonstration in recent years that numerous index-number formulae can be explicitly derived from particular aggregator functions implies that rather than starting the selection process with a number of index-number formulae, an aggregator function with desirable economic properties can be specified and the corresponding index-number procedure derived.

To be more concrete, consider price and quantity data for N commodities for two periods (or economic entities), $P^0 = (P_1^0, P_2^0, \dots, P_N^0)$, $P^1 = (P_1^1, P_2^1, \dots, P_N^1)$, $X^0 = (X_1^0, X_2^0, \dots, X_N^0)$, and $X^1 = (X_1^1, X_2^1, \dots, X_N^1)$. A price index $PI(P^0, P^1, X^0, X^1)$ is defined to be a function of prices and quantities, while a quantity index $QI(P^0, P^1, X^0, X^1)$ is defined to be another function of observable prices and quantities for the two periods.

Aggregator functions

An aggregator function f is a particular formula or procedure for aggregating the price and/or quantity data into some price or quantity index. Thus an aggregator function for the quantity index QI would specify some particular functional form for $QI(P^0, P^1, X^0, X^1)$. The aggregator function for an input-quantity index is essentially a production function, while the aggregator function for an input-price index is a unit-cost function. The aggregator function for outputs

⁵Frisch (1936) distinguishes three approaches to index number theory: (1) "statistical" approaches, (2) the test approach, and (3), the functional approach, now called the economic theory of index numbers by Samuelson and Swamy (1974). The statistical approach assumes that all prices are affected proportionately (except for random errors) by the expansion of the money supply. Therefore it does not matter which price index is used to measure the common factor of proportionality, as long as the index number contains a sufficient number of statistically independent price ratios. Theil (1960) provides a "neostatistical" approach. The test or axiomatic approach initiated by Fisher (1911, 1922) assumes that the price and quantity indices are functions of the price and quantity vectors pertaining to two periods. Tests are *a priori* reasonable properties that the price and quantity vectors should possess. However, not all *a priori* reasonable properties are consistent with each other, that is, there are various impossibility theorems. Moreover, the family of index-number formulae resulting from a consistent restricted set of tests is often not uniquely determined (Diewert and Parkan 1985).

The economic and test approaches to index-number theory can be partially resolved. After assuming explicit functional forms for the underlying aggregator function and cost-minimizing behavior by the producer, certain functional forms for the aggregator function can be associated with certain functional forms for index-number formulae. Many of the index-number formulae, such as Fisher's Ideal formula, have been suggested as desirable in the literature on the test approach to index-number theory (Diewert 1981).

is a unit-revenue function for prices and a factor-requirements function for quantities (Diewert 1974).

An index number, such as a quantity index, is exact for a particular functional form F of the aggregator function if the ratios of the outputs (the values of F) between any two periods or regions are identically equal to the index of outputs: $QI(P^0, P^1, X^0, X^1) = F(X^1)/F(X^0)$ (Diewert 1976).

Diewert (1976a) provides a strong argument for considering only flexible aggregator functions, that is, those aggregator functions which can provide a second-order approximation to an arbitrary aggregator function. Diewert (1976a) terms that class of index numbers that are exactly represented by flexible aggregator functions as superlative.

Fisher's weak factor-reversal test

PI and QI are generally assumed to satisfy Fisher's (1922) weak factor reversal test:

$$PI(P^0, P^1, X^0, X^1)QI(P^0, P^1, X^0, X^1) = \sum_i P_i^1 X_i^1 / \sum_i P_i^0 X_i^0. \quad (4)$$

Fisher's weak factor-reversal test (4) states that the product of the price index multiplied by the quantity index should equal the expenditure ratio between the two periods. PI is to be interpreted as the ratio of the price level in period 1 to the price level in period 0, while QI is the ratio of the quantity levels of the two time-periods (or economic entities). Given either a price index or quantity index, the other function can be defined implicitly by Fisher's weak factor-reversal test (4).

Laspeyres and Paasche (mean-of-order- r) indices

The Laspeyres and Paasche indices have traditionally been the most widely used of all index numbers. They belong to the general class of mean-of-order- r indices.

Define the mean-of-order- r quantity index using period-1 shares as (Allen and Diewert 1981):

$$Z(P^0, P^1, X^0, X^1) = \left(\sum_i S_i^1 (Z_i^1/Z_i^0) \right)^{1/r} \quad \text{for } r \neq 0 \quad (5)$$

$$= \left(\prod_i (Z_i^1/Z_i^0) \right)^{S_{i1}} \quad \text{for } r = 0$$

where $S_i^1 = P_i^1 X_i^1 / \sum_i P_i^1 X_i^1$.

Some well-known indices are special cases of the mean-of-order- r index. The Laspeyres quantity index may be written:

$$Q_L = \sum_i P_i^0 X_i^1 / \sum_i P_i^0 X_i^0 \quad (6)$$

$$= \sum_i S_i^0 (X_i^1/X_i^0),$$

where $S_i^0 = P_i^0 X_i^0 / \sum_i P_i^0 X_i^0$. Since prices are held fixed at their base period (or economic entity) levels, the Laspeyres index indicates how much of the change in value of total quantity resulted from pure quantity changes. Similarly, the Laspeyres price index may be written:

$$P_L = \sum_i P_i^1 X_i^0 / \sum_i P_i^0 X_i^0 \quad (7)$$

$$= \sum_i S_i^0 (P_i^1/X_i^0).$$

The Paasche quantity index is also a specific form of the mean-of-order- r index, and may be written as:

$$Q_P = \frac{\sum P_i^1 X_i^1}{\sum P_i^1 X_i^0} \quad (8)$$

In contrast to the Laspeyres quantity index, prices are held fixed at their new levels. The Paasche price index may be specified as:

$$P_P = \frac{\sum P_i^1 X_i^1}{\sum P_i^0 X_i^1} \quad (9)$$

The Laspeyres quantity index weights with base prices; the Laspeyres price index weights with base quantities; the Paasche quantity index weights with current prices; and the Paasche price index weights with current quantities.

The Laspeyres and Paasche indices are exact for a Leontief or fixed-coefficients aggregator function f :

$$f(X) = \min_i [X_i | a_i : i = 1, 2, \dots, N], \quad (10)$$

where a_i is a fixed constant. The Laspeyres and Paasche indices are also exact for a linear aggregator function f :⁶

$$f(X) = \sum a_i X_i. \quad (11)$$

The Laspeyres and Paasche indices are thus exactly equal to their corresponding true indices if there is either no substitution between commodities or if there is perfect substitution between commodities. The Laspeyres and Paasche indices always offer a first-order approximation to the true index. Therefore, if substitution between commodities lies exactly or close to either no or perfect substitution, the Laspeyres and Paasche indices provide acceptable performances. However, the larger the time period or the greater the difference between the base and comparison levels, the greater the likelihood that substantial price changes may occur (leading to an intermediate substitution case) and the greater the misrepresentation of these indices. Kirkley (Va. Inst. Mar. Sci., Gloucester Point, VA 23062, pers. commun. Feb. 1987) also notes that if the production function from which the Laspeyres and Paasche indices is derived is convex, then the Laspeyres index overstates and the Paasche index understates, while if the production function is concave, then the Laspeyres index understates and the Paasche index overstates.

Geometric indices

The class of geometric indices is also used with some regularity in empirical work. The geometric indices are defined by:

$$Z_G = {}_i I_N (Z_i^1 / Z_i^0)^{S_i}, \quad (12)$$

where $S_i^0 = P_i^0 X_i^0 / \sum_N P_i^0 X_i^0$ and $S_i^1 = P_i^1 X_i^1 / \sum_N P_i^1 X_i^1$. These indices are exact for a Cobb-Douglas aggregator function defined by:

$$f(Z) = A_0 \prod_N Z_i^{A_i}, \quad (13)$$

where $\sum_N A_i = 1$: The geometric mean index is one example of (12) with an aggregator function like (13). Frisch (1936) notes that

⁶Thus more than one index-number formula can be exact for the same aggregator function, and one index-number formula can be exact for quite different aggregator functions (Diewert 1981).

since the geometric index satisfies the circular test, there is no difference between the chain and direct (fixed-base) indices (discussed further below).

The Vartia I (1974, 1976b) index provides another example of one in which the aggregator function is Cobb-Douglas.⁷ The Vartia I price index $P_V(P^0, P^1, X^0, X^1)$ may be written:

$$\begin{aligned} \ln P_V(P^0, P^1, X^0, X^1) & \quad (14) \\ &= \sum_N [L(P_i^1 X_i^1, P_i^0 X_i^0) / L(\sum_N P_i^1 X_i^1, \sum_N P_i^0 X_i^0)] \ln(P_i^1 / P_i^0), \end{aligned}$$

where the logarithmic mean function L introduced by Vartia (1974) and Sato (1976) is defined by $L(a, b) = (a - b) / (\ln a - \ln b)$ for $a \neq b$ and $L(a, a) = a$. Thus for equation (14), $L(P_i^1 X_i^1, P_i^0 X_i^0) = (P_i^1 X_i^1 - P_i^0 X_i^0) / [\ln(P_i^1 X_i^1) - \ln(P_i^0 X_i^0)]$ and $L(\sum_N P_i^1 X_i^1, \sum_N P_i^0 X_i^0) = (\sum_N P_i^1 X_i^1 - \sum_N P_i^0 X_i^0) / [\ln(\sum_N P_i^1 X_i^1) - \ln(\sum_N P_i^0 X_i^0)]$.

The Vartia I quantity index $Q_V(P^0, P^1, X^0, X^1)$ is defined by:

$$\begin{aligned} \ln Q_V(P^0, P^1, X^0, X^1) & \quad (15) \\ &= \sum_N [L(P_i^1 X_i^1, P_i^0 X_i^0) / L(\sum_N P_i^1 X_i^1, \sum_N P_i^0 X_i^0)] \ln(X_i^1 / X_i^0) \\ &= \ln P_V(X^0, X^1, P^0, P^1), \end{aligned}$$

i.e., the price and quantity indices have the same functional form except that the roles of prices and quantities are interchanged. The Vartia I price and quantity indices satisfy the Fisher factor-reversal test (4) and have the property (defined below) of consistency in aggregation (Diewert 1978).

Diewert (1978) further notes that the Vartia I index approximates to the second order any superlative index. Thus the Vartia I (and geometric mean) index $Z_V(P^0, P^1, X^0, X^1)$ will be close to any superlative index $Z_S(P^0, P^1, X^0, X^1)$ provided that P^0 is close to P^1 and X^0 is close to X^1 . Diewert calls this type of index pseudo-superlative. Moreover, this property holds without the assumption of optimizing behavior on the part of economic agents, since this property is founded upon theorems in numerical analysis rather than economics.

Diewert (1978) further suggests that the Vartia I price and quantity indices have serious defects which preclude their empirical applications. The Vartia I quantity index has the property such that rescaling the prices in either period will generally change the index, (i.e., in general $Q_V(TP^0, P^1, X^0, X^1) \neq TQ_V(P^0, P^1, X^0, X^1)$ for $T \neq 1$), while the Vartia I price index has the property such that rescaling the comparison period (or economic entity) price does not in general change the value of the price index by the same scale factor (i.e., in general, $P_V(P^0, TP^1, X^0, X^1) \neq TP_V(P^0, P^1, X^0, X^1)$ for $T \neq 1$).

Quadratic mean-of-order- r (superlative) indices

Quadratic-mean-of-order- r indices are increasingly used in applied economics. These indices improve upon the mean-of-order (Laspeyres, Paasche) and geometric indices without extending the information required. The quadratic mean-of-order- r indices are superlative, because they are exactly represented by flexible aggregator functions. Fundamental to this approach is the quadratic lemma.

⁷Sato (1976) shows that the Vartia II indices are exact for a CES aggregator function.

Quadratic lemma—Diewert (1976a) shows that a superlative index can be expressed in terms of only first-order derivatives. This is the basic property that allows construction of quadratic mean-of-order- r indices with the same information as mean-of-order- r indices. Diewert provides a quadratic approximation lemma which uses the following homogeneous quadratic functional form:

$$f(X) = A_0 + A^T X + \frac{1}{2} X^T A X \quad (16)$$

$$= A_0 + \sum_i A_i X_i + \sum_{i,j} A_{ij} X_i X_j,$$

where A_0 , A_i , and A_{ij} are constants for all i and j , X is an N -dimensional vector, and T represents the transpose operator. Diewert's quadratic approximation lemma states that if the homogeneous quadratic aggregator function f is defined by (16), then:

$$f(X^1) - f(X^0) = \frac{1}{2} [Df(X^1) + Df(X^0)] T [X^1 - X^0], \quad (17)$$

where $Df(X')$ represents the gradient vector of f evaluated at X' , i.e., the matrix of second-order derivatives. Diewert (1981) notes that this lemma follows simply by differentiating f and substituting the partial derivatives into (17). Intuitively, the quadratic lemma states that the difference between the values of a quadratic function evaluated at two points is equal to the average of the gradient (first-order information) evaluated at both points multiplied by the difference between the points.

All first-order approximations satisfy the quadratic lemma. All second-order approximations of the form given in equation (16) also satisfy the quadratic lemma. Moreover, even quadratic functions in which the zero-order and first-order parameters are specific to a data point satisfy the quadratic lemma (the second-order terms are constant across all data points). However, should the second-order parameters be specific to a data point, then the quadratic lemma is not satisfied. Denny and Fuss (1983) provide further discussion on this point.

Contrasting Diewert's lemma with the usual Taylor's series expansion for a quadratic function indicates that knowledge of $D^2 f(X^0)$, i.e., second-order terms, is not required to construct superlative indices.⁸ Moreover, it is not necessary to econometrically estimate the (generally unknown) coefficients which occur in the matrix of coefficients; only the observable price and quantity vectors are required.

Due to the fundamental importance of the quadratic approximation lemma, Appendix 5 provides additional discussion. Particular attention is given to providing intuition into the lemma.

Superlative indices—An index is superlative when it is exact for an aggregator function which provides a second-order approximation to a linear homogeneous function. The superlative indices (and the quadratic mean-of-order- r aggregator function) do not require commodities to be either perfect or zero substitutes. If the relative price of a commodity increases, the economic agent decreases its use (substituting other inputs) until all marginal productivities are

proportional to their new prices. Therefore, the prices from both periods or economic entities enter the superlative index to represent the marginal productivities in both periods or economic entities. Superlative indices also offer a solution to errors of aggregation (which occur because there are changes in the mix of the components making up the aggregate). Use of a superlative index-number procedure on the components of an aggregate will further capture correctly any changes or differences in the quality of the components over time or between economic entities (Christensen 1975).

Quadratic mean-of-order- r —The general class of quadratic mean-of-order- r indices are exact for the quadratic mean-of-order- r aggregator function. For $r \neq 0$, the quadratic mean-of-order- r quantity index can be written:

$$Q_r(P^0, P^1, X^0, X^1) \quad (18)$$

$$= \left[\sum_i S_i^0 (X_i^1/X_i^0)^{r/2} \right]^{1/r} \left[\sum_j S_j^1 (X_j^1/X_j^0)^{-r/2} \right]^{-1/r},$$

where $S_i^0 = P_i^0 X_i^0 / \sum_N P_i^0 X_i^0$ and $S_j^1 = P_j^1 X_j^1 / \sum_N P_j^1 X_j^1$. The quadratic mean-of-order- r price index can similarly be written:

$$P_r(P^0, P^1, X^0, X^1) \quad (19)$$

$$= \left[\sum_i S_i^0 (P_i^1/P_i^0)^{r/2} \right]^{1/r} \left[\sum_j S_j^1 (P_j^1/P_j^0)^{-r/2} \right]^{-1/r}.$$

A multiplicity of superlative price and quantity indices exist, depending upon the value of r . Two superlative index numbers are widely used. The Fisher Ideal index is defined for $r = 2$, and the Tornqvist index is a limiting case as r tends to 0.

Fisher's Ideal—Fisher's Ideal index can be written as:

$$FI_{kl} = \left[\left(\sum S_{li} (Z_{ki}/Z_{li}) \right) / \left(\sum S_{ki} (Z_{ki}/Z_{li}) \right) \right]^{1/2}, \quad (20)$$

where S_{li} and S_{ki} are value share weights for the two economic entities or time periods being compared (k, l), and the Z_i are the corresponding prices or quantities. This index is simply the geometric mean of the Laspeyres and Paasche indices. The Fisher Ideal index is the exact index for the linearly homogeneous quadratic mean-of-order two-aggregator function.⁹ Diewert (1974b) shows that this aggregator function is flexible, implying that the Fisher Ideal index is superlative.

The Fisher Ideal index possesses several nice properties. (Diewert 1976a, 1981) The Fisher Ideal price and quantity indices can be obtained by simply interchanging the quantities and prices in the same general formula. They are also consistent with both a linear aggregator function (perfect substitution) and a Leontief aggregator function (no substitution); no other superlative index-number formula has this rather nice property. The Fisher ideal index numbers are the only pair among the quadratic mean-of-order- r numbers which satisfies the Fisher weak factor-reversal test. As a consequence, the implicit indices equal the direct indices, and no difficulty arises in choosing whether to use an implicit price or quantity index according to the relative variation in price and quantity data going

⁸Denny and Fuss (1983) generalize the quadratic lemma to the case of discrete variables. They demonstrate that the quadratic lemma can still be applied without alteration when a subset of X is a vector of noncontinuous variables. In effect, the discreteness of the variables is ignored. Denny and Fuss (1983) note that the quadratic lemma can be interpreted as resulting from a differencing of two linear approximations. They further note that the quadratic lemma is exact for linear and quadratic functions unless the latter's second-order parameters are not independent of the point of approximation. Appendix 5 discusses this latter topic in greater detail.

⁹The quadratic mean-of-order- r aggregator function f is defined by $f_i(Z) = \left(\sum_N \sum_k A_{ki} Z_k^{r/2} Z_i^{r/2} \right)^{1/r}$. The aggregator function for Fisher's Ideal index number is therefore $f_2(Z) = \left(\frac{1}{2} \sum_N \sum_k A_{ki} Z_k Z_i \right)^{1/2}$, i.e., it is the square root of a homogeneous quadratic function.

from one observation to another. (See Appendices 2 and 3 for additional details.) Moreover, as the degree of data disaggregation increases (so that a quantity may become zero), the Fisher Ideal index formula remains well defined. Kirkley (1984) notes that the Fisher Ideal may be biased unless the biases inherent in the Paasche and Laspeyres exactly counterbalance one another.

Tornqvist—The Tornqvist index in its logarithmic form can be written as (Tornqvist 1936):

$$\ln TI_{kl} = \sum \frac{1}{2}(S_{li} + S_{ki}) \ln(Z_{ki}/Z_{li}), \quad (21)$$

or, without the logarithmic transformation:

$$TI_{kl} = II_i(Z_{ki}/Z_{li})^{1/2}(S_{li} + S_{ki}). \quad (22)$$

The Tornqvist index is superlative, since it is exact for the linearly homogeneous translog aggregator function.¹⁰ The S_{ki} (revenue or cost shares) are the values of the logarithmic derivatives $Df(X^1)$ and $Df(X^0)$ of the quadratic lemma when logarithms of X^1 and X^0 (output or input quantities or prices) are used. Intuitively, the use of these shares as weights incorporates any factor substitution or product transformation which may have occurred. This index requires the assumption of constant share derivatives across comparisons k and l (Denny and Fuss 1983).

Choice among superlative indices—The choice among the various possible superlative indices, i.e., the choice of r in equation (18), for empirical applications may not be important, provided that the variation in prices and quantities is not too great going from period (or economic entity) 0 to 1 (Diewert 1981). This occurs because all superlative indices differentially approximate one other to the second order, provided prices and quantities are the same for the two periods or economic entities. Moreover, the assumption of optimizing consumer or producer behavior is not required to achieve these results. Appendix 3 provides some related discussion in this area.

Maddala (1979) suggests that differences in functional form of the aggregator function (and therefore the choice of index number) produce negligible differences in measures of TFP. Intuitively, the different functional forms suggested in the literature differ in their elasticities of substitution (which depend on the second derivatives of the production function), whereas from the quadratic approximation lemma only the first derivatives matter. Maddala suggests that for productivity measurement, other matters such as disequilibrium, measurement errors, aggregation problems, and economies of scale are more important than functional forms of the aggregator function. The choice of functional form may then be advocated for other reasons.

Denny and Fuss (1983) and Hazilla and Kopp (1984b) provide evidence that suggests caution when using the growth-accounting framework to quantify changes in productivity, outputs, and inputs, since such procedures calculated with the modern theory of index numbers ignore second-order price effects. This topic receives additional attention in Appendix 5.

¹⁰The linearly homogeneous translog aggregator function f is defined as $\ln f(Z) = A_0 + \sum_i A_i \ln Z_i + \frac{1}{2} \sum_{i,j} A_{ij} \ln Z_i \ln Z_j$, where $\sum_i A_i = 1$, $\sum_i \sum_j A_{ij} = \sum_j \sum_i A_{ij} = \sum_j \sum_i A_{ij} = 0$. If input prices are being aggregated then this represents a translog unit-cost function, while if quantities are being aggregated, this represents a translog production function.

Divisia indices

An alternative to approximation is the construction of Divisia indices (Deaton and Muellbauer 1980, Hulten 1973). Divisia indices analyze the continuous effects of price, quantity, or TFP changes instead of comparing two discrete price, quantity, or TFP situations. The Divisia index for period t is defined by:

$$Z(t) = Z(0) \exp \left[\int_0^t \sum_N S_i(t) d \log Z_i / dt dt \right], \quad (23)$$

where $S_i(t)$ is the value share of the i th commodity defined as $S_i(t) = P_i(t)X_i(t) / \sum_N P_i(t)X_i(t)$ and $Z(0)$ is an arbitrary base-period (or economic entity) price, quantity, or TFP level. The Divisia index comparing, say, Z^0 and Z^1 can be written as:

$$\log(Z^1/Z^0) = \int_{Z^0}^{Z^1} \sum_N S_i d \log Z_i. \quad (24)$$

The Divisia index is a line integral, defined with respect to infinitesimal changes in $Z_i(t)$, so that discrete approximations to the Divisia involve approximations to the continuous rate of change of components of the index and to the value share in some infinitesimal interval shares. These approximation errors could accumulate over time causing the index to drift over time. Discrete approximations to the Divisia converge to the Divisia as the discrete units of prices and quantities become small enough and if relative value shares are constant over time (or economic entities). If shares are not constant, the discrete approximation involves an error that depends on the variability of the relative shares and the length of the time period (Trivedi 1981).

Chain and fixed-base indices

Indices may be constructed using either the chain principle or the fixed-base method. The fixed-base method directly compares all changes in prices, quantities, or total-factor productivity to some initial base period or economic entity level. The base period may remain constant or may be changed after some period of time. Two overlapping series of binary comparisons using base periods can also be spliced. The chain method directly compares adjacent observations, while nonadjacent observations are only indirectly compared, using the intervening observations as intermediaries. This practice results in transitive comparisons. Frisch (1936) notes that any chain index satisfies the factor-reversal test and the circular test. Appendix 2 provides further details.

Chain indices make use of all the data from the initial year cumulated to the current year. The concept of using cumulated data leads to the Divisia index in theory and the chain index as its practical realization. In contrast, a base index provides a sequence of direct binary comparisons between the current year and the base year and no reference to the course of prices and quantities in between. Allen (1975) notes that from a statistical angle, fixed-base indices are inefficient in that they do not make full use of all the data as they unfold over time. Fixed-base indices also imply that a price (quantity) index in year t is not influenced by prices (quantities) before year t as well as those achieved in year t .

The difference between chain and fixed-base indices can be intuitively presented. The functional form of the true aggregator function is unknown, so that different index-number formulae provide first- or second-order approximations to the true underlying, but unknown, aggregator function. Approximation errors which arise with these indices are smaller, as are the changes in prices

and quantities from one period to the next. These changes, and therefore approximation errors, are typically (although not always) smaller when the time periods are adjacent to one another than when separated by wide intervals.

To illustrate, suppose the surface of the true aggregator function $f(X)$ is concave to the origin and smooth. Intuitively, over time a chain index "creeps along" the surface of this true aggregator function, providing in effect a piecewise approximation to this surface over the relevant range. The errors of approximation should then be relatively small. In contrast, the fixed-base index compares increasingly divergent points on the true function over time, so that the approximation errors are often increasing over time and are generally larger than with chain indices. Thus the degree of approximation should usually be closer if the chain principle rather than the fixed-base principle is used to construct index numbers. Additional intuition can be developed in terms of the quadratic approximation lemma by reference to Appendix 5, and Appendix Figure 5-1. In this case, the approximation error with chain indices should generally be lower than with fixed-base indices because the two linear functions being averaged lie next to one another in adjacent time periods.

Errors also arise for the Paasche and Laspeyres fixed-base indices, because the base-period quantities or prices (used as weights) reflect a bundle of inputs or outputs whose composition is increasingly likely to change over time. Not a great deal of meaning can be attached to base-period indices which compare distant periods for which the relative quantities or weights may be very different. The reason why Laspeyres and Paasche index numbers (and their derivatives, the Marshall-Edgeworth and the Fisher Ideal indices) do not meet the circular test is because the weights in these indices depend on the period for which the comparisons are being made (Karmel and Polasek 1970).

Norton et al. (1985) use a fixed-base index in which the weights are not changed from period-to-period. Consequently, less information needs to be collected in order to calculate it. However, fixing the weights implies that they are increasingly out-of-date as time passes (i.e., the base-period relative prices at which outputs and inputs are being valued cease to be relevant).

Longer-term comparisons are made with chain indices by a process of chaining direct binary comparisons (also called price relatives). Such an index is called a chain index, and the formula is:

$$P_{0,t}^{ch} = P_{01} \times P_{12} \times P_{23} \times \dots \times P_{t-1,t} \quad (25)$$

where the separate links in the chain are binary comparisons between adjacent periods (two-period base indices where the base is updated each period) made according to some index-number formula. The formula reflects the basic relationship:

$$P_t/P_0 = P_1/P_0 \times P_2/P_1 \times P_3/P_2 \times \dots \times P_t/P_{t-1} \quad (26)$$

For example, let the prices in period-0 be \$1.00, in period-1 \$1.10, in period-2 \$1.32, and in period-3 \$1.19. Then $P_{00} = 1.00$, $P_{01} = 1.10$, $P_{12} = 1.12$, $P_{23} = 0.90$, $P_{03} = 1.19$, and $P_{03} = 1.00 \times 1.10 \times 1.20 \times 0.90 = 1.19$. (Allen (1975), Frisch (1936), Karmel and Polasek (1970), Kirkley, Va. Inst. Mar. Sci., Gloucester Point, VA 23062, pers. commun. Feb. 1987).

Although the precise meaning of a chain index $P_{0,t}^{ch}$ is not simple in character, because it is based on a changing collection of items, nevertheless there is a sense in which weights are kept up-to-date in the chained index; the weights are unlikely to change radically between adjacent periods. The value of a chain index

$P_{0,t}^{ch}$ will not be the same as that of a base-period index P_0 , where a direct point-to-point comparison is involved (Karmel and Polasek 1970).

To compare prices in period 2 relative to period 1 with chain indices, either a direct binary comparison can be made (calculating the price relative to P_{12}) or a figure obtained by dividing P_{02} by P_{01} . This example is actually a specific case of the general problem of changing the base of a series. Suppose that there exists an index for a number of periods with a certain base and it is desired to change the period used as a basis for comparison. The usual practice is to divide through the whole series by the original index number for the new base. In the case of a chained index or an aggregate index with fixed weights, this correctly accomplishes the change in base. For chained indices:

$$P_{2,t}^{ch} = P_{0,t}^{ch}/P_{02}^{ch} \quad (27)$$

$$= [P_{01} \times P_{12} \times P_{23} \times P_{34} \times \dots \times P_{t-1,t}]/[P_{01} \times P_{12}] \\ = P_{23} \times P_{34} \times \dots \times P_{t-1,t}$$

In general, for two time periods s and t , where $s < t$, then the chain index between the two points $P_{s,t}^{ch}$ can be defined as above, but this simply reduces to:

$$P_{s,t}^{ch} = P_{s,s+1} \times P_{s+1,s+2} \times \dots \times P_{t-1,t} \quad (28)$$

Strictly speaking, such a procedure is not valid where indices with changing weights are being used, since a change in the period of reference then requires a change in weights. In practice, this is usually ignored (Allen 1975; Frisch 1936; Karmel and Polasek 1970).

The divergency which exists between a chain index and the corresponding direct or fixed-base index (when the latter does not satisfy the circular test) will often take the form of a systematic drifting (Frisch 1936). This means that with increasing time t , the ratio $P_{s,t}^{ch}/P_{s,t}$ ($t > s$), where $P_{s,t}^{ch}$ denotes the chain index between periods s and t , increasingly departs from unity. The Laspeyres index tends to drift upwards, the Paasche index tends to drift downwards, and the Fischer ideal index tends to drift downwards. Geometric indices should not drift over time, because there are no differences between chain and fixed-base geometric indices. Frisch notes that drifting must not be taken to mean that the fixed base index is right and the chain index wrong. Frisch (1936), Allen (1975), and Karmel and Polasek (1970) provide additional discussion and methods of measuring the amount of drifting.

Diewert (1978, 1981) generally recommends the use of chained rather than fixed-base indices. All superlative, pseudosuperlative, Paasche, and Laspeyres index numbers should coincide quite closely if they are constructed using the chain principle. The chained Paasche, Laspeyres, or any superlative index number can also be regarded as discrete approximations to the continuous-line integral Divisia index, which has some useful optimality properties from the standpoint of economic theory. These discrete approximations will be closer to the Divisia index if the chain principle is used. Moreover, the use of chained indices avoids problems of discontinuities which arise when the base year in the fixed-base indices is changed. The use of chained indices avoids the discontinuities introduced by period changes in the base year.

Consistency in aggregation

Indices of prices and quantities (input and output) and TFP might be constructed from data at the level of the individual firm or consumer (or even region or nation) or from previously constructed subindices (Deaton and Muellbauer 1980; Diewert 1978, 1981). In the first case, an index is constructed in a single step. In the second case, there are two or more stages of construction. This raises the issue of consistency in aggregation. Vartia (1974) defines an index-number formula to be consistent in aggregation if the numerical value of the index constructed in two or more stages necessarily coincides with the value of the index calculated in a single stage. Thus, for example, a discrete Divisia index of discrete Divisia indices would be the discrete Divisia index of the components. Vartia (1976a) notes that the Paasche, Laspeyres, and the geometric indices, including the Vartia I, are consistent in aggregation. Unfortunately, the superlative indices are not consistent in aggregation.

Consistent aggregation providing a perfectly satisfactory overall index that can be applied to individual periods in an intertemporal context, to individual economic entities, or to subgroups of commodities requires homothetic weak separability of the underlying aggregator function.¹¹ Thus to justify the two-stage method of calculating index numbers for any partition of variables requires an aggregator function, such as the Cobb-Douglas, which is homothetically separable in the same partition that corresponds to the two stages. The Paasche and Laspeyres indices are consistent in aggregation since the underlying aggregator function is either linear or Leontief, the Vartia I's underlying aggregator function is the Cobb-Douglas, and the Vartia II's underlying aggregator function is the CES. If the underlying aggregator function is not separable, any attempt to construct an overall or group quantity index by using subgroup indices will result in the group-quantity index varying with variations in quantities of commodities outside of that group. An implicitly separable underlying aggregator function for an index also allows consistent aggregation. Blackorby et al. (1978) (Deaton and Muellbauer 1978; Diewert 1978, 1981; Blackorby et al. 1978), provide further details.

Although superlative indices are not consistent in aggregation when constructing overall indices out of individual subindices, Diewert (1978) shows that they are approximately (second-order differentially) consistent in aggregation. Thus a practical objection to the use of superlative index-number formulae loses its force. Moreover, the degree of approximation will become closer if, for the time-series data, indices are constructed by chaining observations in successive periods rather than by the fixed-base method. To summarize, constructing aggregate indices by aggregating two

¹¹Separability is the relevant property of aggregator functions which allows aggregation. Weak separability requires that the marginal rates of technical substitution (MRTS) between all pairs of variables (e.g., prices) in a particular group of commodities be independent of changes in the levels of variables not in that group. While weak separability of the aggregator function for some group of commodities is necessary and sufficient for the existence of an aggregate, homotheticity is a necessary and sufficient condition for the validity of the multistage aggregation procedure. Homotheticity insures expansion paths for the subgroup of commodities comprising the aggregate which are straight lines emanating from the origin. Homothetic separability for the aggregate exists if the aggregator function is weakly separable in the individual components of the aggregate and the aggregator function is linearly homogeneous.

Homotheticity is a characteristic of production technology which restricts expansion paths to be straight lines from the origin. Marginal rates of substitution or transformation and factor or product shares are therefore constant along any expansion path. In contrast to homogeneity, homotheticity does not place any restrictions on the spacing of isoquants, and returns-to-scale are a function of the initial level of inputs or outputs.

(or more) stages will give approximately the same answer that a one-stage index would, provided that either a superlative index or the Vartia I index is used. Further, given the otherwise superior properties of the superlative index formulae, this procedure is preferred.

Homogeneity and homotheticity

The discussion of superlative index numbers has been developed in the context of aggregator functions which are linearly homogeneous. Moreover, the commodities to be aggregated are implicitly assumed to be separable. Graphically, the set of all isoquants for input quantities to be aggregated into a composite input quantity lie on a straight line from the origin and are equally spaced. The marginal rates of transformation between these inputs are therefore fixed and independent of the particular combinations of other inputs and outputs.

Should the true aggregator function (which the superlative index-number formulae approximate) be homothetic but not linearly homogeneous, then no serious difficulties arise. In the homothetic case, all isoquants (or isoproduct curves) are the same shape and, for any factor ratio, lie on a straight line emanating from the origin (the expansion path). Therefore, the distance between any pair of isoquants is the same on any ray from the origin, and bundles of inputs can be compared directly (Christensen 1975).

If the true aggregator function is nonhomothetic, then isoquants can have different shapes and do not lie on a straight line from the origin. Marginal rates of substitution (and transformation) and factor (and product) shares vary with the aggregate's level. Comparison of input (or output) bundles can be made only by reference to an isoquant corresponding to a particular level of the aggregate. Even if the true aggregator function is nonhomothetic, however, economic theory provides some appealing justification for the use of flexible index numbers. For example, Diewert (1976a) has shown that the Tornqvist index is exact for the nonhomothetic translog aggregator function when the isoquant for the geometric mean output (of the base and comparison-period input bundles) is the basis for comparison. (This is particularly appealing for multilateral indices.) Diewert (1976a) also notes that the Fisher Ideal index will indicate correctly the direction of change in the aggregate, even if the true aggregator function is nonhomothetic. Diewert (1976a, 1981) further shows that the Tornqvist and Fisher Ideal superlative index-number formulae are exact for nonhomothetic aggregator functions. In particular, he shows that any quadratic mean-of-order- r index can approximate an arbitrary nonhomogeneous function to the second order (Diewert 1976a,b, 1981; Christensen 1975; Swamy and Bingswanger 1980).

Bilateral and multilateral indices

Two basic types of output, input, and total-factor productivity (TFP) indices can be developed: bilateral and multilateral. Bilateral indices provide intertemporal comparisons of, say, TFP for any given economic entity or interspatial comparisons among economic entities for any given time period. The Tornqvist bilateral index of TFP may be written as:

$$\ln TFP_k - \ln TFP_l = \sum_i 0.5 (R_{ik} + R_{il}) [\ln Y_{ik} - \ln Y_{il}] \quad (29)$$

$$- \sum_i 0.5 (W_{ik} + W_{il}) [\ln X_{ik} - \ln X_{il}]$$

where k and l are adjacent time periods (or economic entities), the Y_{ij} are output indices for output i of economic entity j , the X_{ij} are input indices, the R_{ij} are output revenue shares, and the W_{ij} are input cost shares. Diewert (1976a) shows that (29) can be derived from a homogeneous translog product-transformation function that is separable in inputs and outputs and exhibits neutral differences in technology. Caves and Christensen (1980) show that separability and Hicks neutral technological change are not required to derive (29) from a homogeneous translog product-transformation function.

The direct use of the bilateral index of TFP is limited for comparisons that are not binary, e.g., TFP of one economic entity in some time period with TFP of another economic entity of a different time period. For example, interpreting k and l as time periods or firms, the total number of possible binary comparisons of the kl time-differentiated firm observations is given by the formula for combinations. There is no guarantee of transitivity in such comparisons. As Caves et al. (1981, 1983) note, in a given year firm k might be found to be more productive than firm l and less productive than firm m ; yet a direct comparison of l and m might indicate that m is less productive than l . This possible lack of transitivity occurs because weights R_{ij} and W_{ij} specific to the firms in question are used. The traditional solution to this problem is to use weights that are not specific to the individual observation. The disadvantage of this solution is that the comparisons lose what is called characteristicity, that is, they are no longer based on economic conditions specific to the two entities being compared (Caves et al. 1981, 1983).

Although transitivity of comparisons and complete characteristicity cannot be simultaneously achieved, transitive results can be achieved in a multilateral setting by the following compromise formula (Caves et al. 1982a; Caves et al. 1981, 1983):

$$\begin{aligned} \ln TFP_k - \ln TFP_l &= \sum_i 0.5 (R_{ik} + R_i^*) [\ln Y_{ik} - \ln Y_i'] & (30) \\ &- \sum_i 0.5 (R_{il} + R_i^*) [\ln Y_{il} - \ln Y_i'] \\ &- \sum_i 0.5 (W_{ik} + W_i^*) [\ln X_{ik} - \ln X_i'] \\ &+ \sum_i 0.5 (W_{il} + W_i^*) [\ln X_{il} - \ln X_i'] \end{aligned}$$

where an asterisk associated with a variable indicates the arithmetic mean and an apostrophe indicates the geometric mean.¹² The use of this Tornqvist multilateral index for binary comparisons results in transitive multilateral comparisons that retain a high degree of characteristicity (revenue or cost-share weights specific to the entities and time periods). The weights used to compute the productivity comparisons reflect the economic conditions faced by all economic entities (through R_i^* and W_i^*), but at the same time more than half

of each weight is specific to k or l . In effect, each economic entity is compared with all others by the multilateral index via a hypothetical entity having the geometric average characteristics of all entities. Transitive comparisons are achieved by using this representative firm as the basis for making all possible binary comparisons, i.e., any two firms are compared with each other by comparing both with the representative firm (the geometric mean).

The issue of bilateral vs. multilateral indices also applies to output and input indices. Consider outputs first. The Tornqvist bilateral output index may be written as:

$$\ln Y_k - \ln Y_l = \sum_i 0.5 (R_{ik} + R_{il}) [\ln Y_{ik} - \ln Y_{il}], \quad (31)$$

where there are say $i = 1, \dots, M$ outputs. The Tornqvist multilateral output index may be written as:

$$\begin{aligned} \ln Y_k - \ln Y_l &= \sum_i 0.5 (R_{ik} + i^*) [\ln Y_{ik} - \ln Y_i'] & (32) \\ &- \sum_i 0.5 (R_{il} + R_i^*) [\ln Y_{il} - \ln Y_i'] \end{aligned}$$

where the definitions are as before. Tornqvist bilateral and multilateral input indices can be derived in exactly the same manner, substituting X for Y and W for R .

Comparisons of the output and input index formulae with the TFP formula readily confirm that productivity comparisons can be interpreted as comparisons of outputs to inputs.

Caves et al. (1982b) discuss the application of multilateral indices in the time-series context. Multilateral indices are applicable to cross-section data, combinations of cross-section and time-series data (panel data), and time-series data. Multilateral methods are attractive for cross-section data because there is generally no natural ordering of the data points. In contrast, time-series data have a natural chronological order. For this reason, adjacent observations in time-series data are usually directly compared, while nonadjacent observations are only indirectly compared, using the intervening observations as intermediaries. This procedure is called chain-linking, and results in transitive time-series comparisons.

Time-series comparisons using superlative bilateral chain-linked indices have the undesirable property that nonadjacent observations are only indirectly compared. Superlative multilateral indices directly compare adjacent and nonadjacent observations, but only by destroying the fixity of historical comparisons. As additional observations are added with time, expanding the set of comparisons, the chain-linked bilateral approach leaves the historical comparisons intact, but the multilateral procedure results in new comparisons for the entire time series. This occurs because the multilateral approach compares one observation with another via a hypothetical entity having the average characteristics (geometric mean) of all entities, and as observations are added over time, the hypothetical average entity changes (Caves et al. 1982a).

The choice between the bilateral and multilateral approaches depends in large part upon the importance attached to the conflicting traits of symmetry of treatment and fixity of historical comparisons. The issue of symmetry becomes important with panel data. The set of time-series comparisons could be linked together through any single cross section, but the results would differ from those obtained by choosing any other cross section. An equally unattractive alternative would construct all the cross-section comparisons and combine them by chain-linking the results through an arbitrarily chosen economic entity (firm, region). The results would then differ from those obtained by choosing any other country. The multilateral approach to panel-data comparisons treats all economic entities

¹²Caves et al. (1981) note that the equation for Tornqvist multilateral TFP comparisons can be derived directly from a translog transformation structure by taking the difference between each firm's transformation function and the function resulting from averaging arithmetically the transformation functions across all observations. Moreover, the use of revenue shares as weights implies that the structure of production exhibits constant returns-to-scale and that the prices of the outputs are proportional to their marginal costs. Relaxation of these assumptions would require extensive econometric estimation. Finally, Caves et al. (1982a,b) note that multilateral comparisons can be obtained from Fisher Ideal bilateral indices in exactly the same way that translog multilateral indices are obtained from translog bilateral indices. However, it is not known whether or not the resulting index can be directly derived from a flexible transformation function that is nonseparable in inputs and outputs and permits non-neutral differences in productivity among economic entities. For these reasons, Tornqvist bilateral and multilateral indices are used in this study rather than the Fisher Ideal index procedure.

(firms, geographical areas) and time periods symmetrically (Caves et al. 1982a).

In conclusion, Caves et al. (1982a,b) state that the superlative multilateral indices are very attractive for cross-section and panel-data comparisons, but that they are not necessarily preferable to chain-linked bilateral indices for time-series comparisons. This follows because chronology provides a natural ordering of time-series data and historical fixity of constructed indices that is lacking for cross-section or panel data.

Productivity measurement and stock effects in marine fisheries

Intertemporal productivity measurement in marine fishing industries faces a measurement problem peculiar to all natural resource industries: variations in the composition, quantity, and quality of the natural resource being exploited. This problem exists in both renewable and nonrenewable resource industries.

In marine fishing industries, variations in the quantity and species composition of the resource stock over time affect the costs and revenue of harvesting for any given quantity of fish landed. These effects on production might in turn obfuscate the efficiency with which inputs would otherwise be converted into outputs, and could lead to biased measures of productivity. For example, fishermen might adopt fish finders which should allow them to locate and target desired species and quantities of fish while reducing search time, fuel costs, risk, spoilage, and other opportunity costs of harvesting. For any given level and species composition of fish stocks, fishermen's productivity should increase. However, if overall resource abundance is declining and lower-valued species are simultaneously increasing as a proportion, fishermen might actually have to increase their search time, fuel costs, and trip length to attain a constant revenue or catch level.

Changes in resource abundance and composition thus shift the fishery-product transformation frontier: increased resource abundance likely causes an expansion outwards, away from the origin, while decreased resource abundance likely causes a contraction of the frontier toward the origin. These shifts can either reinforce or counteract the productivity trend that would otherwise occur with a constant and homogeneous resource. The residual TFP measure might then capture resource changes along with productivity changes.

Resource changes can further and indirectly affect productivity measurement by affecting (multiproduct) economies-of-scale and capacity utilization (the latter by temporary, short-run disequilibrium). Changes in resource composition are less likely to affect TFP measurement, since induced changes in catch or effort composition should be captured by Divisia indices.

Accounting for the effects of the resource stock upon productivity measurement depends upon the approach taken toward productivity measurement. If the structural approach is adopted, a production, cost, or profit function can be econometrically estimated incorporating measures of resource abundance. Resource abundance should be interpreted as a technological constraint, since it is beyond the control of any individual firm but nevertheless affects the environment within which fishing firms operate. Changes in resource abundance may then be viewed as shifts in the technology that relate the generation of outputs to inputs. The firm's production transformation frontier may then be written as $T(Y, X|A)$, with feasibility written as $T(Y, X|A) \leq 0$, where Y refers to a vector of outputs, X refers to a vector of inputs, and A refers to an index

of technology. Resource abundance can then enter the transformation frontier as either a dummy variable or as a parameter (McFadden 1978, Daughety et. al. 1986)¹³ Allowing this measure to interact with the other inputs further allows estimation of the effects of resource abundance upon economic capacity utilization and scale economies.

Current research is focusing upon incorporating the effects of resource abundance into the growth-accounting or index-number procedure. Until this research is completed, the resulting TFP measure is interpreted as not strictly the rate of technological change, but as a measure of both the technical efficiency with which inputs are converted into outputs and the effects of resource availability and composition. The TFP measure may also include changes in technical efficiency, scale effects and effects from changes in capacity utilization (which may change with changes in resource abundance and composition), and effects of public regulation possibly restricting productivity in the short run (but presumably increasing TFP over the long run as resource stocks rebuild to desirable levels).

Acknowledgments

A study of this nature could not be possible without considerable assistance from a number of people. The comments of Dan Huppert, Sam Herrick, Jim Kirkley, Jeff Lee, and an anonymous reviewer are gratefully acknowledged. Chuck Korson and Wes Silverthorne collected and processed a large portion of the cost data. Ed Ueber provided many of the crew sizes. The very helpful cooperation of Will Daspit, Frank Henry, Jim Golding, Jack Taggart, Dale Ward, and Joe Terry was essential for this project. Any errors remain the responsibility of the author.

Citations

- ALLEN, R.G.D.
1975. Index numbers in theory and practice. Macmillan, London.
- ALLEN, R.C., and W.E. DIEWERT.
1981. Direct versus implicit superlative index number formulae. *Rev. Econ. Stat.* 63:430-435.
- BAUMOL, W., J. PANZAR, and R. WILLIG.
1982. Contestable markets and the theory of industrial structure. Harcourt Brace Jovanovich, San Diego, 510 p.
- BELL, F.W., and R. K. KINOSHITA.
1973. Productivity gains in U.S. fisheries. *Fish. Bull.*, U.S. 71:911-919.
- BERNDT, E., and M.A. FUSS.
1986. Productivity measurement with adjustments for variations in capacity utilization and other forms of temporary equilibrium. *J. Econom.* 33:7-29.
- BLACKORBY, C., D. PRIMONT, and R.R. RUSSELL.
1978. Duality, separability, and functional structure: Theory and applications. North-Holland Publ. Co., New York, 395 p.
- CAVES, D., and L. CHRISTENSEN.
1980. The relative efficiency of public and private firms in a competitive environment: The case of Canadian railroads. *J. Political Econ.* 88:958-976.
- CAVES, D., L. CHRISTENSEN, and W.E. DIEWERT.
1982a. Multilateral comparisons of output, input, and productivity using superlative index numbers. *Econ. J.* 92:73-86.
1982b. The economic theory of index numbers and the measurement of input, output, and productivity. *Econom.* 50:1393-1414.

¹³Stollery (1985) and Halvorsen and Smith (1986) provide examples with Canadian mining. Kirkley (1986) discusses measures of fish stock abundance and production technology. Gordon (1954) first discussed resource availability as a technological constraint; Jim Kirkley (Va. Inst. Mar. Sci., Gloucester Point, VA 23062, pers. commun. Feb. 1987) first brought this to the author's attention.

- CAVES, D., L. CHRISTENSEN, and M. TRETHERWAY.
1981. U.S. trunk air carriers, 1972-1977: A multilateral comparison of total factor productivity, Chapt. 3, *In* Cowing, T., and R. Stevenson (eds.), *Productivity measurement in regulated industries*, p. 47-75. Academic Press, New York.
1983. Productivity performance of U.S. trunk and local service airlines in the era of deregulation. *Econ. Inquiry* 21:312-324.
- CHAN, M.W., and D. MOUNTAIN.
1983. Economies of scale and the Tornqvist discrete measurement of productivity growth. *Rev. Econ. Stat.* 65:663-667.
- CHRISTENSEN, L.
1975. Concepts and measurement of agricultural productivity. *Social Systems Res. Inst. Pap.* 294, Univ. Wisc., Madison, WI 53706.
- CHRISTENSEN, L., and D. JORGENSON.
1969. The measurement of U.S. real capital input, 1929-1967. *Rev. Income Wealth* 15:293-320.
1970. U.S. real product and real factor input, 1929-1967. *Rev. Income Wealth* 16:19-49.
- DAUGHETY, A., F. NELSON, and W. VIDOR.
1986. An econometric analysis of the cost and production structure of the trucking industry. *In* Daughety, A. (ed.), *Analytical studies in transport economics*, p. 65-96. Cambridge Univ. Press, Cambridge.
- DEATON, A., and J. MUELLBAUER.
1980. *Economics and consumer behavior*. Cambridge Univ. Press, Cambridge, 450 p.
- DENISON, E.F.
1962. The sources of economic growth in the U.S. and the alternatives before us. *Suppl. Pap.* 13, Committee for Economic Development, New York.
- DENNY, M., M. FUSS, and L. WAVERMAN.
1981. The measurement and interpretation of total factor productivity in regulated industries, with an application to Canadian telecommunications. *In* Cowing, T., and R. Stevenson (eds.), *Productivity measurement in regulated industries*, p. 179-212. Academic Press, New York.
- DENNY, M., and M. FUSS.
1983. A general approach to intertemporal productivity comparisons. *J. Econom.* 23:315-330.
- DEWEES, C.
1986. Technical innovation in the Pacific commercial and salmon trolling fisheries. Unpubl. thesis, Univ. Calif., Davis, CA 95616.
- DIAMOND, P., D. McFADDEN, and M. RODRIGUEZ.
1981. Measurement of the elasticity of factor substitution and bias of technical change. *In* Fuss, M., and D. McFadden (eds.), *Production economics: A dual approach to theory and applications*, Vol. 2, p. 125-145. North-Holland Publ. Co., Amsterdam.
- DIEWERT, W.E.
1974a. Functional forms for revenue and factor requirements functions. *Int. Econ. Rev.* 15:119-130.
1974b. Applications of duality theory. Chapt. 3 *In* Intriligator, M.D., and D.A. Kendrick (eds.), *Frontiers of quantitative methods*, Vol. 2, p. 106-177. North-Holland Publ. Co., Amsterdam.
1976a. Exact and superlative index numbers. *J. Econom.* 4:114-146.
1976b. Harberger's welfare indicator and revealed preference theory. *Am. Econ. Rev.* 66:143-152.
1978. Superlative index numbers and consistency in aggregation. *Econom.* 46:883-900.
1981. The economic theory of index numbers: A survey. Chapt. 7 *In* Deaton, A. (ed.), *Essays in the theory and measurement of consumer behavior in honor of Sir Richard Stone*, p. 163-208. Cambridge Univ. Press, Cambridge.
- DIEWERT, W., and C. PARKAN.
1985. Tests for the consistency of consumer data. *J. Econom.* 30:127-147.
- DUNCAN, A.J.
n.d. New Zealand's inshore fishery: A summary of economic conclusions and management options. N.Z. Fishing Industry Board, Wellington, mimeo.
- FISHER, I.
1911. *The purchasing power of money*. Macmillan, London.
1922. *The making of index numbers*. Houghton Mifflin, Boston.
- FRISCH, R.
1936. Annual survey of general economic theory: The problem of index numbers. *Econom.* 4:1-38.
- GORDON, H.S.
1954. The economic theory of a common-property resource: The fishery. *J. Political Econ.* 62:124-142.
- HALVORSEN, R., and T.R. SMITH.
1986. Substitution possibilities for unpriced natural resources: Restricted cost functions for the Canadian metal mining industry. *Rev. Econ. Stat.* 68:398-405.
- HAZILLA, M., and R. KOPP.
1984a. Intertemporal and interspatial estimates of agricultural productivity. Discussion Pap. QE84-08, Resources for the Future, Wash., D.C.
1985b. The measurement of sectoral productivity: A comparative evaluation of alternative approaches. Discussion Pap. QE84-11, Resources for the Future, Wash., D.C.
- HULTEN, C.
1973. Divisia index numbers. *Econom.* 41:1017-1025.
1975. Technical change and the reproducibility of capital. *Am. Econ. Rev.* 65:956-965.
1986. Productivity change, capacity utilization, and the sources of efficiency growth. *J. Econom.* 33:31-50.
- HUPPERT, D., and C.S. KORSON.
1987. Profile of the groundfish fishery. *In* Huppert, D.D. (ed.), *Limited access alternatives for the Pacific groundfish fishery*, p. 7-11. NOAA Tech. Rep. NMFS 52. Natl. Oceanic Atmos. Adm., Natl. Mar. Fish. Serv., Seattle, WA 98115.
- JORGENSON, D.W.
1974. The economic theory of replacement and depreciation. *In* Sellekaerts, W. (ed.), *Econometrics and economic theory*, p. 189-221. Macmillan, London.
- JORGENSON, D., and Z. GRILICHES.
1967. The explanation of productivity change. *Rev. Econ. Stud.* 34:349-383.
- KARMEL, P.H., and M. POLASEK.
1970. *Applied statistics for economists*, 3d ed. Pitman Publ., London.
- KENDRICK, J.W.
1973. Postwar productivity trends in the United States, 1948-1969. *Natl. Bur. Econ. Res.*, New York.
- KIRKLEY, J.
1984. Productivity in fisheries. Discussion pap., Natl. Mar. Fish. Serv., NOAA, Woods Hole, MA 02543.
1986. Technology, substitution possibilities, and management of a multispecies fishery: A case study of the New England otter trawl industry. Unpubl. thesis, Univ. Maryland, College Park, MD 20742.
- KLOEK, T.
1967. On quadratic approximations of cost of living and real income index numbers. Rep. 6710, Econometric Inst., Netherlands School of Econometrics, Rotterdam.
- KORSON, C.
1981-1985. Annual otter trawl vessel inventory. Natl. Mar. Fish. Serv., NOAA, Southwest Region, Terminal Island, CA 90731, mimeo.
1984. Economic status of the California pink shrimp fishery in 1983. Tech. Memo. NOAA-TM-NMFS-SWR-004, Natl. Mar. Fish. Serv., NOAA, Southwest Fish. Cent., La Jolla, CA 92038.
- KULATILAKA, N.
1985. Are observed technologies at long-run equilibrium? Tests on the validity of the static equilibrium model. *J. Econom.* 15:253-268.
- LAU, L.
1978. Applications of profit functions. *In* McFadden, D., and M. Fuss, (eds.), *Production economics: A dual approach to theory and applications*, Vol. 1, p. 133-216. North-Holland Publ. Co., Amsterdam.
1979. On exact index numbers. *Rev. Econ. Stat.* 61:73-82.
- MADDALA, G.S.
1979. A note on the form of the production function and productivity. *In* Measurement and interpretation of productivity, p. 309-317. Natl. Acad. Sci., Wash., D.C.
- McFADDEN, D.
1978. Cost, revenue, and profit functions. *In* Fuss, M., and D. McFadden, (eds.), *Production economics: A dual approach to theory and applications*, Vol. 1, p. 3-109. North-Holland Publ. Co., Amsterdam.
- MORRISON, C.J.
1985a. Primal and dual capacity utilization: an application to productivity measurement in the U.S. automobile industry. *J. Bus. Econ. Stat.* 3:312-324.
1985b. On the economic interpretation and measurement of optimal capacity utilization with anticipatory expectations. *Rev. Econ. Stud.* 52:295-310.
1986. Productivity measurement with nonstatic expectations and varying capacity utilization: An integrated approach. *J. Econom.* 33:51-74.
- NISHIMIZU, M., and O.J. PAGE, JR.
1982. Total factor productivity growth, technological progress and technical efficiency change: Dimensions of productivity change in Yugoslavia, 1965-78. *Econ. J.* 92:920-936.
- NORTON, V., M. MILLER, and E. KINNEY.
1985. Indexing the economic health of the U.S. fishing industry's harvesting sector. Tech. Memo. NMFS-F/NEC-40, Natl. Mar. Fish. Serv., NOAA, Northeast Fish. Cent., Woods Hole, MA 02543, 42 p.
- OHTA, M.
1975. A note on duality between production and cost functions: Rate of returns to scale and rate of technical progress. *Econ. Stud. Q.* 25:63-65.

- PacFIN (Pacific Coast Fisheries Information Network)
 Management database. Pac. Mar. Fish. Comm., Portland, OR 97201.
 Research database. Natl. Mar. Fish. Serv., NOAA, Southwest Region, Terminal
 Island, CA 90731.
- PACIFIC FISHERY MANAGEMENT COUNCIL (PFMC)
 1985. Pacific coast groundfish plan. Pac. Fish. Manage. Council, Portland, OR
 97201.
- SAMUELSON, P., and S. SWAMY.
 1974. Invariant economic index numbers and canonical duality: Survey and syn-
 thesis. *Am. Econ. Rev.* 64:566-593.
- SATO, K.
 1976. The ideal log-change index number. *Rev. Econ. Stat.* 58:223-228.
- SCHANKERMAN, M., and M.I. NADIRI.
 1986. A test of static equilibrium models and rates of return to quasi-fixed factors,
 with an application to the Bell system. *J. Econom.* 33:97-118.
- SOLOW, R.
 1957. Technical change and the aggregate production function. *Rev. Econ. Stat.*
 23:101-108.
- STAR, S.
 1974. Accounting for the growth of output. *Am. Econ. Rev.* 64:123-135.
- STOLLERY, K.R.
 1985. Productivity change in Canadian mining 1957-1979. *Appl. Econ.*
 17:543-558.
- SWAMY, G., and H. BINGSWANGER.
 1980. Flexible consumer demand with linear estimation equations: Food demand
 in india. Discussion pap. 339, Economic Growth Center, Yale Univ., New
 Haven, CT 06520.
- THEIL, H.
 1960. Best linear index numbers of prices and quantities. *Econom.* 28:464-480.
- TORNQVIST, L.
 1936. The Bank of Finland's consumption price index. *Monthly Bull., Bank
 of Finland* 10:1-8.
- TRIVEDI, P.K.
 1981. Some discrete approximations to Divisia integral indices. *Int. Econ. Rev.*
 22:71-77.
- VARTIA, Y.O.
 1974. Relative changes and economic indices. Licentiate thesis, Univ. Helsinki.
 1976a. Relative changes and index numbers. Research Inst. of the Finnish
 Economy, Helsinki.
 1976b. Ideal log change index numbers. *Scand. J. Stat.* 3:121-126.
- WINSTON, G.
 1974. The theory of capital utilization and idleness. *J. Econ. Lit.* 12:1301-1320.

Table 1
Trawl Landings and Revenue:
Washington, Oregon, northern and central California

Year	Pink shrimp		JV Pacific whiting		Groundfish	
	Tons	Revenue	Tons	Revenue	Tons	Revenue
1981	19923	20.01	48012	6.17	100414	38.06
1982	13913	13.37	74367	9.64	113492	44.33
1983	6613	8.78	79476	9.28	89538	36.71
1984	4818	3.81	86959	10.29	79938	32.26
1985	12779	7.61	34934	3.2	82988	34.86

Note: All values millions of dollars in 1981 dollars
All weights are short tons
Source: PacFIN Management database

Table 2
Number of coastwide otter and shrimp trawl vessels by region and year

Year	Wash.	Oregon	N. Calif.	C. Calif.	Total
1981	87	148	99	72	406
1982	93	169	97	69	428
1983	88	170	98	96	452
1984	97	174	80	63	414
1985	92	146	73	59	370

Note: Vessels employing both otter and shrimp trawl gear are counted only once.
Source: Annual vessel inventory and PacFIN Research database

Table 3
Total number of groundfish and shrimp landings and joint-venture weeks fished by region and year

Year	Wash.	Oregon	N. Calif.	C. Calif.	Total
1981	2488	7410	2687	2529	15114
1982	2107	7097	4181	4470	17855
1983	2302	7723	3565	3984	17574
1984	1976	6071	3441	3544	15032
1985	1908	5686	3959	4857	16410

Source: PacFIN Management database and joint-venture logbooks
Note: Excludes Puget Sound vessels

Table 4
Number of groundfish landings by region and year

Year	Wash.	Oregon	N. Calif.	C. Calif.	Total
1981	1474	4188	2062	2497	10221
1982	1570	4767	3479	4347	14163
1983	1491	6552	3500	3893	15436
1984	1615	5377	3260	3452	13704
1985	1391	4735	3761	4846	14733

Source: PacFIN Management database and Washington Dept. of Fisheries
Note: Excludes Puget Sound vessels

Table 5
Number of pink shrimp landings by region and year

Year	Wash.	Oregon	N. Calif.	C. Calif.	Total
1981	1014	3108	625	31	4778
1982	537	2082	702	75	3396
1983	811	1102	43	48	2004
1984	322	508	118	49	997
1985	488	951	171	4	1614

Source: PacFIN Management database
Note: Excludes Puget Sound vessels

Table 6
Multilateral total-factor productivity

Year	Wash.	Oregon	N. Calif.	C. Calif.	Fleet
1981	1.000000	0.666324	0.988394	0.878428	1.000000
1982	0.969277	0.657095	0.784490	0.780530	0.920491
1983	0.996574	0.619725	0.781322	0.672099	0.883903
1984	1.080838	0.714634	0.869972	0.808189	1.006902
1985	1.186149	0.768584	0.871066	0.737701	1.040016

Note: Tornqvist multilateral chain indices.
Normalized on either 1981 Washington or 1981 fleet.

Table 7
Bilateral total-factor productivity

Year	Wash.	Oregon	N. Calif.	C. Calif.	Fleet
1981	1.000000	1.000000	1.000000	1.000000	1.000000
1982	0.979454	0.984080	0.812086	0.905919	0.928293
1983	0.985601	0.905697	0.811335	0.762710	0.874973
1984	1.086189	0.993388	0.889217	0.919418	0.973892
1985	1.174060	1.097958	0.893955	0.836522	1.015422

Note: Tornqvist bilateral chain indices with 1981 base

Table 8
Tornqvist multilateral TFP growth rates

Year	Wash.	Oregon	N. Calif.	C. Calif.	Fleet
1981	0.385915	-0.02006	0.374241	0.256295	0.232110
1982	-0.03120	-0.01394	-0.23104	-0.11816	-0.08284
1983	0.027773	-0.05855	-0.00404	-0.14956	-0.04056
1984	0.081168	0.142494	0.107473	0.184391	0.130286
1985	0.092975	0.072778	0.001256	-0.09125	0.032357
Avg.	0.042677	0.035693	-0.03159	-0.04364	0.009809

Note: Averages computed for 1982-85.
Computed following conventional productivity practice as $\ln(T+1) - \ln(T)$.
Percentages are obtained by multiplying by 100.

Table 9
Tornqvist bilateral TFP growth rates

Year	Wash.	Oregon	N. Calif.	C. Calif.	Fleet
1981					
1982	-0.02075	-0.01604	-0.20814	-0.09880	-0.07440
1983	0.006256	-0.08300	-0.00092	-0.17207	-0.05915
1984	0.097178	0.092416	0.091659	0.186863	0.107107
1985	0.077792	0.100086	0.005314	-0.09448	0.041759
Avg.	0.040117	0.023363	-0.02802	-0.04462	0.003826

Note: Computed following conventional productivity practice as $\ln(T+1) - \ln(T)$.
Percentages are obtained by multiplying by 100.

Table 10
Tornqvist multilateral-output chain indices for individual species

Year	Wash.	Oregon	N. Calif.	C. Calif.
Dover sole				
1981	1.000000	1.134581	1.235892	0.969079
1982	1.031920	1.213503	1.249156	1.014966
1983	1.043814	1.231094	1.165989	1.029052
1984	1.060182	1.165946	1.178834	1.058754
1985	1.033139	1.155265	1.243946	1.149278
Petrale sole				
1981	1.000000	1.039037	0.998366	1.007475
1982	0.995872	1.077271	0.993645	1.009050
1983	1.017418	1.058683	0.983387	0.983778
1984	1.010374	1.029645	0.990106	0.976597
1985	1.003475	1.020638	1.004848	1.005584
Other flatfish				
1981	1.000000	1.063230	1.021449	1.019490
1982	1.060928	1.080922	1.006804	1.016159
1983	1.039346	1.066653	0.997134	0.978593
1984	1.056870	1.038617	0.986737	0.966470
1985	1.072833	1.049883	1.006727	1.000616
Rockfish				
1981	1.000000	1.135445	0.845552	0.799196
1982	0.987301	1.068244	0.870419	0.960247
1983	0.921153	0.985263	0.791875	0.739806
1984	0.787059	0.940486	0.738121	0.706176
1985	0.746628	0.950586	0.780170	0.706032
Pacific cod and lingcod				
1981	1.000000	0.961548	0.944272	0.950172
1982	0.984703	0.974115	0.946402	0.949718
1983	0.988245	0.981781	0.936098	0.928333
1984	1.015783	0.965144	0.930080	0.928432
1985	1.014086	0.963693	1.013049	0.916561
Sablefish				
1981	1.000000	1.028505	1.065513	1.006452
1982	1.039216	1.061494	1.113609	1.019816
1983	1.026009	1.060395	1.069461	0.997292
1984	1.055970	1.061871	1.058095	0.998186
1985	1.001191	1.069056	1.065822	1.017845
Pacific whiting				
1981	1.000000	1.980067	3.186918	1.266354
1982		3.155684	2.200408	1.465981
1983	1.228060	3.726936	1.827943	1.891140
1984	1.829728	4.331311	1.937834	1.333818
1985	0.569838	2.575733	1.718608	1.083178
Misc. groundfish				
1981	1.000000	0.928832	0.930768	0.925377
1982	0.930179	0.921754	0.927003	0.923938
1983	0.923390	0.924609	0.926919	0.922923
1984	0.941897	0.920984	0.922016	0.921810
1985	0.931889	0.915865	0.926429	0.921569
Pink shrimp				
1981	1.000000	1.455978	0.685070	0.594482
1982	0.776709	1.039321	0.720047	0.562585
1983	0.837948	0.784123	0.537546	0.585361
1984	0.704627	0.723615	0.623711	0.466517
1985	0.918707	0.953571	0.635173	0.376092

Note: Each species normalized on 1981 Washington

Table 11
Tornqvist multilateral total-output chain index

Year	Wash.	Oregon	N. Calif.	C. Calif.	Fleet
1981	1.000000	1.291362	1.021075	0.760186	1.000000
1982	0.949664	1.316348	0.989674	0.843147	1.006640
1983	0.948678	1.301456	0.916246	0.777224	0.979096
1984	0.985571	1.337039	0.912741	0.763521	0.997392
1985	1.042698	1.306008	0.948681	0.789267	1.012015

Note: Normalized on either 1981 Washington or 1981 fleet

Table 12
Tornqvist multilateral total-output growth rates

Year	Wash.	Oregon	N. Calif.	C. Calif.	Fleet
1981	0.209308	0.465006	0.230164	-0.06488	0.298703
1982	-0.05164	0.019163	-0.03123	0.103577	0.006618
1983	-0.00103	-0.01137	-0.07709	-0.08141	-0.02774
1984	0.038151	0.026973	-0.00383	-0.01778	0.018514
1985	0.056346	-0.02348	0.038620	0.033162	0.014554
Avg.	0.010453	0.002819	-0.01838	0.009385	0.002985

Note: Average computed over 1982-85.
Computed following conventional practice as $\ln(T+1) - \ln(T)$.

Table 13
Tornqvist bilateral-output chain indices for individual species

Year	Wash.	Oregon	N. Calif.	C. Calif.
Dover sole				
1981	1.000000	1.000000	1.000000	1.000000
1982	1.030778	1.046032	1.005249	1.044329
1983	1.039376	1.052024	0.919681	1.068231
1984	1.056653	1.007226	0.929270	1.116155
1985	1.032961	0.998675	0.992297	1.234016
Petrale sole				
1981	1.000000	1.000000	1.000000	1.000000
1982	0.997656	1.025436	0.996988	1.001759
1983	1.019186	1.007527	0.991650	0.972668
1984	1.011397	0.984124	0.999488	0.966593
1985	1.003844	0.976959	1.015331	1.007015
Other flatfish				
1981	1.000000	1.000000	1.000000	1.000000
1982	1.050860	1.012182	0.986576	0.993858
1983	1.035842	1.000413	0.982697	0.945585
1984	1.048375	0.980072	0.967930	0.933938
1985	1.058343	0.988099	0.988802	0.979600
Rockfish				
1981	1.000000	1.000000	1.000000	1.000000
1982	0.969789	0.963165	1.025003	1.244426
1983	0.906027	0.903112	0.942332	0.909872
1984	0.772757	0.864293	0.868502	0.881363
1985	0.738265	0.868268	0.919292	0.855615
Pacific cod and lingcod				
1981	1.000000	1.000000	1.000000	1.000000
1982	0.981546	1.008163	1.001947	0.998303
1983	0.985724	1.012166	0.994721	0.972500
1984	1.007928	0.999340	0.988133	0.976240
1985	1.001678	0.998151	1.065126	0.958621
Sablefish				
1981	1.000000	1.000000	1.000000	1.000000
1982	1.038586	1.018711	1.035889	1.011607
1983	1.025752	1.016554	0.982902	0.991006
1984	1.054688	1.016632	0.972060	0.995523
1985	0.996965	1.018245	0.979190	1.022964
Pacific whiting				
1981	1.000000	1.000000	1.000000	1.000000
1982		1.238506	0.836257	1.006033
1983	1.000293	1.288067	0.745557	1.187542
1984	1.847661	1.280507	0.768798	0.919833
1985	0.578058	1.075752	0.729935	0.918422
Misc. groundfish				
1981	1.000000	1.000000	1.000000	1.000000
1982	0.952279	0.998376	0.997840	0.998862
1983	0.946974	0.999049	0.997561	0.998054
1984	0.962312	0.998225	0.994299	0.997104
1985	0.957000	0.997644	0.996707	0.996811
Pink shrimp				
1981	1.000000	1.000000	1.000000	1.000000
1982	0.817113	0.884780	1.032789	0.961276
1983	0.843731	0.713472	0.829293	1.002105
1984	0.754689	0.688080	0.892004	0.858257
1985	0.927733	0.817190	0.899803	0.855178

Year	Wash.	Oregon	N. Calif.	C. Calif.	Fleet
1981	1.000000	1.000000	1.000000	1.000000	1.000000
1982	0.951994	1.017866	0.989733	1.128879	1.012927
1983	0.938590	0.981250	0.919959	1.010007	0.967034
1984	0.983368	0.957565	0.906041	1.003958	0.962535
1985	1.024315	0.966535	0.947134	1.035043	0.988344

Year	Wash.	Oregon	N. Calif.	C. Calif.	Fleet
1981					
1982	-0.04919	0.017708	-0.01031	0.121225	0.012844
1983	-0.01417	-0.03663	-0.07310	-0.11126	-0.04636
1984	0.046604	-0.02443	-0.01524	-0.00600	-0.00466
1985	0.040795	0.009323	0.044356	0.030493	0.026460
Avg.	0.006006	-0.00850	-0.01357	0.008610	-0.00293

Note: Computed following conventional practice as $\ln(T+1) - \ln(T)$.
Percentages are obtained by multiplying by 100.

Year	Wash.	Oregon	N. Calif.	C. Calif.
Dover sole				
1981	0.062378	0.090002	0.239311	0.136368
1982	0.109959	0.116553	0.262092	0.145698
1983	0.107973	0.139658	0.286786	0.188439
1984	0.137110	0.127130	0.308158	0.263875
1985	0.133598	0.133288	0.319552	0.299553
Petrale sole				
1981	0.027498	0.036147	0.024773	0.072473
1982	0.035259	0.057619	0.026134	0.067279
1983	0.057294	0.056145	0.041042	0.054172
1984	0.061184	0.043535	0.047721	0.059309
1985	0.058398	0.039441	0.054731	0.090266
Other flatfish				
1981	0.035638	0.056740	0.073943	0.175925
1982	0.085919	0.062413	0.065854	0.140396
1983	0.052500	0.064428	0.103705	0.110846
1984	0.069220	0.055251	0.083194	0.116338
1985	0.094835	0.060986	0.090399	0.141190
Rockfish				
1981	0.345909	0.258226	0.199817	0.449818
1982	0.430116	0.219123	0.254779	0.532013
1983	0.372185	0.219397	0.356498	0.407205
1984	0.273139	0.254203	0.295879	0.486615
1985	0.286877	0.294933	0.314285	0.391329
Pacific cod and lingcod				
1981	0.060677	0.016345	0.016549	0.050374
1982	0.059709	0.021286	0.019564	0.039840
1983	0.060207	0.030539	0.022901	0.019670
1984	0.087306	0.022967	0.016025	0.028949
1985	0.093287	0.023374	0.044540	0.009347
Sablefish				
1981	0.016350	0.015484	0.057081	0.037595
1982	0.052120	0.029913	0.096263	0.038811
1983	0.036808	0.035748	0.095665	0.026525
1984	0.064435	0.040548	0.084996	0.037598
1985	0.039281	0.058915	0.093637	0.064697
Pacific whiting				
1981	0.000000	0.064884	0.288935	0.000367
1982	0.000000	0.232024	0.138482	0.003749
1983	0.000143	0.298068	0.072521	0.099129
1984	0.160048	0.370051	0.099579	0.000731
1985	0.000161	0.167617	0.032845	0.000012
Misc. groundfish				
1981	0.089734	0.001264	0.003812	0.004731
1982	0.009507	0.000691	0.002876	0.004788
1983	0.001591	0.001229	0.005071	0.004764
1984	0.014890	0.000605	0.001598	0.004165
1985	0.005931	0.000252	0.003622	0.003538
Pink shrimp				
1981	0.361813	0.460904	0.095773	0.072343
1982	0.217406	0.260374	0.133951	0.027422
1983	0.311294	0.154783	0.015806	0.089246
1984	0.132663	0.085705	0.062846	0.002416
1985	0.287628	0.221189	0.046384	0.000063

Note: Columns sum to one for each year

Table 17
Revenue shares for aggregate output by region

Year	Wash.	Oregon	N. Calif.	C. Calif.
1981	0.214827	0.440626	0.241314	0.103231
1982	0.168605	0.500048	0.211973	0.119373
1983	0.223064	0.500821	0.153800	0.122313
1984	0.223811	0.474046	0.183672	0.118469
1985	0.204127	0.433124	0.218770	0.143977

Note: Rows sum to one for each year

Table 21
Laspeyres bilateral total-output fixed-base output index

Year	Wash.	Oregon	N. Calif.	C. Calif.	Fleet
1981	1.000000	1.000000	1.000000	1.000000	1.000000
1982	0.821997	1.153909	0.912882	1.273951	1.061186
1983	0.772182	0.910060	0.539013	1.021493	0.835867
1984	0.727997	0.760679	0.513517	0.797237	0.712299
1985	0.814834	0.786996	0.719087	0.965476	0.803519

Table 18
Revenue shares by species

Year	Flatfish	Rockfish	Sablefish	Pacific whiting	Misc. groundfish	Pink shrimp
1981	0.228841	0.282746	0.027990	0.098352	0.050674	0.311393
1982	0.274514	0.299606	0.048784	0.145825	0.032744	0.198523
1983	0.288509	0.297536	0.044072	0.172589	0.036986	0.160305
1984	0.299684	0.293630	0.053708	0.229618	0.041208	0.082149
1985	0.337891	0.311401	0.063336	0.079819	0.042878	0.164672

Note: Rows sum to one for each year

Table 22
Paasche bilateral total-output fixed-base index

Year	Wash.	Oregon	N. Calif.	C. Calif.	Fleet
1981	1.000000	1.000000	1.000000	1.000000	1.000000
1982	0.847758	1.152493	0.909703	1.255725	1.061971
1983	0.785450	0.800268	0.537646	0.929191	0.772340
1984	0.810597	0.768892	0.559243	0.775773	0.740534
1985	0.707631	0.762588	0.695146	0.931342	0.760912

Table 19
Tornqvist multilateral-chain implicit aggregate real-output price indices

Year	Wash.	Oregon	N. Calif.	C. Calif.	Fleet
1981	1.000000	1.588297	1.100108	0.632123	1.000000
1982	0.875465	0.913271	0.940230	1.452849	0.784505
1983	1.145988	0.632382	0.650732	1.083332	0.646321
1984	0.870856	0.605587	1.119241	1.085154	0.655304
1985	0.881346	0.704910	1.265058	1.551498	0.791198

Note: Formed by Fisher weak factor-reversal test (see text for explanation).
Normalized on 1981 Washington or 1981 fleet.

Table 23
Fisher Ideal bilateral fixed-base total-output index

Year	Wash.	Oregon	N. Calif.	C. Calif.	Fleet
1981	1.000000	1.000000	1.000000	1.000000	1.000000
1982	0.834778	1.153201	0.911291	1.264805	1.061557
1983	0.778787	0.853400	0.538329	0.974249	0.803080
1984	0.768188	0.764775	0.535893	0.786432	0.726065
1985	0.759343	0.774696	0.707016	0.948255	0.781744

Table 20
Tornqvist bilateral-chain implicit aggregate real-output price indices

Year	Wash.	Oregon	N. Calif.	C. Calif.	Fleet
1981	1.000000	1.000000	1.000000	1.000000	1.000000
1982	0.864217	1.168767	0.930372	1.073803	1.055549
1983	1.146229	0.830000	0.641348	0.824957	0.870908
1984	0.863707	0.836760	1.115762	0.816667	0.891656
1985	0.887810	0.942562	1.253913	1.170751	1.032354

Note: Formed by Fisher weak factor-reversal test (see text for explanation).

Table 24
Tornqvist bilateral fixed-base total-output index

Year	Wash.	Oregon	N. Calif.	C. Calif.	Fleet
1981	1.000000	1.000000	1.000000	1.000000	1.000000
1982	0.831325	1.174962	0.906550	1.262224	1.070544
1983	0.751070	0.865508	0.513394	1.150764	0.820717
1984	2.840610	0.763753	0.544107	0.744312	1.185931
1985	0.792929	0.784339	0.666361	0.793430	0.761591

Table 25
Tornqvist multilateral-aggregate input chain index

Year	Wash.	Oregon	N. Calif.	C. Calif.	Fleet
1981	1.000000	1.938041	1.033066	0.865394	1.000000
1982	0.979766	2.003286	1.261553	1.080224	1.093590
1983	0.951940	2.100056	1.172688	1.156414	1.107696
1984	0.911859	1.870943	1.049162	0.944732	0.990555
1985	0.879063	1.699240	1.089104	1.069901	0.973076

Note: Normalized on either 1981 Washington or 1981 fleet

Table 26
Tornqvist bilateral-aggregate input chain index

Year	Wash.	Oregon	N. Calif.	C. Calif.	Fleet
1981	1.000000	1.000000	1.000000	1.000000	1.000000
1982	0.971964	1.034332	1.218754	1.246114	1.091170
1983	0.952302	1.083419	1.133882	1.324235	1.105216
1984	0.905337	0.963938	1.018920	1.091949	0.988339
1985	0.872455	0.880302	1.059488	1.237316	0.973333

Table 27
Tornqvist multilateral-aggregate input growth rate

Year	Wash.	Oregon	N. Calif.	C. Calif.	Fleet
1981	-0.17660	0.485069	-0.14407	-0.32117	0.066592
1982	-0.02044	0.033111	0.199812	0.221738	0.089465
1983	-0.02881	0.047175	-0.07304	0.068155	0.012816
1984	-0.04301	-0.11552	-0.11130	-0.20217	-0.11177
1985	-0.03662	-0.09626	0.037363	0.124420	-0.01780
Avg.	-0.03222	-0.03287	0.013206	0.053034	-0.00682

Note: Average computed for 1982-85.
Computed following conventional practice as $\ln(T+1) - \ln(T)$.
Percentages obtained by multiplying by 100.

Table 28
Tornqvist bilateral-aggregate input growth rate

Year	Wash.	Oregon	N. Calif.	C. Calif.	Fleet
1981					
1982	-0.02843	0.033756	0.197829	0.220030	0.087251
1983	-0.02043	0.046365	-0.07218	0.060804	0.012789
1984	-0.05057	-0.11684	-0.10690	-0.19287	-0.11177
1985	-0.03699	-0.09076	0.039042	0.124980	-0.01529
Avg.	-0.03411	-0.03187	0.014446	0.053236	-0.00675

Note: Computed following conventional practice as $\ln(T+1) - \ln(T)$.
Percentages obtained by multiplying by 100.

Table 29
Tornqvist multilateral-input chain indices for individual inputs

Year	Wash.	Oregon	N. Calif.	C. Calif.
Capital				
1981	1.000000	1.493297	1.016260	0.712498
1982	1.164689	1.725777	1.009586	0.680118
1983	0.965152	1.731570	1.013056	0.926326
1984	1.048221	1.771869	0.806910	0.634371
1985	0.999909	1.467446	0.747832	0.594396
Labor				
1981	1.000000	2.978295	1.079983	1.016479
1982	0.846864	2.852492	1.680466	1.796623
1983	0.925241	3.104099	1.432877	1.601286
1984	0.794212	2.440112	1.383038	1.424437
1985	0.766881	2.338022	1.591238	1.952170
Fuel				
1981	1.000000	1.314194	0.924861	0.914602
1982	0.968086	1.261695	1.007578	1.022959
1983	0.984833	1.316068	0.974642	0.996861
1984	0.952958	1.198347	0.964424	0.969880
1985	0.949028	1.168710	0.994190	1.045427

Note: Normalized on 1981 Washington

Table 30
Tornqvist bilateral-input chain indices for individual inputs

Year	Wash.	Oregon	N. Calif.	C. Calif.
Capital				
1981	1.000000	1.000000	1.000000	1.000000
1982	1.141725	1.145958	0.992528	0.955572
1983	0.847359	1.003822	1.004162	1.336298
1984	1.064866	1.023683	0.803897	0.700222
1985	0.953517	0.837494	0.931737	0.938812
Labor				
1981	1.000000	1.000000	1.000000	1.000000
1982	0.846864	0.957759	1.556010	1.767497
1983	0.925241	1.042240	1.326758	1.575326
1984	0.794212	0.819298	1.280610	1.401344
1985	0.766881	0.785020	1.473390	1.920521
Fuel				
1981	1.000000	1.000000	1.000000	1.000000
1982	0.968045	0.981431	1.078369	1.108289
1983	0.983202	1.018130	1.047477	1.082092
1984	0.956085	0.916900	1.040827	1.056834
1985	0.950329	0.901658	1.070286	1.138541

Table 31
Tornqvist multilateral-aggregate input implicit real-price chain indices

Year	Wash.	Oregon	N. Calif.	C. Calif.	Fleet
1981	1.000000	0.908036	0.951392	0.852814	1.000000
1982	1.203309	0.603971	0.958440	1.123094	0.968105
1983	1.004202	0.512719	0.814385	1.144682	0.860481
1984	1.270734	0.561590	0.898666	0.802676	0.889275
1985	1.161675	0.539658	0.930371	1.013478	0.910484

Note: Formed by Fisher's factor-reversal test (see text for explanation).
Normalized on either 1981 Washington or 1981 fleet.

Table 34
Share of all inputs in total costs by region

Year	Wash.	Oregon	N. Calif.	C. Calif.
1981	0.223180	0.392755	0.219352	0.164711
1982	0.218652	0.394892	0.220400	0.166055
1983	0.196351	0.399427	0.197730	0.206490
1984	0.229767	0.423828	0.188271	0.158132
1985	0.238080	0.394362	0.193572	0.173984

Note: Sum by row

Table 32
Tornqvist bilateral-aggregate input implicit real-price chain indices

Year	Wash.	Oregon	N. Calif.	C. Calif.	Fleet
1981	1.000000	1.000000	1.000000	1.000000	1.000000
1982	1.098898	1.059757	0.898798	0.882023	1.003326
1983	0.909419	0.900371	0.763048	0.905610	0.876076
1984	1.159524	0.987504	0.838317	0.629150	0.942273
1985	1.060399	0.943734	0.866437	0.793935	0.930485

Note: Formed by Fisher's factor-reversal test (see text for explanation)

Table 35
Tornqvist multilateral economic-performance chain index

Year	Wash.	Oregon	N. Calif.	C. Calif.	Fleet
1981	1.000000	1.165505	1.142894	0.651108	1.000000
1982	0.705195	0.993601	0.769585	1.009704	0.745921
1983	1.137283	0.764362	0.624313	0.636077	0.663914
1984	0.740717	0.770621	1.083505	1.092607	0.741983
1985	0.899914	1.003936	1.184420	1.293200	0.903759

Table 33
Share of inputs in total costs

Year	Wash.	Oregon	N. Calif.	C. Calif.
Labor				
1981	0.433897	0.401721	0.435379	0.413456
1982	0.464813	0.412169	0.427663	0.375625
1983	0.466212	0.454291	0.439758	0.450216
1984	0.480218	0.465550	0.445895	0.431485
1985	0.492314	0.470206	0.443233	0.411349
Fuel				
1981	0.180059	0.225103	0.117663	0.153649
1982	0.133997	0.184159	0.157210	0.232437
1983	0.147428	0.178804	0.132978	0.148997
1984	0.109134	0.135108	0.140781	0.180984
1985	0.106073	0.145023	0.164822	0.236210
Capital				
1981	0.386043	0.373175	0.446956	0.432893
1982	0.401188	0.403670	0.415125	0.391936
1983	0.386358	0.366904	0.427262	0.400786
1984	0.410646	0.399341	0.413322	0.387529
1985	0.401612	0.384769	0.391943	0.352440

Note: Sum by column

Table 36
Tornqvist bilateral economic-performance chain index

Year	Wash.	Oregon	N. Calif.	C. Calif.	Fleet
1981	1.000000	1.000000	1.000000	1.000000	1.000000
1982	0.770281	1.085306	0.840614	1.102895	0.976610
1983	1.242249	0.834910	0.681934	0.694784	0.869811
1984	0.809081	0.841746	1.183507	1.193450	0.921575
1985	0.982972	1.096595	1.293736	1.233551	1.126590

APPENDIX 1

Summary of different index formulae

This appendix summarizes the most widely applied indices in order to provide an easily accessible reference of the different formulae. All indices are expressed for quantities, but extension to price indices is straightforward. Deaton and Muellbauer (1980) and Lau (1979) provide fine references. The indices compare two time periods or economic entities (firms, regions) and aggregate over $i = 1, \dots, N$ commodities.

Paasche

$$Q_P = (X^1/X^0) = \frac{\sum P_i^1 X_i^1}{\sum P_i^1 X_i^0}$$

Laspeyres

$$Q_L = (X^1/X^0) = \frac{\sum P_i^0 X_i^1}{\sum P_i^0 X_i^0} = \sum S_i^0 (X_i^1/X_i^0)$$

Tornqvist

$$Q_T = (X^1/X^0) = II_i [X_i^1/X_i^0]^{1/2} (S_{i1} + S_{i0})$$

Fisher Ideal

$$Q_F = (X^1/X^0) = \left[\frac{\sum S_i^1 (X_i^1/X_i^0)}{\sum S_i^0 (X_i^1/X_i^0)} \right]^{1/2}$$

Geometric

$$Q_G = (X^1/X^0) = II_i [X_i^1/X_i^0]^{S_i}$$

Divisia

$$\ln Q_D = \ln(X^1/X^0) = \int_{X^0}^{X^1} S_i d \ln X_i$$

APPENDIX 2

Quadratic mean-of-order- r indices and Fisher's tests

Diewert (1976a, 1981) discusses which of Irving Fisher's tests are satisfied by the quadratic mean-of-order- r quantity and price indices. The quadratic mean-of-order- r quantity index for $r \neq 0$, $X^0 \gg O_N$, $X^1 \gg O_N$, $P^0 > O_N$, and $P^1 > O_N$ may be defined as:

$$Q_r(P^0, P^1, X^0, X^1) \quad (A2.1)$$

$$= \left[\sum_N (X_i^1/X_i^0)^{r/2} S_i^0 \right]^{1/r} \left[\sum_N (X_i^1/X_i^0)^{-r/2} S_i^1 \right]^{-1/r}$$

where $S_i^0 = P_i^0 X_i^0 / \sum_N P_i^0 X_i^0$ and $S_i^1 = P_i^1 X_i^1 / \sum_N P_i^1 X_i^1$. For $r = 2$, equation (A2.1) becomes Fisher's Ideal quantity index, while when r tends to 0, the quadratic mean-of-order- r index becomes the Tornqvist index as a special case. For $r \neq 0$, the (homogeneous) quadratic mean-of-order- r aggregator function may be written:

$$f_r(X) = \left[\sum_N \sum_j A_{ij} X_i^{r/2} X_j^{r/2} \right]^{1/r}, \quad (A2.2)$$

where $A_{ij} = A_{ji}$ for $i \neq j$.

Diewert (1976a, 1981) states that Q_r satisfies the (1) commodity reversal test, i.e., the value of the index number does not change if the ordering of the commodities is changed; (2) identity test, i.e., $Q_r(P^0, P^0, X^0, X^0) = 1$ [also $Q_r(P^0, P^1, X^0, X^0) = 1$] so that the quantity index = 1 if all quantities remain unchanged; (3) commensurability test, i.e., the quantity index remains invariant to changes in units of measurement; (4) determinateness test, i.e., Q_r does not become zero, infinite, or indeterminate if an individual price becomes zero for any $r \neq 0$ and Q_r does not become zero, infinite, or indeterminate if any individual quantity becomes zero if $0 < r \leq 2$ (thus the quantity indices Q_r , for $0 < r \leq 2$, are somewhat more satisfactory than the Tornqvist); (5) proportionality test, i.e., $Q_r(P^0, P^1, X^0, TX^0) = T$ for every $T > 0$; (6) time or point reversal test, i.e., $Q_r(P^0, P^1, X^0, X^1) Q_r(P^1, P^0, X^1, X^0) = 1$.

Define the quadratic mean-of-order- r price index P_r for $P^0 \gg O_N$, $P^1 \gg O_N$, $X^0 > O_N$, $X^1 > O_N$, for $r \neq 0$, as:

$$P_r(P^0, P^1, X^0, X^1) \quad (A2.3)$$

$$= \left[\sum_N (P_i^1/P_i^0)^{r/2} S_i^0 \right]^{1/r} \left[\sum_N (P_i^0/P_i^1)^{-r/2} S_i^1 \right]^{-1/r}$$

where S_i^0 and S_i^1 are defined as before. P_r also satisfies Fisher's tests (1) to (6).

The only Fisher tests not satisfied by the quadratic mean-of-order- r price and quantity indices, P_r and Q_r , are: (7) the circularity test, i.e., $P_r(P^0, P^1, X^0, X^1) P_r(P^1, P^2, X^1, X^2) \neq P_r(P^0, P^2, X^0, X^2)$, so that transitivity is not satisfied by binary comparisons[†]; and (8) the factor-reversal test, i.e., $P_r(P^0, P^1, X^0, X^1) Q_r(P^0, P^1, X^0, X^1) \neq \sum P_i^1 X_i^1 / \sum P_i^0 X_i^0$, except that P_2 and Q_2 , the Fisher Ideal price and quantity indices, satisfy the factor-reversal test (so that implicit indices equal direct indices).

[†]If this test is not passed, the implication is that the price index P_{02} over the time period 0 to 2 does not depend upon how prices develop over time in the intermediate year(s), P_0 to P_2 via P_1 .

APPENDIX 3

Superlative implicit indices

This appendix examines in greater detail the relationship between superlative index numbers and Fisher's factor-reversal test. It borrows heavily from Diewert (1976a, 1981) and Allen and Diewert (1981), and demonstrates that if either a price or quantity direct index is defined, then a corresponding quantity or price index can be defined implicitly by using the weak factor-reversal test [equation (13) of the text].

Define the implicit quadratic mean-of-order- r price index P_r^* as:

$$P_r^*(P^0, P^1, X^0, X^1) = \sum P_i^1 X_i^1 / (\sum P_i^0 X_i^0) (Q_r(P^0, P^1, X^0, X^1)) \quad (A3.1)$$

and the implicit quadratic mean-of-order- r quantity index Q_r^* as:

$$Q_r^*(P^0, P^1, X^0, X^1) = P_i^1 X_i^1 / (\sum P_i^0 X_i^0) (P_r(P^0, P^1, X^0, X^1)) \quad (A3.2)$$

Thus implicit quadratic mean-of-order- r indices defined in this manner will satisfy the weak factor-reversal test of Fisher, equation (13). Then (P_r^*, Q_r) and (P_r, Q_r^*) are both superlative pairs of index-number formula. However, this is not necessarily so for the pair of direct superlative indices (P_r, Q_r) .

The Fisher Ideal index, where $r = 2$, does satisfy the weak factor-reversal test. Therefore, $P_2(P^0, P^1, X^0, X^1) Q_2(P^0, P^1, X^0, X^1) = \sum P_i^1 X_i^1 / \sum P_i^0 X_i^0$. (P_2, Q_2) is a pair of superlative indices, and $(P_2, Q_2^*) = (P_2^*, Q_2) = (P_2, Q_2)$.

The Tornqvist index, where r tends to 0, does not satisfy the weak-factor reversal test in general, i.e., $P_0(P^0, P^1, X^0, X^1) Q_0(P^0, P^1, X^0, X^1) \neq \sum P_i^1 X_i^1 / \sum P_i^0 X_i^0$. This occurs because the quantity index Q_0 is consistent with a homogeneous translog aggregator function, while the price index P_0 is consistent with an aggregator function which is dual to the translog unit-cost function, and the two aggregator functions do not in general coincide. They instead correspond to different (aggregation) technologies, i.e., they are not self-dual. Thus, given Q_0 , the corresponding price index, which satisfies the weak factor-reversal test, is the implicit index defined by $P_0^*(P^0, P^1, X^0, X^1) = \sum P_i^1 X_i^1 / (\sum P_i^0 X_i^0) [Q_0(P^0, P^1, X^0, X^1)]$. Alternatively, given P_0 , the corresponding quantity satisfying the weak factor-reversal test is the implicit index defined by $Q_0^*(P^0, P^1, X^0, X^1) = \sum P_i^1 X_i^1 / (\sum P_i^0 X_i^0) [P_0(P^0, P^1, X^0, X^1)]$. The price-quantity indices P_0, Q_0^* correspond to a translog unit-cost (or revenue) function, while P_0^*, Q_0 correspond to a homogeneous translog aggregator function.

The price-quantity index pair (P_0, Q_0^*) is advocated by Kloek (1967) over the pair (P_0^*, Q_0) . He argues that as data and the level of study are increasingly disaggregated, the individual consumer or producer will utilize positive amounts of fewer and fewer goods (i.e., as N grows, components of the vectors X^0 and X^1 will tend to become zero), but the prices which the producer or consumer face are generally positive irrespective of the degree of disaggregation. Since the logarithm of zero is not finite, Q_0 will tend to be indeterminate as the degree of disaggregation increases, but P_0 will still be well defined (provided that all prices are positive).

The choice between (P_r, Q_r^*) and (P_r^*, Q_r) can alternatively be made by comparing the variation in the N -quantity ratios (X_i^1/X_i^0) to the variation in the N -price ratios (P_i^1/P_i^0) . If there is less variation in the price ratios than in the quantity ratios, i.e., prices are more highly proportional than quantities, then the various types of direct superlative price indices P_r are essentially share-weighted

averages of the price ratios (P_i^1/P_i^0) , and will tend to be in closer agreement with each other than the implicit price ratios P_r^* . In this case, the aggregates generated by direct-price indices should all be numerically close (and they can be approximately justified using Hick's Aggregation Theorem). Thus in this situation, use of a superlative direct-price index and the corresponding implicit-quantity index, (P_r, Q_r^*) , is preferred for some r (Allen and Diewert 1981).

If there is less variation in the quantity ratios than in the price ratios, then the direct quantity indices Q_r are essentially share-weighted averages of the quantity ratios and will tend to be more stable than the implicit-quantity indices Q_r^* . Use of a direct superlative-quantity index Q_r and corresponding implicit-price index P_r may be preferable for some r , and the aggregates generated by these indices should all be numerically close (and can be justified by Leontief's Aggregation Theorem) (Allen and Diewert 1981).

Allen and Diewert (1981) present a simple procedure by which to empirically determine whether prices are more highly proportional than quantities. They suggest individually regressing $\ln(P_i^1/P_i^0)$ and $\ln(X_i^1/X_i^0)$ on constants. The sum of squared residuals of the regressions (SSR) will then be measures of non-proportionality of the vectors P^0 and P^1 and X^0 and X^1 , respectively. Prices are then less proportional than quantities if the SSR from the price-ratio regression is greater than the SSR from the quantity-ratio regression. In this case, the use of the superlative index-number pair (P_r^*, Q_r) for some r is recommended in order to aggregate the data. If the converse holds, then the superlative index-number pair (P_r, Q_r^*) for some r is recommended.

If the proportionality criterion cannot distinguish whether prices are more proportional than quantities, Fisher's Ideal index may be preferred since $P_2 = P_2^*$ and $Q_2 = Q_2^*$ (and thus the formula is approximately consistent with both Hick's and Leontief's aggregation theorems). Moreover, the Fisher Ideal index also lies between the Paasche and Laspeyres indices, since it is the geometric mean of the Paasche and Laspeyres indices.

Tornqvist indices are preferred over Fisher Ideal indices if multilateral indices are employed, since to date it is not known whether or not Fisher Ideal multilateral indices can be directly derived from a flexible transformation function that is nonseparable in inputs and outputs and permits non-neutral differences in productivity among economic entities, while this has been demonstrated for the Tornqvist multilateral index by Caves et al. 1981. Moreover, because the data for this study are available in quantity and revenue values, the Tornqvist implicit index for prices and the Tornqvist direct index for quantities, (P_0^*, Q_0) , is used.

APPENDIX 4

Aggregation of outputs

Aggregation of individual outputs into a composite output is accomplished by a factor-requirements function for quantities and a unit-revenue function for prices. A factor-requirements function relates the minimum amount of aggregate input required to produce the vector of outputs, while the unit-revenue function provides the maximum amount of total revenue for a given level of an aggregate input. Kirkley (1986) provides additional theoretical background on revenue functions.

The quadratic mean-of-order- r functional form can be used to provide superlative price and quantity indices by aggregating individual prices and quantities, respectively. For example, the linearly homogeneous translog functional form provides a second-order approximation to an arbitrary twice-differentiable factor-requirements function:

$$\ln Y = A_0 + \sum_i A_i \ln Y_i + \frac{1}{2} \sum_{i,j} A_{ij} \ln Y_i \ln Y_j, \quad (\text{A4.1})$$

where $\sum_i A_i = 1$, $\sum_i \sum_j A_{ij} = \sum_j \sum_i A_{ij} = \sum_{i,j} \sum_{i,j} A_{ij} = 0$, and the producer is producing M outputs, $Y = (Y_1, Y_2, \dots, Y_M)$.

The Tornqvist output quantity index may then be specified as:

$$g(Y^1)/g(Y^0) = \prod_M [Y_i^1/Y_i^0]^{S_i^1 + S_i^0}, \quad (\text{A4.2})$$

where $S_i^1 = P_i^1 Y_i^1 / \sum P_i^1 Y_i^1$ and $S_i^0 = P_i^0 Y_i^0 / \sum P_i^0 Y_i^0$.

Similarly, if the unit-revenue function $R(P)$ is translog over the relevant range of data and the producer is maximizing revenue, the Tornqvist product-price index can be defined as:

$$R(P^1)/R(P^0) = \prod_M [P_i^1/P_i^0]^{S_i^1 + S_i^0}, \quad (\text{A4.3})$$

which in log form becomes:

$$\ln(P^1/P^0) = \sum_i \frac{1}{2} (S_i^1 + S_i^0) \ln(P_i^1/P_i^0). \quad (\text{A4.4})$$

Diewert (1974a, 1976a) provides additional details, including the dual theoretical relationship between revenue functions and factor requirement functions.

APPENDIX 5

Quadratic approximation lemma

Denny and Fuss (1983) provide intuition into the quadratic approximation lemma given in equation (17). In particular, they show that the quadratic lemma of Diewert (1976a) can be interpreted as resulting from a differencing of two linear approximations. The discussion in this appendix follows their discussion quite closely.

Consider the true aggregator function $Y = f(X)$, where $X = (X_1, X_2, \dots, X_N)$, with m th-order continuous partial derivatives with respect to the X_i . $Y = f(X)$ can then be approximated by an m th-order Taylor-series expansion around the point X^r :

$$Y = Y^r + \sum_i f'_i [X_i - X_i^r] + \frac{1}{2!} \sum_{i,j} f''_{ij} [X_i - X_i^r] [X_j - X_j^r] + \dots + \frac{1}{m!} \sum_{i_1, \dots, i_m} f^{(m)}_{i_1 \dots i_m} [X_{i_1} - X_{i_1}^r] \dots [X_{i_m} - X_{i_m}^r] + R_{m+1}^r, \quad (\text{A5.1})$$

where R_{m+1}^r is the $(m+1)$ -order remainder term and the superscript r denotes evaluation at r .

Replace X_i by X_i^s . Then the right-hand side of the Taylor-series expansion is just equal to Y^s . If the roles r and s are reversed (i.e., evaluate the derivatives at s and replace X_i^r by X_i^s and X_i by X_i^r), an expression is obtained for Y^r . Subtracting the two expressions and dividing by 2 provides an exact representation of the difference $Y^r - Y^s$. This difference, with suitable interpretation, is the general growth-accounting equation.

Consider now a linear approximation to the true aggregator function $Y = f(X)$ and that two data points are observed, $\{X^r\}$ and $\{X^s\}$. These data points and the corresponding true values of the true aggregator function, Y^r and Y^s , are labelled A and B in Appendix Figure A5.1 (again, adapted from Denny and Fuss 1983).

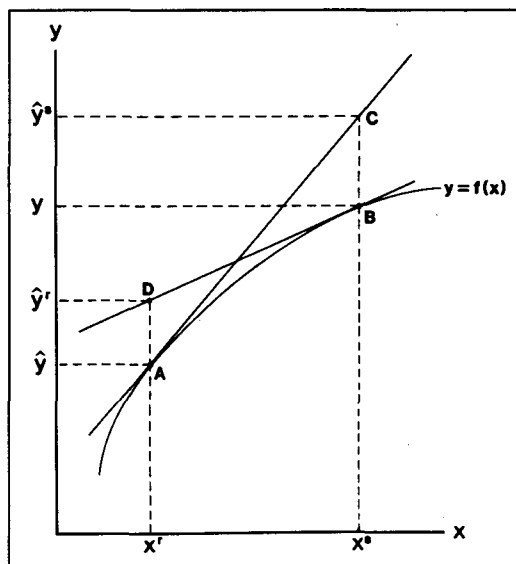


Figure A5.1
Quadratic approximation lemma.

It is assumed that the true function $f(X)$ is unknown and hence Y^s and Y^r and the difference $Y^s - Y^r$ are also unknown.

To approximate Y^s , take a linear approximation of $f(Y)$ around A and evaluate it at X^r . This point is called C and the corresponding value on the vertical axis, Y^{*s} . The approximation can be written as:

$$Y^{*s} = Y^s - R_2^s = Y^r + \sum_i f_i^s [X_i^s - X_i^r]. \quad (A5.2)$$

Similarly, to approximate Y^r , take the linear expansion of $f(X)$ around B and evaluate it at X^s , denoting the point D and the relevant value Y^{*r} . This approximation can be written as:

$$Y^{*r} = Y^r - R_2^r = Y^s + \sum_i f_i^r [X_i^r - X_i^s]. \quad (A5.3)$$

The unknown difference $Y^s - Y^r$ can be approximated by the approximate difference $Y^{*s} - Y^{*r}$, where:

$$\begin{aligned} Y^{*s} - Y^{*r} &= (Y^s - Y^r) + (R_2^s - R_2^r) \\ &= (Y^s - Y^r) + \sum_i [f_i^s + f_i^r][X_i^s - X_i^r]. \end{aligned} \quad (A5.3)$$

Utilizing the last set of equalities and solving for $Y^s - Y^r$ provides:

$$\begin{aligned} Y^s - Y^r &= 0.5 \sum_i [f_i^s + f_i^r][X_i^s - X_i^r] \\ &\quad + 0.5[R_2^r - R_2^s]. \end{aligned} \quad (A5.4)$$

By ignoring the error of approximation $0.5[R_2^r - R_2^s]$, this last equation becomes the quadratic approximation lemma.

All first-order approximations satisfy the quadratic lemma. All second-order approximations of the form given in equation (16) also satisfy the quadratic lemma. Moreover, even quadratic functions in which the zero-order and first-order parameters are specific to a data point (e.g., s) satisfy the quadratic lemma. However, should the second-order parameters be specific to a data point, then the quadratic lemma is not satisfied.

APPENDIX 6

Constructing index numbers on electronic spreadsheets with personal computers

This appendix suggests ways of constructing Tornqvist chain index numbers on electronic spreadsheets using personal computers. After the data is placed on a spreadsheet (e.g., LOTUS), create a table composed of only the share-weighted binary logarithmic changes from time-period to time-period. Unfortunately, the relative copy command of LOTUS cannot be used to make copies of a general formula. These numbers are binary period-to-period changes. Next, create a second table composed of the exponents of the logarithmic changes. The copy command of LOTUS can be used to create this table. This table represents the Tornqvist binary index numbers for period-to-period changes, that is, as if the base period is updated with each time period.

To create chain indices, create a third table in the electronic spreadsheet. Index the initial time period 0, the second time period 1, the third time period 2, and so forth. Then the value of the chain index in time T is simply $P_{0T}^{chain} = P_0 \times P_{01} \times P_{12} \times \dots \times P_{T-1,T}$. Thus the value of the chain index in period 1 is simply the value of the first period's binary index P_{01} (from the second of the created tables) multiplied by the initial base-period value P_0 . The value of the chain index in period 2 is then the value of the second period's binary index P_{12} (from the second of the tables created) multiplied by the product $P_0 \times P_{01}$, which is the value of period 1's chain index and which has already been created in the third table (the one currently being worked in). Once a single column of chain indices has been created, the relative copy command can then be used for other economic entities if more than one is being analyzed.

The initial base-period value P_0 differs, depending upon whether or not a bilateral or multilateral index is constructed. The base-period value for a bilateral index is simply 1.0. The base-period value for a multilateral index is the logarithmic change of the base period not from a preceding period as created in the first of the three tables, but from the geometric mean. This value for an output index of a single species is simply $0.5(R_{i0} + R_i^*)(\ln Y_{i0} - \ln Y_i^r)$; for a single input it becomes $0.5(W_{j0} + W_j^*)(\ln X_{j0} - \ln X_j^s)$; and for total-factor productivity it is simply the difference between these two, where R indicates a revenue share, W indicates a cost share, $*$ indicates the arithmetic mean, $'$ indicates the geometric mean, Y indicates output, X indicates input, and \ln indicates natural logarithm. These logarithmic changes are placed in the first table created (which is composed of binary, share-weighted period-to-period changes). Because of the interpretation of the first period as deviations from the geometric mean, the initial period's revenue ratio used in Fisher's factor-reversal test used to construct implicit indices is the initial period's revenue to the arithmetic mean of all revenues.