A BIOECONOMIC MODEL OF THE PACIFIC WHITING*

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The Pacific whiting (*Merluccius productus*) is a highly migratory fish occupying the continental shelf and slope off the west coast of North America. The species spawns in January off southern California and northern Mexico. During spring and summer the older and larger fish will migrate as far north as central Vancouver Island. Recruitment is highly variable, with strong year classes often supporting the commercial fishery during several years of low recruitment. The level of recruitment appears to be independent of the size of the spawning population.

A simple bioeconomic model of the Pacific whiting is constructed with independent recruitment. Fishery production functions are estimated from data on U.S. catch, average annual biomass and the number of vessels in the U.S. fleet. A stochastic optimization problem, seeking to maximize the expected value of industry profit, is formulated. Its solution would require a joint distribution on future recruitment and other bioeconomic parameters. Such a distribution is problematic. As an alternative, the certainty-equivalent problem is solved yielding solution values for the stochastic equilibrium and an approximately-optimal rule that sets allowable catch based on an estimate of current-year biomass.

Adaptive management can result in large changes in fleet size and allowable catch from year to year. The whiting fishery might be characterized as an opportunistic fishery, requiring a generalist fleet to expand or contract as bioeconomic conditions warrant. It is possible that long-run conditions would not support a profitable fishery, but that short-run fishing is profitable based on previous years of strong recruitment. The situation is not dissimilar to that facing the owner of a marginal gold mine that opens or closes depending on the price of gold. In the case of the whiting fishery, the optimal level of short-run fishing will depend not only on price, but on current biomass, the annual cost of fishing, the discount rate and vessel productivity. A simple interactive program is provided for would-be managers.

1. Introduction. With the development of a joint-venture fishery, the Pacific whiting (*Merluccius productus*) has become a commercially valuable species. Trawlers from California, Oregon, Washington and the province of British Columbia harvest whiting (also called hake) and then off-load the cod-end of their nets to a foreign factory vessel where the whiting is quickly processed to preserve freshness and texture. In 1989, the U.S. fleet delivered approximately 204 000 metric tons of whiting to foreign processing vessels, earning about $21 million in revenues.

The Pacific whiting is a highly migratory species, spawning off the coasts of southern California and northern Mexico in January (Bailey *et al.*, 1982).

During the spring and summer the population migrates northward, with the older and larger fish crossing into Canadian waters in August. Joint-venture arrangements have proved profitable to both U.S. and Canadian trawlers, and the distribution of allowable catch between the U.S. and Canada has taken on greater importance. While there is no formal treaty, fisheries managers from both countries have met to work out a long-term plan for binational allocation.

Recruitment in the whiting fishery appears to be independent of spawning biomass, but is positively correlated to surface temperature during spawning. Temperature in the month of January is affected by Eckmann transport, a process where warmer nearshore surface waters are pushed offshore, followed by an upwelling of deeper, cooler water (Bailey, 1981).

By August the whiting stock is distributed along the coast by age. While the location of a cohort in a particular year will depend on temperature, cohorts aged two to six are likely to be found off northern California and Oregon, while cohorts 7 to 14 are likely to be found off the coasts of Washington and British Columbia. In September and October whiting begin their southward migration from the feeding grounds to the spawning areas, and the cycle repeats itself.

The age structure of the resource and its reasonably stable migratory pattern have lead previous researchers to develop cohort models with population dynamics, migration and trophic interactions (Francis, 1983), stochastic recruitment (Swartzman *et al.*, 1983), and a game-theoretic approach to U.S.–Canadian management (Swartzman *et al.*, 1987). Dorn and Methot (1989) also employ a cohort model with recruitment randomly generated by iterative resampling from estimates of recruitment for the period 1959–1986. Constant and variable effort strategies are examined by averaging yields from 10 replicate, 1000-year simulations. Estimates of average yield ranged from 178 000 to 244 000 tons for the constant-effort strategy and from 205 000 to 251 000 tons for the variable-effort strategy. They recommend that total allowable catch be split 80 and 20% for the U.S. and Canada, respectively.

A simpler approach is taken in this paper. All the numerical results can be derived from the nine observations on catch, mean annual biomass and effort (vessel numbers) in Table 1, and by using the 20-line program (in BASIC) listed in Table 3. Analytical expressions for stochastic equilibrium and the approximately-optimal policy rule for adaptive management require some calculus and a fair amount of tedious algebra.

While the model is simple, it incorporates economic elements which have been absent in all the previous modeling of the Pacific whiting. Specifically, the program in Table 3 will employ estimates of a vessel productivity parameter, natural mortality, annual cost per vessel, dockside (or exvessel) price, the real rate of discount (interest) and long-run average recruitment to calculate what

has been called the stochastic equilibrium. More relevant to short term management is the adaptive-management rule which, given an updated set of bioeconomic parameters and an estimate of current-year biomass, will suggest levels for allowable catch and fleet size. The issue of distributing allowable catch between the U.S. and Canada is left for resolution by managers from both countries.

The rest of this paper is organized as follows. In the next section we constuct a bioeconomic model and derive equations defining stochastic equilibrium and the adaptive-management rule. In the third section we estimate production functions for the Pacific whiting fishery and calibrate the model for price and cost *circa* 1988. Section 4 examines stochastic equilibria and the performance of the adaptive-policy rule for allowable catch under a range of values for the bioeconomic parameters. The paper concludes with a discussion of the implications and limitations of the model.

2. Bioeconomics: Stochastic Equilibrium and Adaptive Management. Let X_t denote the average biomass of Pacific whiting in year t, E_t the level of fishing effort in year t and Y_t the level of harvest or catch. We assume there exits a production function relating annual catch to biomass and effort and write $Y_t = F(X_t, E_t)$, where the partial derivatives of $F(X_t, E_t)$ are denoted with subscripts and assumed to have the following signs: $F_X > 0$, $F_E > 0$, $F_{X,E} = F_{E,X} > 0$, $F_{X,X} \leqslant 0$ and $F_{E,E} \leqslant 0$. If p denotes the exvessel price per unit of catch (say, \$/metric ton) and c the cost of effort (say, cost/vessel/year), then we may write net revenue or profit in year t as:

$$\pi_t = pF(X_t, E_t) - cE_t. \tag{1}$$

Average annual biomass is assumed to change according to the following first-order difference equation:

$$X_{t+1} = (1 - M)[X_t - F(X_t, E_t)] + R_t \tag{2}$$

where M is annual natural mortality and R_t is a random variable denoting recruitment in year t. Maximization of the present value of expected profits subject to the dynamics of mean annual biomass may be stated mathematically as:

$$\text{Maximize } E\left\{ \sum_{t=0}^{\tau} \rho^t [pF(X_t, E_t) - cE_t] \right\}$$

$$\text{Subject to } X_{t+1} = (1 - M)[X_t - F(X_t, E_t)] + R_t$$

where $\rho = 1/(1 + \delta)$ is a discount factor and δ is the real rate of discount (or real annual interest rate).

This stochastic optimization problem might be solved by dynamic programming if a distribution for future recruitment were known. If other bioeconomic parameters are also random variables then one would need a joint distribution over all random variables. Such a distribution is problematic. As an alternative we consider what is called the "certainty-equivalent problem". The name is a bit of a misnomer, because the solution to the certainty-equivalent problem will not be the same as the solution to the stochastic dynamic programming problem (when the necessary distribution is known). The actual degree of suboptimality associated with the solution to the certainty-equivalent problem will depend on the specifics of the problem, the functional forms, the presence of irreversibilities, and the degree to which initial conditions differ from the long-run "stochastic equilibrium".* Before discussing the issue of suboptimality further, it may be useful to pose and solve the certainty-equivalent problem.

Let the expected value of R_t be denoted by R. The certainty-equivalent problem is the deterministic problem obtained by substituting the expected value for its random variable. This results in a problem with a Lagrangian expression that may be written as:

$$L = \sum_{t=0}^{\infty} \rho^t \{ pF(X_t, E_t) - cE_t$$

$$+ \rho\lambda_{t+1}[(1-M)[X_t - F(X_t, E_t)] + R - X_{t+1}]\} \tag{3}$$

where λ_{t+1} is the Lagrange multiplier associated with biomass in period $t+1$, and may be interpreted as the marginal value of an additional unit (say, metric ton) of fish in the water in year $t+1$. The Lagrange multiplier is also called the "shadow-price" of the fish stock. Note that R becomes a parameter in the certainty-equivalent problem.

In the Appendix we derive the first-order necessary conditions for this problem. They can be evaluated in steady state and are shown to imply the following two equations:

$$\frac{c(1-M)F_X}{(pF_E - c)} = \delta + M \tag{4}$$

$$R = MX + (1-M)F(X, E). \tag{5}$$

Equation (4) is a special case of what has been called the "fundamental equation of renewable resources" (see Conrad and Clark, 1987, p. 34). With independent recruitment the first derivative of the net growth function vanishes

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* Perhaps a more accurate name would be "certainty-equivalence equilibrium" since the concept of stochastic equilibrium is usually associated with a stationary probability distribution.

and we are left equating the "marginal stock effect" to the sum of the rate of discount and natural mortality. The marginal stock effect measures the incremental cost savings from larger biomass relative to the immediate benefit if that increment in biomass were harvested this year.

Equation (5) requires that expected (or long-run average) recruitment offset the reduction in biomass from natural mortality plus that portion of biomass that would have survived had it not been harvested. Equations (4) and (5) collectively define what Burt (1967) refers to as the stochastic equilibrium. Burt was concerned with the optimal management of a groundwater stock when recharge (form rain or melting snow) was stochastic. He notes that the stochastic equilibrium is "always approached, but rarely experienced".

The stochastic equilibrium for our problem is portrayed in Fig. 1. From the implicit function theorem, equation (4) will define a curve in $X - E$ space. Totally differentiating equation (4) and making use of the partials of $F(X, E)$, we can show that along this curve dE/dX is positive. Depending on the form of $F(X, E)$ it may be possible to solve for an explicit relationship, $E = E(X)$, that is positively sloped.*

Equation (5) also implies a curve in $X - E$ space. Total differentiation and the signs for F_X and F_E will imply that along this curve $dE/dX < 0$. If an explicit relationship, $E = R(X)$, can be obtained from equation (5), it will be negatively sloped. Thus, the partials of $F(X, E)$ imply that a nonzero stochastic equilibrium, (X_R, E_R) in Fig. 1, will be unique.

While the stochastic equilibrium may be of interest in determining the long-run effects of changes in the bioeconomic parameters, it is not very useful for short-term management. When fish biomass is not at its long-run equilibrium we would need to solve the deterministic certainty-equivalent problem, or a finite-horizon stochastic dynamic programming problem to determine the first step along an "approach path". With $F(X_t, E_t)$ nonlinear, this is not a trivial problem.

Instead of taking this tack we make use of an "approximately-optimal" technique proposed by Burt (1964, 1967) for groundwater management and more recently examined by Kolberg (1990) for management of a fishery. This approach makes use of equation (4) by noting that it can be regarded as defining a relationship between X_t and E_t in the vicinity of long-run equilibrium. Could we use this relationship for short-run management? If we do, how inferior would the resulting decisions be, relative to the solution

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* The curve $E(X)$ has nothing to do with the expectation operator. It is a smooth, positively-sloped curve obtained from equation (4) and will be used to identify the approximately optimal level of effort given an estimate of biomass, X. The curve implied by equation (5) will be denoted $E = R(E)$, since this curve will depend on parameters of the production function, the annual mortality rate and the long-run, expected level of recruitment, R. This latter curve is used in defining the stochastic or certainty-equivalence equilibrium, but only $E(X)$ is needed for short-run, adaptive management.
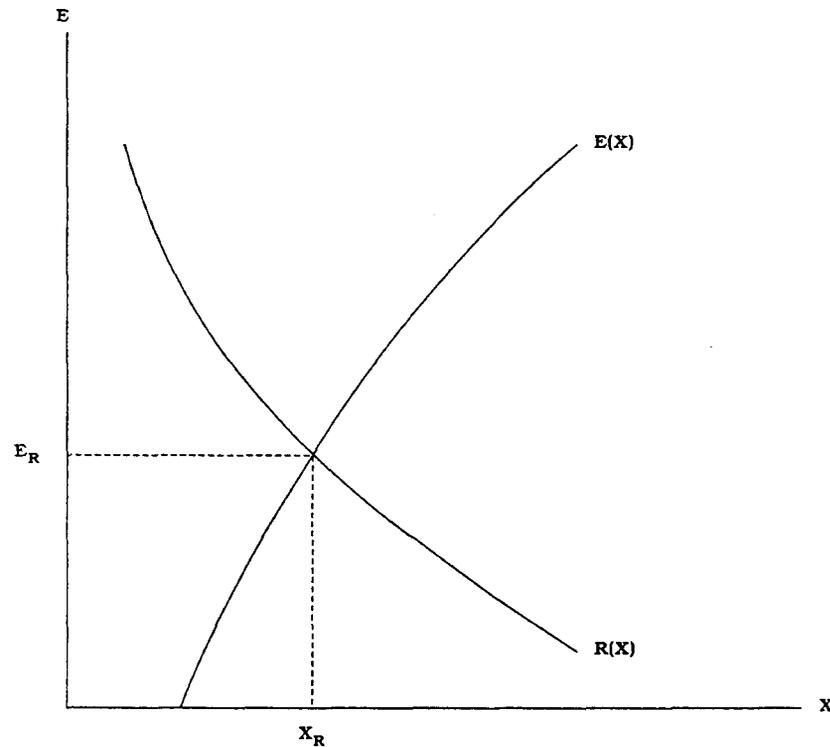
Figure 1. The long-run stochastic equilibrium.

obtained for a stochastic dynamic programming problem (with a known distribution for recruitment)? We will take these two questions in order.

The procedure for using equation (4) as an adaptive management rule is shown in Fig. 2. In the northeast quadrant we have redrawn the $E(X)$ curve from Fig. 1. Its position depends on all of the bioeconomic parameters except R, expected recruitment, which only appeared in equation (5). Suppose that biologists, using data from a series of scientific trawls or through a cohort model taking into account last year's total (U.S. plus Canadian) harvest, could provide managers with an estimate of biomass for the forthcoming year. With an estimate of X we could project up to the $E(X)$ curve to determine the recommended level of effort. The estimate of current biomass will also imply a specific production function in $E - Y$ space drawn in the northwest quadrant. Projecting E over to the appropriate production function results in a catch rate which might be used as allowable catch for the forthcoming year. Because recruitment is stochastic and because fishermen may exceed or fail to harvest allowable catch in a particular year, the subsequent estimates of X may bounce around. From Fig. 2 we can get a qualitative feel for how recommended effort
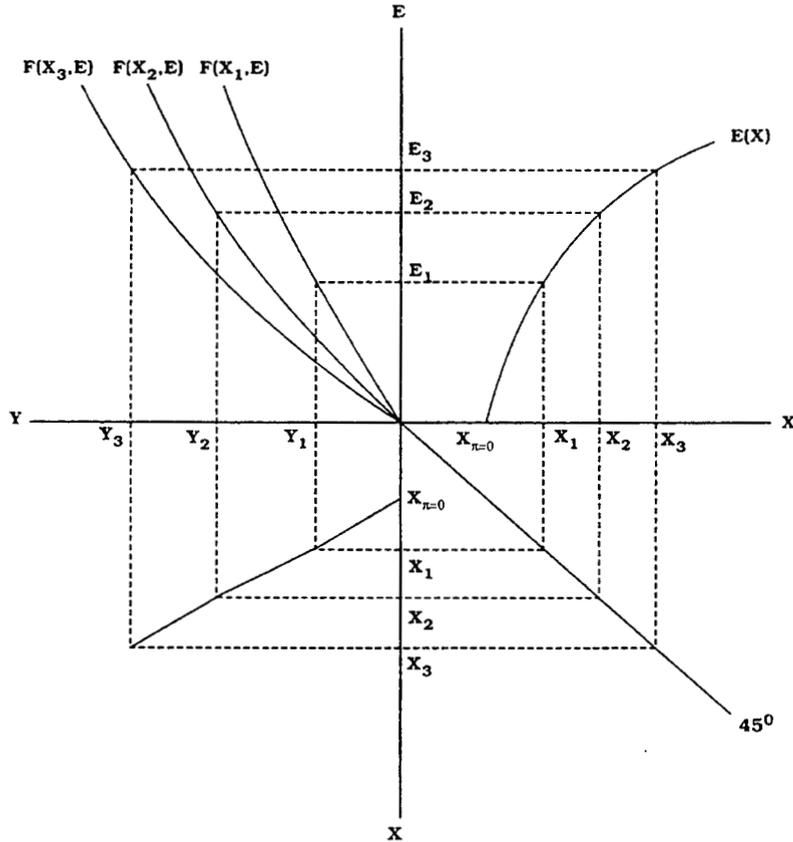
Figure 2. A depiction of the approximately-optimal feedback policy.

and allowable catch vary with X. First, note that there is likely to be an intercept of the $E(X)$ curve on the X axis. This has a straightforward interpretation. For a given set of bioeconomic parameters (per unit price, annual cost, the rate of discount, natural mortality, and perhaps catchability) there is likely to be some stock level below which fishing today would reduce present value. This is denoted by $X_{\pi=0}$. As X increases we see a less than proportional increase in E. The resulting change in Y is less easy to assess qualitatively because the production function shifts upward with increases in X. With a particular form for $F(X, E)$, and given estimates of the bioeconomic parameters, we could numerically examine the change in Y for a change in X. (We will do this for the whiting fishery in Section 4.) If we wish, we could collect the (X, Y) pairs by constructing a 45° transfer line in the southeast quadrant, project X downward, across, and then pair it with the corresponding Y projected downward from the Y-axis of the northwest quadrant. This is done in

the southwest quadrant and the four "dots" have been (arbitrarily) "connected" by a series of line segments.

Burt compared the level of groundwater pumping recommended by such a procedure to the level of pumping recommended when using stochastic dynamic programming, taking the estimate of current X as an initial condition, with all other parameters the same. In his study he found that the pumping rates differed by less than 2% when the current groundwater stock was within 42% of the stochastic equilibrium. As the current stock got closer to the stochastic optimum, the difference went to zero. On the basis of this relatively small departure from the optimal pumping rate, Burt dubbed this rule "the approximately-optimal" pumping rule.*

Burt and Cummings (1977), in considering this rule for other renewable resources, found that the difference between the approximately-optimal harvest rate and the optimal rate obtained via stochastic dynamic programming was likely to exhibit a consistent and perhaps attractive bias. When $X < X_R$ the harvest rate from the approximate rule was likely to be less than the harvest rate from the optimal rule. When $X > X_R$ harvest was likely to be slightly more than optimal. This would lead to a more rapid approach to equilibrium in a deterministic model. The slightly lower levels for recommended harvest when the resource stock was less than its stochastic equilibrium caused Burt and Cummings to regard the approximate rule as also being "conservative". Managers may find this built in conservatism (when stock is low) to be attractive.

In a recent study and application to the anchovy fishery in northern California, Kolberg (1990) analyzed the above approximate procedure and compared it to the optimal solution (obtained via dynamic programming) and two other approximate solutions obtained from first- and second-order Taylor approximations to the value function at the steady state optimum. Burt's original approximate rule [equation (4) in this paper] is equivalent to the first-order approximation of the value function. Kolberg finds that both first- and second-order rules result in harvesting decisions that produce a stream of discounted profits within 1% of the maximum.

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* The approximate optimality of the $E(X)$ curve as employed in Fig. 2 can be derived from the first-order conditions in the Appendix. Suppose that the optimal approach from some stock level X, involves values for the Lagrange multipliers of $\lambda_t = \lambda + \varepsilon_t$ and $\lambda_{t+1} = \lambda + \varepsilon_{t+1}$, where λ is the value of the multiplier at the certainty-equivalence equilibrium. Then the first and second of the first-order conditions can be shown to imply:

$$\frac{(pF_E - c)[\delta + M + (1 - M)F_X]}{(1 - M)F_E} = pF_X + \varepsilon_{t+1} + \varepsilon_t.$$

If the sum of ε_{t+1} and ε_t is small (which seems especially likely if convergence is oscillatory), then the above equation will be closely approximated by equation (4) in the text, and hopefully not too far from the solution to the underlying stochastic optimization problem.

It is difficult to make such comparisons in the whiting fishery. Without a joint distribution for recruitment and other bioeconomic parameters we do not have the necessary ingredients for the appropriate stochastic problem. As we will see in the next section, many of the estimates for equilibrium stock are around 1.0 million metric tons. This is within 30% of the 1989 estimate of 1.3 million metric tons for mean annual biomass (Dorn and Methot, 1989; Table 12). It would appear, at least superficially, that the approximately-optimal decision procedure described above can be appropriately applied to the Pacific whiting fishery.

3. Calibration of the Model for the Pacific Whiting Fishery.

In the general model of the preceding section the production function, $Y_t = F(X_t, E_t)$, took on central importance in defining the stochastic equilibrium and the adaptive-management rule. When one attempts to specify and estimate such a function, one encounters at least two problems. First, where does one obtain a time series of estimates for average annual biomass, and second, how should one define effort?

In calibrating the model to the Pacific whiting fishery the author was fortunate to have estimates of average annual biomass from a stock-synthesis model developed by Dorn and Methot (1989). This time series seemed the best available and would also allow a comparison of yield levels from two otherwise disparate modeling perspectives.

The definition of effort has always proven difficult. Ideally, one would like as precise a measure as possible of the actual volume of water "strained" per unit time. The closest practical measure might be the number of hours that a vessel had net in the water fishing. In a bioeconomic model, the analyst is further removed from the ideal measure because of the need to estimate the unit cost of effort. The measure adopted here is the number of vessels in the fishery. This measure is open to criticism because it may not correspond to the volume of water strained during a season, but it is a measure for which we have some data on unit annual cost.

Table 1 contains data on catch by U.S. vessels, estimates of mean annual biomass, and the number of vessels in the U.S. whiting fleet from 1981 to 1989. From 1985 onward the fleet has increased, with a jump from 42 vessels in 1988 to 65 in 1989. The estimate of mean annual biomass has declined from 2.225 million metric tons in 1986 to 1.315 million metric tons in 1989. Dorn and Methot believe that this reflects the "mining" of the strong 1980 and 1984 year classes that were recruited into the fishery in 1982 and 1986, respectively. (Note the jump in average annual biomass in those years.)

Table 2 contains the regression results when the data in Table 1 were used to estimate Cobb–Douglas and exponential production functions. The Cobb–Douglas function takes the form $Y = qX^{\alpha}E^{\beta}$, and is linear logs. It contains, as a

Table 1. Data on catch, mean annual biomass and effort in the
U.S. Pacific whiting fishery

Year (t)	Catch (Y_t)*	Biomass (X_t)†	Effort (E_t)‡
1981	44 395	1 384 000	21
1982	68 488	2 000 000	17
1983	73 150	1 805 000	19
1984	81 610	1 742 000	21
1985	35 586	1 685 000	17
1986	85 103	2 225 000	25
1987	110 792	2 012 000	31
1988	142 657	1 688 000	42
1989	204 038	1 315 000	65

*U.S. catch is measured in metric tons and is the sum of joint-venture and domestic catch from Table 1 of Dorn and Methot (1989).

†Biomass is measured in metric tons and is the mean annual estimate of biomass from Table 12 of Dorn and Methot (1989).

‡Effort is measured as the number of vessels in the U.S. Pacific whiting fleet as listed in the fax from D. E. Squires. NMFS, Southwest Fisheries Center, La Jolla, California, 18 June 1990.

special case, the standard catch-per-unit-effort production form (when $\alpha = \beta = 1$).

The exponential function takes the form $Y = X(1 - e^{-\alpha E})$. With this form it is impossible to catch more than current biomass, a logical characteristic, unfortunately not exhibited by the Cobb–Douglas production function. (Note: With the Cobb–Douglas form, as effort goes to infinity, so does catch.)

The exponential function may be estimated by regressing the natural log of the fraction of surviving biomass on effort. Ideally one would like to obtain an intercept not significantly different from zero and a significantly negative coefficient on effort. Alternatively, one can force the regression through the origin by suppressing the intercept.

The regression results for Cobb–Douglas and exponential production functions are given in Table 2. For the Cobb–Douglas form the coefficient for the natural log of q is not significantly different from zero, implying that q is not significantly different from one. The estimates of α and β are significant at the 5% level for a one-tail test. The adjusted R^2 is 0.8369. The Durbin–Watson statistic would appear to be in the inclusive range. The small sample size, however, makes determination of autocorrelation difficult.

In the least squares regression for the exponential function, the estimates of both the constant and the effort coefficient were significant. The adjusted R^2 was 0.9568. Because the Durbin–Watson seemed at the lower end of the inconclusive range the regression was run correcting first for first-order autocorrelation, then for second-order autocorrelation. Neither $AR(1)$ nor

Table 2. Regression results for the Cobb–Douglas and exponential production functions

A. Cobb–Douglas: $\ln Y_t = \ln q + \alpha \ln X_t + \beta \ln E_t + \varepsilon_t$
Least squares

Variable	Coefficient	Standard error	t-statistic	Two-tail significance
$\ln q$	−7.4226	7.4350	−0.9983	0.357
$\ln X$	1.0274	0.4979	2.0631	0.085
$\ln E$	1.2240	0.1878	6.5169	0.001
$R^2 = 0.8776$,	Adjusted $R^2 = 0.8369$,	$F = 21.5285$,	$D - W = 1.4724$	

B. Exponential: $\ln(1 - Y_t/X_t) = \gamma + \alpha E_t + \varepsilon_t$
Least squares

Variable	Coefficient	Standard error	t-statistic	Two-tail significance
γ	0.0215	0.0068	3.1732	0.016
E	−0.0028	0.00021	−13.35	0.000
$R^2 = 0.9622$,	Adjusted $R^2 = 0.9568$,	$F = 178.232$,	$D - W = 1.1146$	

Least squares, $AR(1)$

Variable	Coefficient	Standard error	t-statistic	Two-tail significance
γ	0.0272	0.0187	1.4575	0.205
E	−0.0030	0.00043	−7.0358	0.001
$AR(1)$	0.5962	0.5030	1.1852	0.289
$R^2 = 0.9708$,	Adjusted $R^2 = 0.9591$,	$F = 83.209$,	$D - W = 0.9544$	

Least squares, $AR(2)$

Variable	Coefficient	Standard error	t-statistic	Two-tail significance
γ	0.0225	0.0121	1.8498	0.138
E	−0.0028	0.00036	−7.7592	0.001
$AR(2)$	−0.0783	0.6942	−0.1128	0.916
$R^2 = 0.9651$,	Adjusted $R^2 = 0.9477$,	$F = 55.368$,	$D - W = 0.9676$	

$AR(2)$ were significant and in fact the Durbin–Watson statistics became smaller. The constant term became insignificant in both the $AR(1)$ and $AR(2)$ regressions but the estimate of α was essentially unchanged and remained significant.

The exponential form, with $\alpha > 0$ implies that production is strictly concave in effort, while the Cobb–Douglas function with $\beta > 1$ is not concave and would cause the stochastic equilibrium to be locally unstable. For this reason, and others noted above, we adopt the exponential form and run sensitivity analysis on α over the interval [2.0E-2, 3.0E-2].

In an analysis of the tax returns of 13 vessels participating in the whiting fishery in 1988, Squires (1990) estimates annual variable costs per vessel to be approximately $150 000. More difficult to estimate is the portion of fixed costs that should also be included when estimating annual operating costs. Squires calculates annual fixed costs by adding the costs of insurance, rent, association

dues, professional services and 7% of vessel acquisition costs (for vessels bought in 1978–1986, inclusive), for a total of approximately $237 000 in 1988. The sum of annual variable and fixed cost payments comes to $387 000.

It is difficult to argue that all the costs filed (*ex post*) on a tax form are relevant when a fisherman chooses to fish whiting, as opposed to some other species. One also suspects that there is an incentive to report as high a cost as possible (to reduce taxable income). In the numerical analysis of the next section we restrict our estimate of c to the interval ($200 000, $300 000).

Francis (1983), in fitting a cohort model to survey data, concluded that annual mortality was likely to be age-dependent, with rates varying from 0.195 for 5-year-old fish, to 0.757 for 11-year-old fish. Dorn and Methot use a constant rate of 0.20 for all cohorts. An average annual mortality rate of 0.25 is used in the Base-Case, with values of $M = 0.20$ and $M = 0.30$ also examined.

The price per metric ton for whiting has fallen since the early 1980s, when it peaked at slightly over $151 in 1982. From 1986 to 1989 the price has been relatively stable between $106 and $110 per metric ton. Stochastic equilibria and adaptive management are examined for prices of $100, $110 and $120 per metric ton.

Modeling by Dorn and Methot also provided estimates of recruitment, measured as billions of age two fish entering the fishery. They construct a time series from 1958 to 1988. There is a large range, from a low of 0.017 in 1987 to a high of 5.16 in 1963. The average over this 31-year period was 0.991 billion fish. An average 2-year-old whiting will weigh about 250 grams, transforming the 0.991 billion fish into an average recruitment of approximately 250 000 metric tons per year.

Though imprecise, we set $R = 250\,000$ metric tons in the Base-Case parameter set. It is important to emphasize that while recruitment is highly variable, mean annual biomass is much less variable. Adaptive management does not depend directly on recruitment, only on an estimate of mean annual biomass. This has ranged from a high of 3.695 million metric tons in 1965, to low of 1.315 in 1989; with most year to year changes being less than 15%.

The final parameter required for both stochastic equilibrium and adaptive management is an estimate of the real (inflation-free) rate of discount. There has been a long standing debate among economists as to the appropriate rate of discount to employ when evaluating public investments or managing publicly held resources. There appears to be no simple answer. It depends on where the funds are coming from (whether they are displacing private investment or consumption) and whether the beneficiaries of the project derive a significant portion of their income from the investment or resource.

The question is perhaps more easily answered when managing a fishery resource. If there are a large group of fishermen, or if the species being managed constitutes only a small portion of the total income derived from fishing, then

the discount rate should be risk-free as well. Discount rates of 2, 4 and 6% will be evaluated.

4. Results. Restricting our analysis to the exponential production function, $Y = X(1 - e^{-\alpha E})$, we note that $F_X = (1 - e^{-\alpha E})$, and that $F_E = \alpha X e^{-\alpha E}$. Substitution into equation (4) results in:

$$\frac{c(1 - M)(1 - e^{-\alpha E})}{(p\alpha X e^{-\alpha E} - c)} = \delta + M.\qquad (6)$$

Equation (5) takes the form:

$$R = MX + (1 - M)X(1 - e^{-\alpha E}).\qquad (7)$$

It is possible to solve equation (6) for an explicit expression for E, yielding:

$$E = -\ln\left[\frac{c(1 + \delta)}{[p\alpha(\delta + M)X + c(1 - M)]}\right]\bigg/\alpha.\qquad (8)$$

This is our $E = E(X)$ curve in Figs 1 and 2. It will be used in the adaptive-management program.

Using equation (7) it is possible to eliminate E from equation (6) and obtain a quadratic expression in X. The positive root gives an expression for the optimal (stochastic) equilibrium stock. This expression is tedious to derive but some careful algebra should reveal:

$$X_R = (-B + \sqrt{B^2 - 4N})/2\qquad (9)$$

where:

$$B = -\frac{[p\alpha R(\delta + M) + c(1 - M)\,\delta]}{[p\alpha(\delta + M)]}\qquad (10)$$

and

$$N = -\frac{c(1 - M)R}{[p\alpha(\delta + M)]}.\qquad (11)$$

With X_R we can calculate long-run optimal effort as:

$$E_R = -\ln\{(X_R - R)/[(1 - M)X_R]\}/\alpha.\qquad (12)$$

From the production function we know $Y_R = X_R(1 - e^{-\alpha E_R})$.

In the program in Table 3 we define and read the parameters α, c, δ, M, p and R and then calculate the stochastic equilibrium X_R, E_R and Y_R. You are then

asked if you would like to adaptively manage. If you answer yes, you are asked
for an estimate of current-year biomass. Using this as the value of X in
equation (8), and the same bioeconomic parameters as specified in line 10, the
program calculates the approximately-optimal E (lines 140–150), then catch
(line 160), and finally prints the results.

Table 3. A listing of the BASIC program to calculate the stochastic equilibrium and to
adaptively manage based on estimates of current biomass

```
 10  DATA 0.25E-2.250000,0.04,0.25,110,250000
 20  READ A,C,D,M,P,R
 30  B= -(P*A*R*(D+M)+C*(1-M)*D)/(P*A*(D+M))
 40  N= -C*(1-M)*R/(P*A*(D+M))
 50  XR=(-B+SQR(B^2-4*N))/2
 60  ER= -LOG((XR-R)/((1-M)*XR))/A
 70  YR=XR*(1-EXP(-A*ER))
 80  PRINT:PRINT "Long-Run Average Biomass=";XR
 90  PRINT:PRINT "Long-Run Average Effort=";ER
100  PRINT:PRINT "Long-Run Average Catch=";YR
110  PRINT:INPUT "Do you want to Adaptively Manage? Yes=1, No=0.";W
120  IF W=0 GOTO 200
130  PRINT:INPUT "Current Biomass=";X
140  NUM=C*(1+D):DEN=P*A*(D+M)*X+C*(1-M)
150  E= -LOG(NUM/DEN)/A
160  Y=X*(1-EXP(-A*E))
170  PRINT:PRINT "Current Biomass=";X
180  PRINT:PRINT "Recommended Effort=";E
190  PRINT:PRINT "Recommended Catch=";Y
200  END
```

The Base-Case parameter set is $\alpha = 0.25E-2$, $c = \$250\,000$, $\delta = 0.04$, $M = 0.25$,
$p = \$110$ and $R = 250\,000$. Table 4 reports the calculated values for stochastic
equilibrium and the approximately-optimal values for effort and allowable
catch when the current biomass is 1.0E6, 1.5E6 and 2.0E6 metric tons. Each
parameter (with the exception of R) is varied above and below its Base-Case
value to determine its effect on the stochastic equilibrium and the adaptively-
managed levels of effort and allowable catch. The results are presented in the 10
subcases (B through K) also contained in Table 4. Increases in long-run
expected recruitment, R, will increase the equilibrium levels for biomass, effort
and yield, but given the imprecise estimate of this value and the previously
noted fact that the stochastic equilibrium is "seldom experienced", we do not
present these results.

For the Base-Case parameter set the stochastic equilibrium occurs at a mean
annual biomass of 957 748 metric tons, supporting a fleet of six vessels

Table 4. Stochastic equilibria and adaptive management

A. Base-Case parameter set

	$\alpha = 0.25\text{E-}2$	$c = \$250\,000$	$\delta = 0.04$
	$M = 0.25$	$p = \$110$	$R = 250\,000$ mt
$X_R = 957\,748$ mt		$E_R = 6$ vessels	$Y_R = 14\,083$ mt
When	$X = 1.0\text{E}6$	$E = 11$	$Y = 27\,128$
	$X = 1.5\text{E}6$	$E = 67$	$Y = 230\,159$
	$X = 2.0\text{E}6$	$E = 115$	$Y = 501\,441$

B. $\alpha = 0.20\text{E-}2$

No commercial fishery in the long run. Vessels not sufficiently productive.

When	$X = 1.0\text{E}6$	No commercial fishery	Stock too low
	$X = 1.5\text{E}6$	$E = 43$	$Y = 122\,881$
	$X = 2.0\text{E}6$	$E = 96$	$Y = 349\,730$

C. $\alpha = 0.30\text{E-}2$

$X_R = 883\,051$ mt		$E_R = 15$ vessels	$Y_R = 38\,983$ mt
When	$X = 1.0\text{E}6$	$E = 28$	$Y = 81\,921$
	$X = 1.5\text{E}6$	$E = 81$	$Y = 321\,930$
	$X = 2.0\text{E}6$	$E = 126$	$Y = 627\,606$

D. $c = \$200\,000$

$X_R = 867\,361$ mt		$E_R = 21$ vessels	$Y_R = 44\,213$ mt
When	$X = 1.0\text{E}6$	$E = 40$	$Y = 94\,668$
	$X = 1.5\text{E}6$	$E = 104$	$Y = 342\,837$
	$X = 2.0\text{E}6$	$E = 159$	$Y = 655\,897$

E. $c = \$300\,000$

No commercial fishery in the long run. Fishing too costly.

When	$X = 1.0\text{E}6$	No commercial fishery	Stock too low
	$X = 1.5\text{E}6$	$E = 40$	$Y = 142\,002$
	$X = 2.0\text{E}6$	$E = 84$	$Y = 377\,113$

F. $\delta = 0.02$

$X_R = 958\,887$ mt		$E_R = 6$ vessels	$Y_R = 13\,704$ mt
When	$X = 1.0\text{E}6$	$E = 10$	$Y = 25\,788$
	$X = 1.5\text{E}6$	$E = 64$	$Y = 220\,201$
	$X = 2.0\text{E}6$	$E = 110$	$Y = 482\,143$

G. $\delta = 0.06$

$X_R = 956\,702$ mt		$E_R = 6$ vessels	$Y_R = 14\,432$ mt
When	$X = 1.0\text{E}6$	$E = 12$	$Y = 28\,414$
	$X = 1.5\text{E}6$	$E = 70$	$Y = 239\,956$
	$X = 2.0\text{E}6$	$E = 120$	$Y = 519\,553$

Table 4—*continued*

H. $M = 0.20$		
$X_R = 1\ 075\ 564$ mt	$E_R = 17$ vessels	$Y_R = 43\ 609$ mt
When $X = 1.0E6$	$E = 9$	$Y = 22\ 556$
$X = 1.5E6$	$E = 56$	$Y = 195\ 652$
$X = 2.0E6$	$E = 98$	$Y = 433\ 735$

I. $M = 0.30$		
No commercial fishery in the long run. Natural mortality too high.		
When $X = 1.0E6$	$E = 13$	$Y = 31\ 657$
$X = 1.5E6$	$E = 77$	$Y = 262\ 887$
$X = 2.0E6$	$E = 132$	$Y = 563\ 536$

J. $p = \$100\ mt^{-1}$		
No commercial fishery in long run. Price too low.		
When $X = 1.0E6$	$E = 0$	$Y = 0$
$X = 1.5E6$	$E = 52$	$Y = 183\ 544$
$X = 2.0E6$	$E = 98$	$Y = 436\ 090$

K. $p = \$120\ mt^{-1}$		
$X_R = 921\ 131$ mt	$E_R = 12$ vessels	$Y_R = 26\ 289$ mt
When $X = 1.0E6$	$E = 22$	$Y = 52\ 823$
$X = 1.5E6$	$E = 81$	$Y = 273\ 585$
$X = 2.0E6$	$E = 132$	$Y = 561\ 549$

Vessel numbers are rounded to nearest whole vessel. Catch is rounded to nearest whole metric ton. Catch is calculated before rounding effort. Thus, fractional effort less than 0.5 vessels may give rise to slightly different catch for same biomass (Subcase H to J when $X = 2.0E6$).

harvesting 14 083 metric tons per year. These values are significantly below those observed in the previous decade (see Table 1).

When the current biomass increases from 1.0 to 2.0 million metric tons the adaptive rule recommends that fleet size increase from 11 to 115 vessels and that catch be allowed to increase from 27 128 to 501 441 metric tons. When current biomass is 1.5 million metric tons, a recommended fleet of 67 vessels would harvest 230 159 metric tons. These latter values are very similar to the "observed" values for catch, biomass and effort in 1989 from Table 1.

From this single piece of analysis we might hazard a characterization of the whiting fishery. It is a fishery that will be strongly influenced by current bioeconomic conditions. It should be managed opportunistically. When stochastic recruitment "deals a full house", maximization of expected present value says the fleet should significantly expand to harvest the windfall. The downside, of course, is that when recruitment deals nothing, the fleet must "fold 'em" and walk. To quote the Kenny Rodgers song, fisheries managers have to

"know when to hold 'em and know when to fold 'em". In the U.S. and elsewhere, unfortunately, managers and fishermen have been slow to walk, trying to stay in the game when bioeconomic conditions indicate one should leave (at least temporarily).

The program in Table 3 indicates when fishing would reduce present value by returning a negative value for effort and catch. This can occur in the long-run stochastic equilibrium or in the short-run under adaptive management. In fact, for a given set of bioeconomic parameters a fishery that is unprofitable in the long-run may continue to be fished if strong recruitment or favorable prices prevail. Conversely, a fishery which is profitable in the long-run (stochastic equilibrium) may be shut down in the short-run because biomass has declined below a level that would support positive effort and catch along the optimal approach path. Recall the interpretation of $X_{\pi=0}$ in Figs 1 and 2.

The first situation is shown in Subcase B where, when vessel productivity declines from $\alpha = 0.25\text{E-2}$ to $\alpha = 0.20\text{E-2}$, there is no fishing in the stochastic equilibrium. If a run of strong recruitment (or a temporary moratorium) pushes biomass up to 1.5E6 metric tons a fleet of 43 would be allowed to harvest 122 881 metric tons. In Subcase C, where $a = 0.30\text{E-2}$, the stochastic equilibrium has a biomass of 883 051 metric tons supporting 15 vessels and an annual yield of 38 983 metric tons. If recruitment pushes biomass up to 1.5E6 metric tons, adaptive managers would send out 81 vessels to harvest 321 930 metric tons.

In Subcase E, with an annual vessel cost of $300 000 there would be no fishing in the stochastic equilibrium. A biomass level of 1.0E6 is still below $X_{\pi=0}$. At a biomass of 1.5E6 a fleet of 40 vessels is allowed to harvest 142 002 metric tons.

The value for $X_{\pi=0}$ when $p = \$100$ is precisely 1.0E6 metric tons (see Subcase J). The fishery is not profitable in the long run at this price, but short-run biomass levels of 1.5E6 and 2.0E6 would support fleets of 52 and 98 vessels.

A systematic analysis of the results in Table 4 will reveal:

(i) an increase in α will reduce equilibrium biomass while increasing fleet size and catch (Subcase A to C);

(ii) an increase in cost, c, will increase equilibrium biomass, reducing effort and catch (Subcase D to A);

(iii) an increase in the discount rate has relatively little impact, reducing equilibrium biomass slightly, causing a fractional increase in effort and a slight increase in catch (Subcase F to G);

(iv) an increase in natural mortality might shut down the fishery in the long run and has the effect (similar to an increase in the discount rate) of increasing effort and catch in the short run (before fish die of natural causes; see Subcase H to I), finally;

(v) an increase in price may make the fishery tenable in the long run and will increase effort and yield when adaptively managed at the same level of biomass (Subcase J to K).

It is a bit difficult to compare the results of Table 4 to the results of Francis (1983), Swartzman *et al.* (1983, 1987) and Dorn and Methot (1989). All of the models are cohort models and none are bioeconomic, in the sense of maximizing a present value measure. Perhaps the only common denominator is average yield. This is difficult to calculate in anything but a naive way because the cohort models are frequently run with constant fishing mortality or with constraints that prevent the biomass from declining below some bound. With that caveat in mind, we note that a simple average of yields listed in the first row of Table 3 from Swartzman *et al.* (1983) is 193 666 metric tons. The average yield from Table 2 of Swartzman *et al.* (1987) is 184 000 metric tons. From Dorn and Methot (1989) we have previously noted that average yield ranges from 178 000 to 244 000 metric tons for low risk runs and from 205 000 to 251 000 metric tons for high risk runs. If one averages the 44 yields (including zero yield when the fishery is shut down in the long or short run) from Table 4 in this paper one obtains 195 552 metric tons. While the models are very different in their biological and economic details, from the perspective of average yield they would appear to be in the same ballpark.

5. Conclusions. The Pacific whiting has become an important commercial species for both the U.S. and Canada. Both countries participate in joint-venture fisheries, where domestic trawlers capture whiting and off-load onto foreign processing vessels. Several papers published in the 1980s have examined population dynamics within age-structured models. Recruitment is thought to be independent of spawning biomass, and has been treated as a random variable. Because older and larger fish migrate further north, the age-structure of the resource can influence the availability of fish in Canadian waters.

While these models have been rich in biological detail, they have not adequately incorporated the economic factors which affect the commercial value of the resource, nor have they tried to determine optimal fleet size. The biological detail present in these models necessitates numerical analysis, such as Monte Carlo simulation, to determine the properties of the model and to develop average yields that might be used in making recommendations for allowable catch.

In this paper we have traded-off the biological detail of a cohort model in order to incorporate some of the economic factors thought to be important in the Pacific whiting fishery. The simple bioeconomic model of Section 2 permitted us to: (1) pose a stochastic optimization problem that sought to

maximize the present value of expected net revenue; (2) solve the certainty-equivalent problem for the stochastic equilibrium and an approximately-optimal rule for adaptive management; and (3) portray the equilibrium (Fig. 1) and show how the adaptive-management rule would operate (Fig. 2).

Data on catch, mean annual biomass and vessel numbers allowed for the direct estimation of a fishery production function. Cobb–Douglas and an exponential function both gave reasonable fits. The exponential form makes more sense biologically, gave a slightly better fit and for the parameter estimate was strictly concave in effort. This form was used and a range of values for the other bioeconomic parameters was obtained from previous biological and economic research.

In the bioeconomic model, long-run (stochastic) equilibrium depended on the production parameter, annual vessel cost, the discount rate, natural mortality, exvessel price and long-run average recruitment. In the short-run, using the adaptive-management rule, fleet size and allowable catch depended on the first five parameters and current biomass (instead of recruitment). Recommendations for short-run fleet size and allowable catch could fluctuate widely depending on the bioeconomic parameters, especially current biomass. From the Base-Case parameter set we observed that a current biomass of 1.0E6 metric tons would commend a fleet of only 11 vessels harvesting 27 128 metric tons. If current biomass were 1.5E6 metric tons, 67 vessels could harvest 230 159 metric tons and, if biomass increased to 2.0E6 metric tons (perhaps in the vicinity of "pristine equilibrium"), 115 vessels could harvest a 501 441 metric tons.

Such results characterize what might be called an opportunistic fishery, requiring a flexible fleet of generalist vessels able to respond to windfall recruitment and to shift to other fisheries when bioeconomic conditions are no longer favorable. Such flexibility has not been present in the U.S. or Canadian fishing industry, where effort seems quick to expand, but slow to contract. Managers and the fishing industry need to explore ways of increasing flexibility.

To use the adaptive-management rule we need an estimate of current-year biomass. The cohort models, especially the stock-synthesis model of Dorn and Methot (1989), can provide such an estimate. The age-structured models also have the advantage of being able to project changes in the abundance of particular cohorts. Such information might be important in determining spawning potential and the availability of whiting in Canadian waters.

This model should not be viewed as a replacement or even as a competitor for the niche occupied by the more complex biological models within the current "management landscape". Rather, it should be used to complement the analysis of such models in seeking the economically efficient and equitable distribution of the Pacific whiting resource.

APPENDIX

The Lagrangian for the certainty-equivalent problem has first-order conditions requiring:

$$\frac{\partial L}{\partial E_t} = \rho^t\{pF_E - c - \rho\lambda_{t+1}(1-M)F_E\} = 0$$

$$\frac{\partial L}{\partial X_t} = \rho^t\{pF_X + \rho\lambda_{t+1}(1-M)[1-F_X]\} - \rho^t\lambda_t = 0$$

$$\frac{\partial L}{\partial[\rho\lambda_{t+1}]} = \rho^t[(1-M)[X_t - F(X_t, E_t)] + R - X_{t+1}] = 0.$$

In steady state these conditions imply:

$$\rho\lambda = (pF_E - c)/[(1-M)F_E]$$

$$\rho\lambda[(1-M)[1-F_X] - (1+\delta)] = -pF_X$$

$$R = MX + (1-M)F(X, E).$$

The second steady state equation can be further simplified to:

$$-\rho\lambda[(\delta + M) + (1-M)F_X] = -pF_X.$$

Multiplying through by -1 and substituting the first steady-state expression for $\rho\lambda$ yields:

$$(pF_E - c)[(\delta + M) + (1-M)F_X] = pF_X[(1-M)F_E].$$

This last expression can be further simplified to:

$$\frac{c(1-M)F_X}{(pF_E - c)} = \delta + M$$

which is given as equation (4) in the text. Equation (5) in the text is the third of the steady-state equations listed above.

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