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Divergency between average and frontier production technologies: an empirical investigation for Bangladesh

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Applied economic analysis and policy formation often relies on estimated production technologies. Primal representations of production technologies can be specified as the average production function or as the stochastic frontier. This paper uses nested and non-nested specification tests and assessment of economic variables, including elasticities and marginal products, to evaluate systematic differences between the average production function and three different specifications of the stochastic frontier. Bangladesh serves as an illustrative example. The importance emerges of first beginning any analysis of production technologies with nested hypothesis testing of inefficiency and non-nested hypothesis testing for systematic but unknown differences between the average and stochastic frontier functions followed by nested testing of the final form of the production technology.

I. INTRODUCTION

In empirical work, the production function is usually estimated by regression analysis and, so, provides only an average relationship between inputs and output, since the regression line is fitted through the mean of the data set. In contrast, the estimation of production frontier corresponds to the formal economic definition of a production function (Tisdell 1982).[1] Applied policy formation based upon estimation of these two alternative representations of the production technology, the 'average' production function (hereafter the production function) and the production frontier, might lead to different policies and conclusions. Hence, an appreciation of the extent of differences of these estimates of the production function can be useful in practice.

The issue of possible differences in the assessment of the production technology by production functions and production frontiers is addressed in this paper. The approach adopted is an empirical one, using a cross-section of data for industries from Bangladesh, to estimate econometrically both an aggregate production function and an aggregate production frontier.[2] The results are used to assess differences in the substitution possibilities between labour and

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[1] For a comprehensive discussion of conceptual issues in the measurement of economic and productive efficiencies see Tisdell (1985).

[2] The current analysis, like the similar analysis of Bairam (1988), does not really identify the (outer) production frontier of the economy, since the output, capital and labour quantities are amalgams. In most industries firms will vary in their technical efficiency, but this is 'concealed' when industry averages are used. Unfortunately, we do not have the data to estimate this 'outer' production frontier for the economy. But it is quite possible, for instance, that employment-generating/displacing effects of new capital when this outer frontier is considered could differ even more substantially from those suggested by the average aggregate production than do the estimates given here for the aggregate production frontier. This suggests that while the type of analysis may give some useful first approximations for policy purposes, that it needs to be supplemented by in-depth studies in industry economics. Furthermore, it may be important to take into account natural resource and sustainability aspects, e.g. role of natural resource stocks in production, before reaching firm policy conclusions from this type of production technology estimation.

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capital, differences in their marginal products, and any final form of the technology found from a series of nested hypothesis tests of constant returns to scale (CRTS) and functional form. Both nested and non-nested specification tests evaluate whether the frontier or production function is more appropriate. It is found that selection between the frontier and production function hinges upon the type of significance tests employed, nested or non-nested, and the initial null hypothesis.

II. APPROACH AND METHODOLOGY

Technical efficiency and procedures for specification of the production frontier, the functional form, measurement of marginal products and measurement of factor substitution possibilities are discussed in this section.

Technical efficiency and production frontiers

A sector's technical efficiency measures the ability to produce the maximum output possible from a given set of inputs and production technology. Technical efficiency is a relative concept since each sector's production performance is compared with a best-practice input–output relationship. The best-practice performance, or production frontier, is established by the practices of the most efficient sectors. Technical efficiency is then measured as the deviation of the individual sectors from this best-practice frontier.

This best practice is assumed stochastic, with a corresponding two-sided error term, to capture exogenous shocks beyond the control of firms. Since all sectors do not produce the frontier output, an additional one-sided error term is introduced to represent technical inefficiency. This approach contrasts with the production function which provides an average relationship between output and inputs over the entire sample, and in which a single, two-sided error term captures the stochastic influences and unobserved input usage.

The production frontier is estimated by stochastic frontier approach, which may be written (Aigner *et al.*, 1977) as

$$Y_i = h(X_1, X_2, \ldots, X_N, A)\, e^{\Phi_i} \tag{1}$$

where Y_i is the output of the ith of the 47 sectors in the Bangladesh economy, X_j is the jth of N inputs, A represents a vector of parameters, e is the exponential operator, and Φ_i is a sector-specific error term. The error term Φ_i is composed of two independent components: $\Phi_i = V_i - U_i$. The systematic component, V_i, represents random variation in output due to factors outside the sector's control (such as weather), measurement error and statistical noise. It allows the deterministic production frontier to be stochastic. The technical efficiency relative to the stochastic frontier, $e^{-U_i} = Y_i/[h(X_1, X_2, \ldots, X_N, A)\, e^{-V_i}]$, is captured by the one-sided error component $U_i \geqslant 0$. When $U_i = 0$, production lies on the stochastic frontier and is technically efficient and, when $U_i > 0$, production lies below the frontier and is technically inefficient.

It is assumed that the symmetric error V_i is independently and identically distributed as $N(0, \sigma_V^2)$ and that the non-negative error U is distributed as the absolute value of a normal distribution, $|N(0, \sigma_U^2)|$, i.e. half-normal, and that $\sigma^2 = \sigma_U^2 + \sigma_V^2$. Define $\delta = \sigma_U/\sigma_V$. The greater δ is from unity, the more production is dominated by technical inefficiency, while the closer to zero, the more the discrepancy between the observed and frontier output is dominated by random factors beyond the control of the sector.

The inefficient component of Φ in the stochastic frontier, U, might be distributed other than half-normal. One possibility is the exponential distribution. A second possibility, proposed by Stevenson (1980), attempts to add an additional degree of flexibility to the one-sided error distribution. Stevenson proposed a truncated (at zero) normal distribution with non-zero mean for the distribution of U. This adds an additional parameter to the model, say μ.

Translog production technology

The production technology was specified as translog, which, following Berndt and Christensen (1973), can be written as

$$\begin{aligned} \ln Y = {} & \alpha + \alpha_K \ln K + \alpha_L \ln L + \alpha_{KK} \ln K \ln K \\ & + \alpha_{KL} \ln K \ln L + \alpha_{LL} \ln L \ln L \end{aligned} \tag{2}$$

where Y represents output, K is capital, and L is labour. The translog production technology was interpreted as a second-order approximation to an underlying, unknown technology rather than as a direct representation of technology.

Marginal products

Marginal products can be derived from the translog form. The logarithmic marginal product for capital may be written as

$$\delta \ln Y/\delta \ln K = (\delta Y/\delta K)/(K/Y) = \alpha_K + \alpha_{KK} \ln K + \alpha_{KL} \ln L.$$

Rearranging yields:

$$\delta Y/\delta K = [\alpha_K + \alpha_{KK} \ln K + \alpha_{KL} \ln L][K/Y].$$

Similarly, $\delta Y/\delta L = [\alpha_L + \alpha_{KL} \ln K + \alpha_{LL} \ln L][L/Y]$. A production function is usually considered to be well-behaved only if output increases monotonically with all inputs. Monotonicity holds if $\delta Y/\delta K > 0$ and $\delta Y/\delta L > 0$.

Elasticities of substitution

From the translog production technology (Equation 2), one can compute the elasticity of substitution between inputs. Substitution elasticities are measured using Hicks' elasticity of complementarity (HEC) rather than the widely applied

Allen elasticity of substitution (AES) because of biases introduced in the latter when derived from a translog production function (Field 1988).[3] The cross HEC may be written as

$$h_{ij}=(\alpha_{ij}+M_iM_j)\ /M_iM_j \tag{3}$$

where M_i is the logarithmic marginal product of factor i, $\ln(\delta Y/\delta X_i)$. A positive (negative) value of h_{ij} indicates that inputs i and j are q-complements (q-substitutes), so that increased usage of one increases (decreases) the usage of the other. With the Cobb–Douglas form, $\alpha_{ij}=0$ and $h_{ij}=1$.

The own HEC may be written as

$$h_{ii}=(\alpha_{ii}+M_i^2-M_i)\ /M_i^2 \tag{4}$$

Own HECs should be negative. Unlike the AESs, any maintained separability restrictions are directly imposed.

The HECs or AESs do not provide direct information on the behaviour of relative factor shares, but the direct elasticity of substitution (DES) and Morishima elasticity of substitution (MES) do (Sato and Koizumi, 1973; Blackorby and Russell, 1989; Kang and Brown, 1981). Because computation of the MES from the translog technology faces the same computational bias as the AES, attention has been restricted to the DES.

The DES measures the substitution between two inputs along an isoquant with all other inputs and output held constant. Following McFadden (1963), the DES between X_i and X_j is defined as

$$d_{ij}=[\mathrm{d}(X_i/X_j)/\mathrm{d}(f_i/f_j)][(X_i/X_j)/(f_i/f_j)] \tag{5}$$

where $f_i=\delta f(X)/\delta X_i$ and i is not equal to j. The DES is a generalization of the two-factor elasticity of substitution formula applied to each pair and is symmetric. The DES lies between zero and infinity, and grows larger as the substitution becomes easier between two inputs. The DES is a two-factor, two-price elasticity of substitution. The DES is a short-run elasticity since it holds all other inputs constant, but under weak (or strong) separability (or only two inputs), the DES, a short-run measure, equals the long-run two-factor, two-price elasticity of substitution for the separable inputs (Mundlak, 1968).[4]

The DES for the translog production technology at the point of approximation may be written as (Boisvert, 1982)

$$d_{KL}=\frac{-[\alpha_K+\alpha_L]}{-[\alpha_K+\alpha_L]+\dfrac{\alpha_K^2\alpha_{LL}-2\alpha_K\alpha_L\alpha_{KL}+\alpha_L^2\alpha_{LL}}{\alpha_K\alpha_L}} \tag{8}$$

III. ESTIMATION

In comparison to the maximum likelihood estimates of the frontier model, 'Ordinary least-squares estimates are inefficient and the estimate of the constant term is inconsistent (at least within the assumptions of the frontier model)' (Greene, 1990, p. 329). The differences between the average production function and the frontier should be intercept differences (Cowing *et al.*, 1988). This can be seen from V_i in the frontier error term V_i-U_i. In the average function this would be absorbed in the constant term, shifting it neutrally downward, leaving the slope coefficients the same. However, the slope coefficients might also display statistically significant differences since the OLS estimates are inefficient (Greene, 1990).

However, as noted by Cowing *et al.* (1988), the statistical and the economic differences (e.g. elasticities, scale economies, marginal products) must be distinguished. In their own words, 'Even if frontier estimates are statistically different from the OLS estimates, it may be the case that such differences are of little economic importance. This question is of interest, for, ..., frontier estimation is considerably more difficult than least squares' (Cowing *et al.*, 1988, p. 65).

Model specification

An industry-level production technology was estimated for 47 sectors of the Bangladesh economy during 1976–77.[5] The output Y was defined as the sectoral outputs in millions of Taka. Two inputs per sector were specified: K for capital in millions of Taka and L for labour in thousands of man-years. As discussed above, the stochastic production frontier has an additive two-sided error term normally distributed and representing random disturbances. The other component represents deviations in technical efficiency. Three alternative specifications are evaluated: (1) half-normal (2) exponential and (3) the generalized form of Stevenson (1980). The production function is estimated with a single additive, normally distributed error term, which represents stochastic disturbances.

Nested hypothesis testing and final form of the technology

The general translog form of the production technology in Equation 2 might not be the final form, which in turn has important implications for the elasticities of substitution, marginal products, and measures of economies of scale.

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[3] Field (1988) notes that AESs estimated from the (primal) translog production function may not be desirable because the matrix of the estimated coefficients must be inverted to derive an AES. If one coefficient has a large standard error, all AESs are affected.

[4] Hence, the results for the separable inputs hold for both partial and full static equilibrium and any output level. In addition, the (symmetric) DES provides information on the relative shares of inputs i and j, S_i and S_j, where sign is $\delta(S_i/S_j)\ /\delta(X_i/X_j) >|=|<0$ according as $d_{ij}>|=|<1$ (Sato and Kazomi, 1973). Thus, if capital K is substituted for labour L, so that the ratio of capital to labour increases, the share of capital increases/remains constant/decreases relative to the share of labour as $d_{KL}>|=|<1$.

[5] The existence of an aggregate production technology is assumed, which in turn requires the maintained hypothesis of additive separability among firms (van Daal and Merkies, 1984).

Restrictions on the production possibilities can be tested by a series of nested econometric restrictions on the production technology, Equation 2. The translog production technology might be characterized by CRTS or even the more restrictive Cobb–Douglas form. In the latter case, the technology is globally homogeneous, strongly separable in the inputs, and with constant unitary elasticities of substitution. In contrast, the technology with the translog form is not restricted to homogeneity and the elasticities of substitution are not *a priori* restricted constant or strongly separable in the inputs.

These hypothesis tests follow a nested sequential procedure, starting with a test for constant returns to scale and proceeding to tests for the Cobb–Douglas form and Cobb–Douglas with CRTS. Each succeeding hypothesis is tested given that the previous hypothesis is maintained. The testing procedure ends whenever a hypothesis is rejected. Following Denny and Fuss (1977), the overall significance of the nested hypothesis tests is approximately the sum of the individual test's significance. A significance of 0.025 is assigned for each test, giving an overall significance of 0.075. Because the translog function is interpreted as a second-order approximation to an underlying production technology, the tests hold only at the point of approximation and approximately in the neighbourhood of the point of expansion (Denny and Fuss, 1977).

CRTS requires the following econometric restrictions on the translog production function of Equation 2 (Berndt and Christensen, 1973):

$$\alpha_K+\alpha_L=1 \quad \text{and} \quad \alpha_{KK}+\alpha_{LL}+\alpha_{KL}=0 \tag{7}$$

The restriction for the Cobb–Douglas functional form is

$$\alpha_{KL}=0 \tag{8}$$

If the null hypothesis (Equation 8) is not rejected, homogeneity, strong separability and unitary elasticities of substitution among all inputs are maintained hypotheses. The restriction for constant returns to scale with Cobb–Douglas is (given $\alpha_{KL}=0$)

$$\alpha_K+\alpha_L=1 \tag{9}$$

The data

The data used in this paper are taken from Alauddin (1986) and Alauddin and Tisdell (1988) and the 47-sector input–output table prepared by the Bangladesh Planning Commission (BPC, 1980a, b). The details of the sector classification scheme, the methodology of estimates and their limitations have been discussed elsewhere by Alauddin (1986) and Alauddin and Tisdell (1988) so we do not wish to repeat them here. Estimates of capital requirements were obtained by taking the product of the fixed capital coefficients for various industries and their corresponding output levels as reported by the Bangladesh Planning Commission (BPC 1980a, pp. 12–24). Depending on the sectors, these coefficients were estimated on three different bases, the details of which appear in BPC (1980a, p. 24). Clearly, the estimates involve conjectures and may not be entirely reliable and, therefore, they may have serious limitations for policy purposes. It is well-known that many conceptual difficulties and statistical pitfalls surround the estimation of capital stock (see, for example, Harcourt, 1973). Nevertheless, given that the primary objective of the paper is to illustrate how there can be a divergence between average and frontier production technology estimates using the same data base, the accuracy of the data estimates can be considered to be of secondary importance.

Note that the data, since they relate to the year 1976–77, are quite old. Although a 53-sector input–output table for a later year, 1981–82, is available (BPC, 1990), the employment series for that particular year is not yet available. The 1981–82 data could not be used for this analysis. Fortunately, the 1976–77 data still enable the main methodological issues to be investigated empirically.

Econometric estimation

Econometric estimation of the frontier form of the translog production technology given in Equation 2 used maximum likelihood under the behavioural assumption of the expected profit maximization for the three distributions of U_i.[6] Symmetry of the second-order coefficients follows from Young's theorem on the order of differentiation of a continuous function, and the symmetry is imposed as a maintained hypothesis in Equation 2, giving: $\alpha_{KL}=\alpha_{LK}$.

Estimation of the standard translog production function given in Equation 2 was by OLS under the hypothesis of the expected profit maximization. The R^2 was 0.855, indicating a good fit for the cross-sectional data. The Lagrange multiplier test of Breusch and Pagan (1979) for a null hypothesis of heteroscedasticity of the form Var $[\varepsilon]=h(\beta'X)$ gave a chi-square test statistic of 6.422 with five degrees of freedom, thereby not rejecting the null hypothesis at the conventional levels of significance.[7] As a consequence, White's (White,

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[6] Zellner *et al.* (1966) showed that a direct estimation of the primal production technology without this behavioural assumption gives biased and inconsistent estimates because of simultaneity between inputs and output. Under the behavioural assumption of the expected profit maximization, inputs may be assumed exogenous rather than endogenous.

[7] Interpreting the production technology as a second-order approximation, rather than as a direct representation, of technology and use of an additive error term introduces heteroscedasticity because the remainder term of the second-order Taylor series approximation is incorporated into the residuals (Kulatilaka, 1987). In addition, the use of data aggregated by arithmetic means can also introduce heteroscedasticity. Heteroscedasticity is also often found in cross-sectional data. Hence, a very general form of heteroscedasticity from a number of sources is allowed .

Table 1. *Nested hypothesis tests for final form of production technology*

	Chi-square or F-test			
	Stochastic production frontier			
Hypothesis test	Half-normal	Exponential	Generalized	Production function
CRTS translog	193.225(2)	194.340(2)	144.877(2)	1.2941 (2,41)
Cobb–Douglas functional form				4.402 (3,41)
CRTS Cobb–Douglas form				206.60 (1,44)

Note: Wald test for production frontier and F-test for production function. Level of significance for each individual test is 0.025 and overall significance is 0.075. Degrees of freedom in parenthesis.

1980) procedure was applied to obtain heteroscedastic-consistent estimates of the parameters.[8]

A direct estimation of the production technology, under the hypothesis of the expected profit maximization, was elected rather than the estimation of the production technology and its first-order conditions (less one), since the latter approach requires constant returns to scale for the cost shares to sum to unity and thereby avoid singularity of the variance–covariance matrix of residuals. Moreover, estimation of Equation 2 with its first-order conditions without scaling of the data by sample means (to maintain consistency between the production frontier and production function) places the residuals from the objective function and the first-order conditions on the same measurement scale. Finally, estimation with cross-sectional, rather than time-series, data and with the small number of explanatory variables should minimize any multicollinearity that might arise from the direct estimation rather than the systems approach of Equation 2 and its first-order condition.[9]

Final form of the production technology

The results of the nested hypothesis testing for the final form of the production technology for the production frontiers and production function are summarized in Table 1. The overall level of significance for each nested testing sequence was 0.075, while the level of significance for each test within each nested testing sequence was 0.025. The three production frontiers are considered first and then the production function. Wald tests (Godfrey, 1988) were used for the production frontiers and F-tests were used for the production function. The Wald test is distributed as chi-square with degrees of freedom equal to the number of independent restrictions.

The Wald test rejected the null hypothesis of constant returns to scale in all the specifications of the stochastic frontier. Because the testing is nested, it was terminated at this point. In sum, the unrestricted translog frontier was not rejected at an overall level of significance of 0.075 (and lower) as the final form of the frontier. The parameter estimates are reported in Table 2.

The null hypothesis of CRTS for the translog production function was not rejected at both the 0.025 and 0.01 levels of significance, since the F-statistic was $F(2,41)=1.2941$, with numerator and denominator degrees of freedom in parenthesis. The null hypothesis of Cobb–Douglas functional form for the production function was rejected at a 0.025 level of significance by an F-test with $F(3,41)=4.402$, but was marginally rejected at 0.01 significance. The null hypothesis of CRTS for the Cobb–Douglas production function was rejected. The Breusch–Pagan test for heteroscedasticity gave a chi-square test statistic with two degrees of freedom of 47.2501, decidedly rejecting the null hypothesis of no heteroscedasticity. The estimates were corrected for heteroscedasticity (heteroscedastic-consistent estimates), following White (1980).

IV. COMPARISON OF PRODUCTION TECHNOLOGY

The nested hypothesis tests for the final forms of the stochastic production frontiers and production function indicated that, at the 0.075 level of overall significance, both representations of production possibilities were translog, with the production function displaying CRTS. Moreover, there was an indication that, at a lower level of overall significance, the final form of the production function would be Cobb–Douglas.

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[8] Heteroscedasticity of the production *frontier* cannot be tested by the Breusch–Pagan test and corrected with current econometric packages. Hence, it is not known if the frontier's parameter estimates are heteroscedastic or not and the parameter estimates of the *frontier* function are not necessarily heteroscedastic-consistent.

[9] The condition index detects multicollinearity and is the square root of the ratio of the largest to the smallest characteristic root of $X'X$, where X is the vector of regressors (Belsley *et al.*, 1980). The computed value of 0.0159 indicates that multicollinearity does not pose a serious problem.

Table 2. *Parameter estimates of production technology*

	Production function			Stochastic frontier		
Variable	Cobb–Douglas	Unrestricted translog	CRTS translog	Half-normal	Exponential	Generalized likelihood
Intercept	3.793*	1.989*	2.776*	2.317*	2.147*	2.261
	(0.868)	(0.391)	(0.146)	(0.682)	(0.685)	(13.804)
Capital	0.271*	0.817*	0.868*	0.826*	0.822*	0.817*
	(0.111)	(0.195)	(0.185)	(0.260)	(0.258)	(0.300)
Labour	0.443*	0.572*	0.132*	0.559	0.565	0.572
	(0.095)	(0.219)	(0.185)	(0.325)	(0.323)	(0.373)
Capital squared		−0.027*	−0.035	−0.027	−0.027	−0.027
		(0.024)	(0.027)	(0.033)	(0.033)	(0.038)
Labour squared		0.065	0.111*	0.067*	0.066*	0.065*
		(0.030)	(0.018)	(0.018)	(0.018)	(0.020)
Capital* labour		−0.082*	−0.076	−0.082	−0.082	−0.082
		(0.036)	(0.039)	(0.047)	(0.047)	(0.054)
$\sigma^2=\sigma_U^2+\sigma_V^2$				0.394*	0.541*	0.662
				(0.123)	(0.165)	(7.365)
μ						0.0002
						(63.259)
Log-likelihood				−39.919	−39.893	−40.111

Note: Translog functional form unless otherwise stated. Standard errors in parentheses. Heteroscedastic-consistent estimates for standard production function.
*Denotes significance at 5% level.

Statistical tests for equality of coefficients

A direct comparison of parameter estimates for the translog production function and the stochastic production frontier reported in Table 2 indicates fairly close similarity between the intercepts and first-order and second-order coefficients.[10]

As discussed above, the differences between the stochastic production frontier and the production function should be intercept differences, so that the stochastic frontiers and production function merely represent neutral shifts from one another. The slope coefficients, however, might display statistically significant differences due to the inefficient estimates of OLS. Following Cowing *et al.* (1988), statistical tests of these differences were conducted. An F-test on the null hypothesis that all coefficients except the intercept are the same in the stochastic frontiers was first run against both the unrestricted and CRTS translog production functions as the null. The estimated F for the unrestricted (CRTS) translog production function as the null was: (1) $F(5,41)=0.2043$ (3.836), for the half-normal; (2) $F(5,41)=0.2040$ (3.938) for the exponential; and (3) $F(5,41)=0.5639$ (3.813) for the generalized likelihood, indicating no statistically significant difference. The null hypothesis that all intercept coefficients were the same in both models was tested by a Wald test with the stochastic frontier as the null. The chi-square statistic of the Wald test with five independent restrictions for the unrestricted (CRTS) translog production function gave a value of: (1) 23.0505 (467.402) for the half-normal; (2) 23.005 (470.848) for the exponential; and (3) 0.042598 (456.161) for the generalized likelihood, rejecting the null hypothesis for the half-normal and exponential forms but not for the generalized likelihood with the unrestricted translog production function and uniform rejection for the CRTS translog production function.

The second-order coefficients play a crucial role in flexible functional forms, giving them their 'flexibility'. Test of significance for the null hypothesis of no differences in the second-order coefficients against the unrestricted (CRTS) translog production function as the null gave: (1) $F(3,41)=0.0002$ (2.905) for the half-normal; (2) $F(3,41)=0.0003$ (2.980) for the exponential; and (3) $F(3,41)=0.0002$ (3.058) for the generalized likelihood, all implying no differences for the unrestricted translog but differences for the CRTS translog. The Wald test of no differences in the second-order coefficients against the stochastic frontier as the null gave chi-squares with three degrees of freedom for the unrestricted (CRTS) translog of: (1) 269.165 (14.903); (2) 399.00 (15.456) and (3) 292.008 (14.142), respectively, for the half-normal, the exponential and the generalized likelihood, rejecting the null hypothesis of no differences for both the unrestricted and CRTS translog production function.

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[10] We may have omitted variable bias in the estimated coefficients, since land and human capital, the omitted variables, are likely to be correlated with the included variables.

Table 3. *Hicks elasticities of complementarity*

	Production function			Stochastic frontier		
Variable	Cobb–Douglas	Unrestricted translog	CRTS translog	Half-normal	Exponential	Generalized likelihood
Capital	−2.696	−0.264	−0.196	−0.250	−0.257	−0.264
Labour	−1.258	−0.549	−0.207	−0.630	−0.563	−0.550
Capital–labour	1	0.825	0.337	0.822	0.823	0.825

Note: Calculated at point of approximation.

Table 4. *Direct elasticities of substitution*

	Production function			Stochastic frontier		
Variable	Cobb–Douglas	Unrestricted translog	CRTS translog	Half-normal	Exponential	Generalized likelihood
Capital–labour	1.000	1.207	8.078	1.294	1.211	1.207

Note: Calculated at point of approximation.

In sum, mixed information was found on the statistical differences between the coefficients of the production function and the three stochastic frontiers.

Economic differences

Since parameter estimates from second-order functional forms, whether interpreted as direct or second-order approximations of technology, have little meaning in themselves, the factor substitution possibilities, marginal products and scale economies are examined next in order to make more meaningful comparisons about the production technology and implications for economic policy formation. The elasticities were evaluated at the point of approximation.

The Hicks elasticities of complementarity (HEC) for the translog form are reported in Table 3. The HECs are negative for both the production frontiers and the production function, as expected for a well-behaved production technology. The own HECs are inelastic for both the production frontiers and translog production function, with the production function having slightly larger absolute values for own HECs. The HECs for the CRTS translog production function were more inelastic than the unrestricted translog or Cobb–Douglas forms. The HECs between capital and labour indicated inelastic q-complementarity, so that increased usage of one increases the demand or marginal product of the other. Complementarity among inputs, or 'cooperant' factors, is generally expected to prevail (Hicks, 1946; Sakai, 1974). Policy formation based upon an accurate measurement of the cross HEC should give similar results (unless the production function is interpreted as Cobb–Douglas, where $\mathrm{HEC}_{KL} = 1$).

The direct elasticities of substitution (DES) for capital and labour are reported in Table 4. The DESs for the production frontier and the production function are all around 1.207, except for the CRTS translog production function, which has a DES of 8.078. Hence, the capital–labour isoquants for the production frontiers and unrestricted translog production function are similar and are more elastic and flatter in shape than that for the Cobb–Douglas form and the CRTS translog production function's DES is considerably more elastic than any other. Moreover, if capital K is substituted for labour L, so that the ratio of capital to labour increases, the share of capital will increase relative to the share of labour for both the production frontiers and function if the latter is interpreted as translog. If the production function is Cobb–Douglas, then the income shares are constant. In sum, policy targeting income distribution and relative factor shares could provide very dissimilar results, depending upon the specification of technology.

The marginal products for the production frontiers and the production function are reported in Table 5. They are calculated for mean values of capital, labour and output. They are all positive, indicating that the monotonicity condition of a well-behaved production technology is satisfied. In this case, marginal products for the production frontiers and the production function are virtually the same. Hence, policies requiring knowledge of marginal products, such as those targeting allocative efficiency, would provide virtually identical results.

Economies of scale can be measured as the sum of the production coefficients: $\delta \ln Y/\delta \ln K + \delta \ln Y/\delta \ln L$. At the point of approximation, this measure becomes simply $\alpha_K + \alpha_L$. This value is 1.386, 1.388 and 1.389, respectively, for the half-normal, exponential and generalized production

Table 5. *Marginal products*

	Production function			Stochastic frontier		
Variable	Cobb–Douglas	Unrestricted translog	CRTS translog	Half-normal	Exponential	Generalized likelihood
Capital	0.355	1.126	1.196	1.138	1.133	1.126
Labour	0.027	0.056	0.013	0.057	0.858	0.058

Note: Calculated at point of approximation.

frontiers, is 1.389 and 1.000 for the unrestricted and CRTS translog production functions, respectively, and 0.714 for the Cobb–Douglas. Hence, economic policies targeting scale economies would be affected by the specification of the production technology.

Fitted values

The fitted values from the translog stochastic frontiers and the translog production function should match one another if the two give equivalent representations of technology. The fitted values were regressed from the unrestricted translog production function upon those of the stochastic frontier with the half-normal error term in the relationship $Y=\alpha+\beta X$ to evaluate slope intercept differences between the two series. Ideally, the intercept should be zero and the slope unity if the two data series are the same. The results of the regressions were:

	coefficient	t-ratio
α	-0.0179	-2.060
β	1.0024	723.549

with $R^2=0.999941$. The intercept α clearly differs from zero given the t-ratio of -2.060. This result is expected if the intercepts between the translog stochastic frontiers and production function are different due to the technical inefficiency term U_i in the stochastic frontier's disturbance term. The F-test on the linear restriction that the slope is unity, i.e. $\beta=1$, gave the result $F(1,45)=4.4782$, marginally rejecting the null hypothesis that $\beta=1$ at 0.05 but not at 0.025 or 0.001. In sum, there will be systematic, statistically significant differences between the two due to the disturbance term, affecting the intercepts, and possibly the slope coefficients.

Nested and non-nested tests

The relationship between the unrestricted translog stochastic frontiers and unrestricted translog production function can be tested formally. The production function can be viewed as a restricted version of the stochastic frontier through the error term $\Phi_i=V_i-U_i$. The econometric restriction is $\delta=\sigma_u/\sigma_v=1$. Hence, as inefficiency becomes insignificantly small, the frontiers' probability density functions become indistinguishable from the normal for U_i distributed half-normal, exponential or following the generalized likelihood. The estimated values of δ are: (1) $\delta=0.646$ with a standard error of 2.242 for the half-normal; (2) $\delta=0.5977$ with a standard error of 5.784 for the generalized likelihood; and (3) $\delta=6.196$ with a standard error of 20.312 for the exponential. The results indicate that there are no statistical differences between the different translog stochastic frontiers and unrestricted translog production function estimated by OLS.

The test on δ and on the equality of coefficients specifies the alternative to the null and contains the null as a special case. Hence, this is a nested specification test. When the alternative is fully specified but does not contain the null model as a special case, specification tests are known as non-nested or encompassing hypotheses (Godfrey, 1988).

These non-nested tests can be applied to the translog stochastic frontier and the unrestricted translog production function as a check that a systematic but unknown difference does not exist. The MacKinnon et al. extension (MacKinnon *et al.*, 1983) of the J-test (Davidson and MacKinnon, 1981) was applied. The t-ratio for the test against the stochastic frontier as the null was 0.606 and the log-likelihood was -39.826. The t-ratio for the test against the unrestricted translog production function as the null was -0.340 and the log-likelihood was -39.816. The results lend weight to the conclusion that there are no statistical differences between the two unrestricted translog models.

V. CONCLUDING REMARKS

It has been shown that depending on the method of hypothesis testing and the initial null hypotheses chosen, significant differences or no differences between the estimated average production function and the stochastic production frontier and various functional forms may be observed. Because of these the following procedures are suggested.

It may be preferable to first begin any analysis of production technologies with nested hypothesis testing of technical inefficiency and non-nested hypothesis testing for systematic but unknown differences between the unrestricted forms of the average and frontier production functions. Nested specification tests on the final form of the production technology would follow after first selecting between the average and stochastic frontier functions.[11] This hypothesis testing sequence starts from the most general question, that of the average or stochastic frontier representation of technology, and proceeds to specific forms for the selected representation of technology (average or frontier). While this sequence is not formally and explicitly nested, it does proceed from the most general to the most specific.[12] Alternatively, beginning hypothesis testing for the final form of the technology could give misleading results without first testing between the average and stochastic frontier functions. In turn, economic policy formation, such as that based on relative income shares, factor substitution possibilities, and technical efficiency differentials could differ.

For the Bangladesh data, nested hypothesis tests on the final form of the production technology indicated a significant difference between the average production function and stochastic production frontier; the average production function was CRTS translog and possibly even Cobb–Douglas and all three stochastic frontiers were unrestricted translog. In contrast, nested hypothesis tests of technical inefficiency and non-nested hypothesis testing for systematic but unknown differences indicated no significant differences. Certainly, for the Bangladesh data we examined, economic policy could have built inappropriately upon a translog stochastic production frontier rather than the appropriate CRTS translog or perhaps Cobb–Douglas average production function. In this case, there is little scope for policies to improve management efficiency by ensuring that all sectors adopt frontier management practices, since these practices are already being adopted and deviations are due to random factors.

The frequency of divergent results, such as those obtained on the basis of the Bangladesh data, is difficult to anticipate *a priori*. Instead, it seems to be an empirical issue, which must be examined on a case-by-case basis.

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[11] Selection of the average production function and estimation by OLS would not imply inefficient estimates in comparison to the frontier model, since that inefficiency is predicated on a distinguishable difference between the two models.

[12] The tests of technology and pooling would not be nested as a group. Hence, the statistical significance of the tests would not be the (approximate) sum of the individual tests' significances (cf. Denny and Fuss, 1977). However, Godfrey (1988) notes that when there is a high degree of dependence between the individual significance tests, the actual overall significance level may be somewhat smaller than its upper bound of $s\alpha$, where s is the number of tests and α is the significance level of each test.

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