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ON THE MEASUREMENT OF ECONOMIC CAPACITY UTILIZATION FOR MULTI-PRODUCT INDUSTRIES*

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This paper considers capacity utilization measures for the multi-product firm. The single-product dual measure of capacity utilization easily extends to the multi-product case. Three possible extensions of the single-product primal measure are considered. Although each has its limitations because of the restrictions embodied in it, each provides different, yet potentially useful information about capacity utilization in a multi-product industry. The dual and primal measures of multi-product capacity utilization are applied to the multi-species New England fishing industry to evaluate the potential for capacity expansion under a regulatory program of license limitation.

1. Introduction

Measures of capacity utilization (CU) have been used for many years to analyze the current 'state' of the economy and the expansionary or contractionary forces that might exist. However, it is only recently that economists have attempted to develop CU measures that are closely tied to the economic theory of firm behavior. Pioneering studies in this area include the work by Klein (1960) and Hickman (1964), and more recently by Morrison (1985, 1986) and Berndt and Fuss (1986). These studies have defined CU using the concept of the firm's short-run cost function where one or more inputs are treated as quasi-fixed. Morrison (1985) proposes two alternative definitions, a primal measure defined in terms of the firm's output level and a dual measure defined in terms of the firm's costs.

To date, however, both the theoretical and the empirical development of these theory-based CU measures has been confined to the case of a single-product firm, i.e., a firm that produces a single output. In reality, many firms

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produce two or more outputs and thus should instead be viewed as multi-product firms. The growing body of literature devoted to developing a theory of the multi-product firm [Baumol, Panzar, and Willig (1982), Laitinen (1980), Lau (1978), Sakai (1974)] attests to their importance in empirical work. It would therefore be useful to have economic CU measures that can be applied to the case of a multi-product firm.

The purpose of this paper is to propose and illustrate empirically several alternative definitions of CU that are derived from the theory of the multi-product firm. We consider one dual measure and three alternative primal measures that differ in terms of the restrictions imposed. All of the measures can be viewed as generalizations of the single-product measures to the case of a multi-product firm.

The paper is organized as follows. We begin with a brief review of the single-product measures proposed by Morrison (1985) since these provide a point of departure for our analysis. The next section discusses a dual CU measure for the multi-product case. It is shown that the single-product dual measure can be easily extended to apply to the multi-product case. The extension of the single-product primal measure is more problematic because a scalar measure of output does not generally exist for multi-product firms. Section 4 considers three alternative generalizations. The first requires that the technology be homothetically separable in outputs, and is therefore applicable only to a subset of multi-product firms. The second does not require separability but assumes that all outputs change proportionately. It therefore can be thought of as a 'ray' CU measure. The final measure defines CU in terms of one of the outputs, holding the other outputs constant. Although this numerical CU measure will vary depending upon which output is used, we show that under fairly weak assumptions the conclusion about whether capacity is currently under- or overutilized will be unique.

The theoretical discussion of alternative CU measures is followed in section 5 by an empirical illustration. One industry that is characterized by multi-product production is the multi-species fishing industry, in which fishing vessels combine a vector of inputs (capital, labor, and fuel) to produce a vector of multiple products (different fish species). We use data from the New England otter trawl industry to illustrate an application of the use of our alternative measures with a translog variable cost function. This multi-product industry is not only one of the world's most valuable fishing industries, but as we discuss below, measures of capacity utilization can make an important contribution to regulatory programs designed to correct the market failure arising from an open-access resource.

2. Single-product measures

Consider a cost-minimizing firm that produces an output level $\bar{y}$ using n variable inputs $(x_1, \ldots, x_n)$ and one input (K) that is quasi-fixed, i.e., fixed in the short run but variable in the long run. Let the firm's variable cost function be given by $G(\bar{y}, w, K)$ where w is the vector of variable input prices. Short-run total costs $C = C(\bar{y}, w, K, P_K)$ are then $G + P_K K$, where P_K is the price of K. Given K, the capacity level y^* is defined to be the output level at which the short-run average cost ($SRAC$) curve is tangent to the long-run average cost ($LRAC$) curve. This can be found by differentiating C with respect to K and solving for $y^*(K, w, P_K)$, i.e., y^* solves

$$G_K(y, w, K) + P_K = 0. \tag{1}$$

Given this capacity output level y^*, the primal CU measure is then defined to be $CU_q \equiv \bar{y}/y^*$. If $CU_q > 1$, then $\bar{y} > y^*$ and there is pressure to increase investment in K. Likewise, $CU_q < 1$ implies disinvestment incentives. When $CU_q = 1$, K is the cost-minimizing level of capital for producing $\bar{y}$ and the firm has no incentive to change the level of K.

The primal CU measure captures the output gap that exists when actual output differs from capacity output. Alternatively, capacity utilization can be measured in terms of the cost gap that exists when $\bar{y}$ is not equal to y^*. If the firm were in long-run equilibrium, i.e., $\bar{y} = y^*$, then by (1), it must be true that

$$-G_K(\bar{y}, w, K) = P_K. \tag{2}$$

Since $-G_K(\bar{y}, w, K)$ can be interpreted as the shadow value of K (Z_K), this states that the firm is in long-run equilibrium if the shadow value of K is equal to the price of K. Thus, if $\bar{y}$ were the long-run capacity output level, the firm's cost would be given by the shadow cost

$$C^* = C(\bar{y}, w, K, Z_K) = G(\bar{y}, w, K) + Z_K K. \tag{3}$$

The cost gap that results from not being in long-run equilibrium then gives a dual CU measure, namely,

$$CU_c \equiv \frac{C^*}{C} = \frac{C(\bar{y}, w, K, Z_K)}{C(\bar{y}, w, K, P_K)} = 1 + \frac{(Z_K - P_K)K}{C}. \tag{4}$$

As before, $CU_c > 1$ implies $Z_K > P_K$ and investment incentives exist, while $CU_c < 1$ means $Z_K < P_K$ and the firm would like to disinvest.

3. Dual CU measures for multi-product firms

To extend the above CU measures to the multi-product firm let $\bar{y} = (\bar{y}_1, \ldots, \bar{y}_m)$ be an m-dimensional vector of output levels with m greater than or equal to one. Clearly $m = 1$ corresponds to the single-output case, and thus any CU measure derived for an arbitrary m could be used for the single-output firm. We can define the firm's variable cost function $G(\bar{y}, w, K)$ and short-run total cost function $C(\bar{y}, w, K, P_K) \equiv G(\bar{y}, w, K) + P_K K$ as before, where now y is a vector rather than a scalar. We will assume $\partial^2 C/\partial y_i \partial K \equiv G_{iK} \leq 0$, i.e., that an increase in K will not increase the short-run marginal cost of any output.[1]

Although a multi-product firm does not have a scalar measure of output, the cost of producing any vector of output is a scalar, and thus the dual measure of CU used for the single-product firm can also be used for the multi-product firm. More specifically, treating y as a vector in (1), we can still define the shadow value of the quasi-fixed input $Z_K = -G_K$ and the analogous shadow cost. The dual CU measure for the multi-product firm is then

$$CU_c \equiv \frac{C^*}{C} = \frac{G(\bar{y}_1, \ldots, \bar{y}_m, w, K) + Z_K K}{G(\bar{y}_1, \ldots, \bar{y}_m, w, K) + P_K K}$$

$$= 1 + \frac{(Z_K - P_K)K}{C(\bar{y}_1, \ldots, \bar{y}_m, w, K, P_K)}, \tag{5}$$

where (5) is a straight-forward extension of (4).[2] Thus, the single-product dual concept of CU carries over directly to the multi-product case.

4. Primal CU measures for multi-product firms

Unfortunately, the primal CU measure does not extend as readily. Because output is no longer a scalar, it is not possible to form the ratio of actual to capacity output to obtain a scalar CU measure. It is possible, however, to define analogous output-based measures of CU for the multi-product firm. In this section, we propose three alternative measures.

4.1. CU under homothetic separability

Suppose that the firm's technology is homothetically separable in outputs, i.e., the transformation function $F(y, x, K) = 0$ takes the form $\tilde{F}(h(y), x, K) = 0$ for some function h: $R^m \rightarrow R$, where h is homogeneous of degree one

[1] This assumption always holds at the cost-minimizing level of K if K is a normal input [Hof et al. (1985)]. However, it is not guaranteed to hold for all K. The results of Lau (1976) imply that the variable cost function G must be convex in (y, K). This only requires that $G_{ii}G_{KK} - G_{iK}^2 \geq 0$, where $G_{ii} \geq 0$ and $G_{KK} \geq 0$. It does not restrict the sign of G_{iK}. Furthermore, assuming K is a normal input is not sufficient to guarantee $G_{iK} \leq 0$ if $G_{ij} \neq 0$ for some j. Nonetheless, we maintain this assumption since it seems reasonable to assume that the firm operates in a region where the marginal cost of any output is a nonincreasing function of K.

[2] See Squires (1987b) for a discussion of a similar measure based on the restricted multi-product profit function.

after normalization. This is essentially equivalent to assuming that the outputs can be aggregated into a single measure of aggregate output $h(y)$. Then the corresponding variable cost function will take the form

$$G(y_1,\ldots,y_m,w,K) \equiv \tilde{G}(h(y),w,K), \tag{6}$$

for some function $\tilde{G}$: $R^3 \to R$ [Denny and Pinto (1978), McFadden (1978)].

Given any output vector $\bar{y}$, the long-run cost-minimizing level of K (K^*) can be found by differentiating total short-run costs $C = \tilde{G} + P_K K$ with respect to K and solving for K, i.e., $K^* = K^*(h(\bar{y}), w, P_K)$ solves

$$\tilde{G}_K(h(\bar{y}),w,K) + P_K = 0. \tag{7}$$

Although long-run and short-run average costs are not defined here as in the single-product case, K^* can still be interpreted as the value of K for which the long- and short-run total cost functions are tangent at $\bar{y}$.

Alternatively, (7) could be solved for $h(y)$ as a function of w, K, and P_K. We denote this solution $h^* = h^*(w, P_K, K)$. h^* can be interpreted as the value of aggregate output for which the long- and short-run cost functions are tangent given K. Thus, it is the multi-product analogue of y^* and CU_q can be generalized by using $h(\bar{y})$ and h^* instead of $\bar{y}$ and y^* in the definition. More specifically, we define a primal measure of capacity utilization under homothetic separability, CU_s, to be

$$CU_s \equiv h(\bar{y})/h^*. \tag{8}$$

Clearly, if $CU_s > 1$, i.e., $h(\bar{y}) > h^*$, then by (7) $-G_K > P_K$ and incentives for investment exist. Likewise, $CU_s < 1$ implies disinvestment incentives exist, while $CU_s = 1$ if the firm is in long-run equilibrium. Finally, if $m = 1$, then CU_s reduces to CU_q since h is proportional to y in this case.

4.2. *A ray measure of CU*

Since the assumption of homothetic separability is quite restrictive, it would be useful to provide a more general measure of multi-product CU. In the literature on multi-product firms it has often been convenient to define concepts assuming that all outputs increase or decrease in fixed proportions. For example, the concepts of ray average cost (RAC) and multi-product returns to scale are both based on this assumption [Baumol, Panzar, and Willig (1982)]. Although a profit-maximizing firm may not choose to change all outputs proportionately in response to a parameter change, the assumption of proportional changes in output is convenient, since it essentially converts the multi-product problem into a single-product one. In this subsection, we propose a CU measure based on the assumption that outputs move along a ray.[3]

We begin by writing the firm's short-run total cost function as

$$\hat{C}(\bar{y}, k, w, K, P_K) = G(k\bar{y}, w, K) + P_K K, \tag{9}$$

where $k > 0$ is a scalar. This gives the cost of producing any vector $k\bar{y} = (k\bar{y}_1, \ldots, k\bar{y}_m)$. $\tilde{C}(\bar{y}, 1, w, K, P_K)$ is then the total cost of producing the output vector $\bar{y}$, i.e., the firm's actual costs.

As before, long-run equilibrium requires that, for a given output vector $k\bar{y}$, K must be at its cost-minimizing level. Equivalently, it must be true that

$$G_K(k\bar{y}, w, K) + P_K = 0. \tag{10}$$

Eq. (10) can be solved for the cost-minimizing level of K, $K^*(k\bar{y}, w, P_K)$. Alternatively, treating K as fixed, we can solve (10) for k to get the optimal 'scale' of operation $k^*(\bar{y}, K, w, P_K)$. If $k^* > 1$, then to be in long-run equilibrium with the given level of K the firm would have to expand its level of operation, i.e., move out along the ray through $\bar{y}$. Equivalently, $k^* > 1$ implies that at the current output level $\bar{y}$ (corresponding to $k = 1$) $-G_K < P_K$ and incentives to disinvest exist.[4] Alternatively, $k^* < 1$ means that the firm would like to scale back its level of operation given K, or increase its level of K given its output levels.

This interpretation of k^* suggests a multi-product generalization of the primal measure CU_q based on the movement of all outputs along a ray. We propose the following measure:

$$CU_r \equiv 1/k^*. \tag{11}$$

This measure clearly sends the right 'signals' about the state of the economy since investment incentives exist when $CU_r > 1$ and disinvestment incentives exist when $CU_r < 1$. In addition, if $m = 1$, CU_r reduces to CU_q.

Finally, it should be noted that our definition of capacity utilization under homothetic output separability, CU_s, is a special case of the ray measure CU_r. This result can be seen in the following way. Suppose the technology is homothetically separable in outputs. Then $CU_s \equiv CU_r$ if and only if

[3]It should be recognized from the start, however, that such a measure could be biased since it requires a firm's optimal expansion path to lie along a ray through the origin in output space so that all products are produced in fixed proportions. A firm may respond to exogenous shocks (e.g., changes in product prices) by altering its product mix, thereby producing along a different product ray. An analogous bias can exist in estimates of multi-product economies of scale [Bailey and Friedlaender (1982), Shaffer (1984)].

[4]This follows from the fact that G_K is a decreasing function of k as long as $G_{iK} \leq 0$ for all i and $G_{iK} < 0$ for some i.

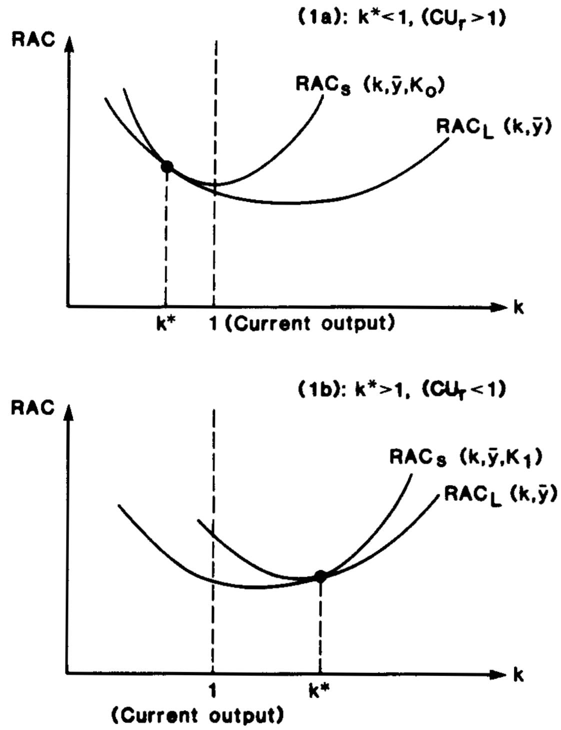

Fig. 1

$k^*h(\bar{y}) = h^*$. By the definitions of h^* and k^*, $-\tilde{G}_K(h^*, K, w) = P_K$ and $-\tilde{G}_K(h(k^*\bar{y}), K, w) = P_K$, which implies that $h^* = h(k^*\bar{y})$. Thus $CU_s \equiv CU_r$ if and only if $k^*h(\bar{y}) = h(k^*y)$, but this holds by linear homogeneity of h. The equivalence of the two measures is as expected since under homotheticity the expansion path for outputs is linear.

The ray measure of capacity utilization can be illustrated graphically using fig. 1. RAC_s is the firm's short-run ray average cost defined as $RAC_s \equiv C(k\bar{y}, K)/k$ (other arguments of C have been suppressed to simplify notation). It is drawn here as the familiar U-shaped curve. Likewise, RAC_L is the firm's long-run ray average cost defined by $RAC_L \equiv C(k\bar{y}, K^*(k\bar{y}))/k$ where $K^*(k\bar{y})$ is the solution to (10). Again, we allow it to be U-shaped. It reflects a cross-section of the firm's long-run total cost surface above the ray through $\bar{y}$. Note that the horizontal axis measures k rather than a scalar measure of output. Fig. 1a illustrates the case where the relationship between the level of K (here K_0) and the level of output $\bar{y}$ is such that capital is overutilized, i.e., $CU > 1$. If capital were at a different level, say K_1, capacity might be underutilized. This is illustrated in fig. 1b.

4.3. Partial CU measures

Both CU_s and CU_r as defined above allow all outputs to change in measuring the long-run equilibrium position of the firm. However, they impose restrictions on the technology (in the case of separability) or the way in which outputs can change (for the ray measure of CU). An alternative approach would vary only a single output in defining capacity output. This approach is discussed in this subsection.

Interpreting y in (1) as the vector $y = (y_1, \ldots, y_m)$, this equation can be solved for any given y_i in terms of the other $m - 1$ output levels. In other words, if G_{iK} nonzero for all i, then (1) implicitly defines a set of explicit functions,

$$y_i^* = f_i(\bar{y}_{-i}, K, w, P_K), \qquad i = 1, \ldots, m, \tag{12}$$

where $\bar{y}_{-i}$ denotes the $(m-1)$-dimensional vector $(\bar{y}_1, \ldots, \bar{y}_{i-1}, \bar{y}_{i+1}, \ldots, \bar{y}_m)$. We can then interpret y_i^* as the capacity level of output for the ith product given the actual output levels for all other products. A partial CU measure can then be defined as

$$CU_i \equiv \bar{y}_i / y_i^* \quad \text{for any given } i. \tag{13}$$

Of course, the numerical value of this CU measure will vary across products, and therefore it is not unique for a given firm. However, it can be shown that, if $G_{iK} < 0$ for all i,[5] then these measures provide a consistent indication of whether the firm's capacity is currently under- or overutilized. We state this result in the form of a theorem.

Theorem 1. If $G_{iK} < 0$ for all i, then exactly one of the following holds:

(i) $CU_i > 1$ *for all i,*
(ii) $CU_i < 1$ *for all i,*
(iii) $CU_i = 1$ *for all i.*

Proof. It is sufficient to show that under the above condition CU_i is greater than/ less than/ equal to one if and only if CU_j is greater than/ less than/ equal to one for all i and j; or equivalently, that $\bar{y}_i$ is greater than/ less than/ equal to y_i^* [defined in (12)] iff $\bar{y}_j$ is greater than/ equal to/ less than y_j^* for all i and j.

[5]From the proof of the theorem, it is clear that this condition could be weakened to requiring that G_{iK} simply have the same sign for all i. However, assuming $G_{iK} > 0$ for all i seems unreasonable.

Consider any two products i and j. Let $\bar{y}_{-ij}$ denote the $(m-2)$-dimensional vector of actual output levels excluding $\bar{y}_i$ and $\bar{y}_j$. We begin by noting that, since both $(y_i^*, \bar{y}_j, \bar{y}_{-ij})$ and $(\bar{y}_i, y_j^*, \bar{y}_{-ij})$ solve (1), and (12) is simply an explicit form of the implicit relationship in (1), it must be true that

$$y_i^* = f_i(\bar{y}_j, \bar{y}_{-ij}) \quad \text{and} \quad \bar{y}_i = f_i(y_j^*, \bar{y}_{-ij}). \tag{14}$$

Thus, to prove the theorem, it is sufficient to show that f_i is a strictly monotonically decreasing function of y_j. Implicitly differentiating (1) gives

$$\partial f_i / \partial y_j = \partial y_i^* / \partial y_j = -G_{Kj}/G_{Ki}, \tag{15}$$

which is negative if $G_{iK} < 0$ for all i. Thus f_i is strictly monotonically decreasing in y_j and the proof is complete. Q.E.D.

The intuition behind the theorem is the following. From (1) it is clear that, given the levels of all other outputs, if $\bar{y}_i$ is less than y_i^*, then $-G_K < P_K$ and there is an incentive to disinvest, i.e., capacity is underutilized. This is true regardless of which product we consider, i.e., which one we allow to adjust to equate the shadow value and the price of capital. Thus, the question of whether the firm faces expansionary or contractionary forces has the same answer regardless of which product is used to measure capacity utilization.

5. Empirical application

One industry where questions regarding capacity utilization have been raised is the fishing industry. Capital is generally measured in terms of vessel size (in gross registered tons), gear, and equipment. For any individual firm (often comprised of a single vessel) the level of capital can be viewed as quasi-fixed since in the short run vessel size cannot be easily varied. Measures of capacity utilization then provide an indication of whether the current level of capital is the long-run equilibrium level, and thus whether pressures for expansion or contraction of the industry exist.

In addition, measures of capacity utilization can provide an important contribution to designing regulatory programs which correct the market failure arising from the open-access nature of the resource. One of the most widely applied regulations, license limitation, restricts the number of vessels in the industry that can harvest the resource. As resource stocks recover and incomes eventually rise, fishermen invariably take advantage of any unused production potential or expand their capacity through additional investment to further increase their harvests and incomes. This, in turn, tends to drive the industry back to its original open-access condition. However, a multi-product CU measure calculated while planning for license limitation which includes vessel reductions would help adjust the number of vessels to both the resource conditions and each vessel's production capacity. Thus optimal utilization of both the resource stock and each individual vessel would be advanced. Participation could even be keyed by a vessel's extent of capacity utilization. Alternatively, if all vessels currently in the industry are grandfathered into the license limitation program, CU measurement would indicate if ancillary regulations are required to prevent full utilization of any unused production capacity or prevent further investment.

For these reasons, we choose a multi-product fishing industry to illustrate the proposed CU measures. The particular industry we consider is the New England otter trawl industry, which simultaneously harvests cod, flounder, haddock, redfish, pollock, and other species. An overview of the industry is provided by Squires (1987a).

To reduce the dimensionality of the problem, the many species harvested by the industry were aggregated (using Divisia indices) into three outputs: roundfish (cod and haddock), flatfish (flounders), and all others. These outputs are assumed to be produced (harvested) using two variable inputs, labor and fuel, and one quasi-fixed input, capital (represented by the vessel's gross registered tonnage).[6] The data set includes a cross-section of 42 vessels (firms) for 1980.[7] Outputs are endogenous and all firms are assumed to be profit maximizers.

The firm's variable cost function $G(y, w, K)$ is assumed to be of the translog form, which provides a second-order Taylor series approximation to

[6]Capital is specified as quasi-fixed, because it is lumpy and difficult to adjust over short time periods. (Typically an existing vessel is sold and another one purchased and then overhauled and refitted or a new one is built.) Squires (1987b) applied Kulatilaka's (1985) test for full static equilibrium of capital with a profit function and found capital to be in full equilibrium. Yet, specifying capital as quasi-fixed in a cost function is nonetheless appropriate in this study, because translog profit and cost functions, when interpreted as approximations, are not self-dual. Moreover, when the observed levels of the quasi-fixed factor coincide with the desired levels, the restricted and full static equilibrium models are equivalent and either specification is appropriate.

[7]Squires (1987a) discusses the construction of the data in detail, including the firm-level prices for capital services. These were based upon vessel-acquisition prices (including hull, gear, equipment, and engine) obtained for vessels purchased in the period 1976 through 1979. Most of the prices were exact and compiled from bills-of-sale; the remainder were from federal tax returns. In most instances, information was available for additional gear and equipment purchased subsequent to the vessel's purchase. All values were deflated by the GNP implicit price deflator at the time of acquisition. Effects from capital vintage were assumed negligible during this period, since vessel and gear design remained constant. Both new and used vessels were included, although the majority were new vessels. All costs, output, and revenue data are confidential at the level of the individual firm.

The *ex ante* capital services price was derived from the capital stock acquisition price, and equals the sum of the depreciation and the opportunity cost of capital. Upon the recommendation of marine financial specialists, a 7% depreciation rate was applied. The opportunity cost of capital was assumed equal to Moody's long-term bond rate for utilities, which is 10.94% in 1980 and 15.72% in 1981. The *ex ante* capital services price (in $1972) ranged from $6,100 to $123,107 with a mean of $40,706 and standard deviation of $22,603.

an arbitrary variable cost function. Thus,

$$\begin{aligned}\ln G = A_0 &+ \sum_{i\in I} A_i \ln w_i + \sum_{j\in J} A_j \ln y_j + A_K \ln K \\ &+ \tfrac{1}{2}\sum_{i\in I}\sum_{h\in I} A_{ih} \ln w_i \ln w_h + \tfrac{1}{2}\sum_{j\in J}\sum_{k\in J} A_{jk} \ln y_j \ln y_k \\ &+ \tfrac{1}{2}A_{KK}(\ln K)^2 + \sum_{i\in I}\sum_{j\in J} A_{ij} \ln w_i \ln y_j \\ &+ \sum_{i\in I} A_{iK} \ln w_i \ln K + \sum_{j\in J} A_{jK} \ln y_j \ln K, \end{aligned} \tag{16}$$

where $I = \{L$ (labor), Z (fuel)$\}$ and $J = \{R$ (roundfish), F (flatfish), O (other)$\}$. The restricted share equations for the variable inputs are then obtained by Hotelling's Lemma:

$$\begin{aligned} S_i &= \partial \ln G/\partial \ln w_i \\ &= A_i + \sum_{h\in I} A_{ih} \ln w_h + \sum_{j\in J} A_{ij} \ln y_j + A_{iK} \ln K, \end{aligned} \tag{17}$$

for $i = L, Z$. Finally, the assumption of profit maximization implies marginal-cost pricing for all of the outputs, i.e.,

$$p_j = \partial G/\partial y_j \quad \text{for} \quad j = R, F, O, \tag{18}$$

where p_j is the price of output j. This implies that

$$\begin{aligned} S_j &= p_j y_j/G = \partial \ln G/\partial \ln y_j \\ &= A_j + \sum_{k\in J} A_{jk} \ln y_k + \sum_{i\in I} A_{ij} \ln w_i + A_{jK} \ln K, \end{aligned} \tag{19}$$

for $j = R, F, O$. Eqs. (16), (17), and (19) were jointly estimated using the iterative Zellner estimator, with symmetry and linear homogeneity in prices imposed and with the fuel share equation dropped due to singularity.[8] The

[8]An anonymous referee has noted that, under the assumption of profit maximization, output quantities are endogenous. Because outputs were specified as exogenously determined, a simultaneous equations problem arises. This issue will be addressed in future work by use of an instrumental variable estimator such as iterative three-stage least squares. Nonetheless, as brought to our attention by Arnold Zellner, the finite-sample properties of instrumental variable estimators are not entirely known, and thus there is no assurance that they will be better than those of our current estimates given our relatively small sample.

Table 1

Parameter estimates from translog variable cost function.[a]

Parameter	Estimate	Standard error
A_0	0.216	0.213
A_R	1.208	0.072
A_F	1.127	0.083
A_O	0.808	0.096
A_L	0.483	0.027
A_K	−0.362	0.297
A_{RR}	0.319	0.036
A_{RF}	0.020	0.032
A_{RO}	−0.056	0.030
A_{RL}	−0.014	0.011
A_{RK}	−0.003	0.106
A_{FF}	0.669	0.068
A_{FO}	−0.025	0.035
A_{FL}	−0.041	0.021
A_{FK}	−0.441	0.099
A_{OO}	0.076	0.032
A_{OL}	−0.030	0.011
A_{OK}	0.107	0.163
A_{LL}	0.166	0.153
A_{LK}	−0.090	0.030
A_{KK}	0.245	0.313

[a]All variables scaled by sample arithmetic mean.

resulting parameter estimates are reported in table 1.[9] The generalized R^2, which considers goodness of fit for the entire system of equations, is 0.999. This is computed as $1-\exp[2(L_0-L_1/N]$, where L_0 (L_1) is the sample maximum of log-likelihood when all slope coefficients are zero (unconstrained) and N is the sample size [Baxter and Cragg (1970)]. The R^2's for the ordinary least squares estimation of each individual equation are as follows:

Equation	R^2
$\ln G$	0.954
S_L	0.514
S_R	0.836
S_F	0.828
S_O	0.762

[9]As a check on the soundness of these parameter estimates, the sign of the implied shadow price of capital was calculated. It was negative at the arithmetic sample mean and for 32 of the individual vessels. The 10 positive shadow prices of capital could reflect deterioration in performance of the second-order approximation when evaluated away from the point of approximation (the arithmetic sample mean in this case), which as noted by Wales (1977), can occur even if the data come from a well-behaved technology. Alternatively, the incorrect algebraic signs could reflect violations of correct model performance.

Table 2

Alternative capacity utilization measures.[a]

Dual measure:	CU_c	0.955	(0.045)
Ray measure:	CU_r	0.962	(0.052)
Partial measures:	CU_i for $i=R$	0.564	(0.102)
	$i=F$	0.714	(0.062)
	$i=O$	0.221	(0.237)

[a] Evaluated at arithmetic sample mean; linearized standard errors in parentheses.

The estimated parameters were used to calculate the dual, ray, and partial measures of capacity utilization. The CU measure under homothetic output separability was not calculated, since the data clearly rejected the separability hypothesis.[10]

The alternative CU measures are reported in table 2. Each is calculated at the arithmetic mean of the explanatory variables given in table 3. Linearized standard errors are given in parentheses.[11] All of the CU measures except for the third output's partial measure are statistically significant at 5%. Both the dual and ray CU measures suggest that in 1980 the New England otter trawl industry was very close to being in long-run equilibrium. This is consistent with Squires' (1987b) results using the test for long-run equilibrium suggested by Kulatilaka (1985). In interpreting CU_r, it should be remembered that this measure is subject to bias due to the maintained hypothesis that outputs are produced in fixed proportions (see footnote 3). Nonetheless, the fact that it provides results that are basically consistent with the dual CU measure and with previous work suggests that it may still be a useful indication of the current state of the industry.

The partial CU measures, the CU_i's, consistently indicate overcapitalization in the industry in 1980. Having partial CU measures that suggest overcapitalization is not necessarily inconsistent with the dual measure CU_c being close to one. Of course, $CU_c=1$ implies $CU_i=1$ for all i. However, if these measures converge at different rates (e.g., if costs are relatively insensitive to changes in output), then having short-run quantities that differ considerably

[10] In addition, imposing homothetic (homogeneous) output separability on the translog variable cost function implies that variable costs are proportional to aggregate output $h(y)$, i.e., that there are short-run constant returns to scale in the production of aggregate output. This implies increasing returns to scale in the long run. However, this is not necessarily inconsistent with perfect competition, because as discussed by Baumol, Panzar, and Willig, multi-product industry structure depends not only upon the overall scale of production, but on cost economies arising from joint production and product diversity. Moreover, this seems to be a problem with the translog specification that would not exist with other flexible functional forms such as the normalized quadratic.

[11] The linearized standard errors were calculated, following the delta method, as first-order Taylor's series approximations [Efron (1981)]. Analytical derivatives were used for the linearized standard error of the dual measure. Numerical derivatives were used for the linearized standard errors of the ray and partial measures, where each parameter was altered by 0.1%.

Table 3
Sample statistics of explanatory variables ($1972).

w_L	$7837.67/person
w_Z	$0.574/gallon
w_K	$40,705.90/vessel
y_R	451,817 pounds
y_F	296,046 pounds
y_O	841,202 pounds
K	120.38 gross registered tons

from their long-run equilibrium levels will not necessarily imply having actual costs that differ considerably from shadow costs. In other words, output disequilibrium may not impose a very large cost penalty. Thus, the empirical results in table 2 seem to suggest that, in terms of its output levels, the New England otter trawl industry was overcapitalized in 1980, although this overcapitalization did not impose considerable costs on the industry.

Finally, the partial CU measures suggest that the degree of overcapitalization in the industry varied considerably across products. This highlights the importance of allowing for the multi-product nature of the industry in measuring capacity utilization. Treating the firm as a single-product firm (as is done implicitly in the calculation of the ray measure, CU_r) masks this variability. However, knowledge of the variability across products may have important implications for analyzing the expansionary or contractionary forces within an industry if these forces are product-specific. For example, based on our 1980 estimates for the otter trawl industry, there appears to be much more slack in the industry in its production of the residual ('other') category of fish than there is in its production of flatfish. Thus, the future demand for flatfish is likely to be of more importance in determining the future expansionary or contractionary forces in the industry than is the demand for the other types of fish.

A limited license program in the New England otter trawl industry would be able to match the number of vessels with the resource stock conditions without giving undue concern to expansion of production capacity and wasteful inputs ('capital stuffing') as the resource stock recovers and fishermen's incomes increase. Moreover, the partial CU measures suggest a supplementary production quota placed upon the flatfish species assemblage could help forestall the future expansions in fishing activity that plague license limitation.

6. Summary and conclusions

We have proposed several alternative measures of capacity utilization that could be used for a multi-product firm. Each is in some way an extension of the single-product measures that have been proposed elsewhere. The single-product dual measure of capacity utilization based on a comparison of actual and shadow costs extends easily to the multi-product case because it is a scalar regardless of the number of products produced. Extension of the single-product primal measure is more difficult. We have proposed three possible extensions. Although each has its limitations because of the restrictions embodied in it, each provides different yet potentially useful information about capacity utilization in a multi-product industry. Moreover, our application of these measures to the New England otter trawl industry highlights the importance of explicitly incorporating the multi-product nature of some industries into studies of capacity utilization.

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