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TWO COMPUTER PROGRAMS TO PROJECT POPULATIONS WITH TIME-VARYING VITAL RATES

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U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
National Marine Fisheries Service
Southwest Fisheries Center

NOAA Technical Memorandum NMFS

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TWO COMPUTER PROGRAMS TO PROJECT POPULATIONS WITH TIME-VARYING VITAL RATES

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TWO COMPUTER PROGRAMS TO PROJECT POPULATIONS WITH TIME-VARYING VITAL RATES

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ABSTRACT

Two computer programs which project age-structured populations using time-varying survival and fecundity rates are described and illustrated with a numerical example from human demography. Detailed instructions for the use of both programs are provided, and FORTRAN source code listings are given in appendices.

INTRODUCTION

This memorandum describes and illustrates the application of two computer programs developed to estimate future population size and age structure when survival and fecundity rates are variable. Due to our less-than-perfect ability to forecast future environmental conditions, there will be some uncertainty about future population size, even if we have estimated the vital rates accurately under present conditions. We compute the future population vectors using a projection matrix (Leslie 1945). However, the elements of the Leslie matrix are assumed to be random variables. With variable vital rates, the population vector and total population size in the future are no longer fixed quantities, but variates with some (often unknown) distributions. The programs described in this memorandum compute the means and variances for a variety of demographic parameters when vital rates vary randomly. In particular, confidence intervals on population size and the probability that the population will increase are computed. We will briefly describe the two programs, provide instructions for their operation, and give a numerical example of their application. The work was supported by NOAA Contract 80-ABC-00207.

PROGRAM DESCRIPTIONS

FORTRAN program SPP (Stochastic Population Projection) is based on a formula derived by Sykes (1969). This program uses the third of three models presented by Sykes; in this model the elements of the Leslie matrix (the effective fecundity rates and the survival rates) are random variables, each

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*Present address: National Marine Fisheries Service, NOAA, Southwest Fisheries Center, P. O. Box 271, La Jolla, CA 92038 with a specified mean and variance, and with specified covariances between all elements of the Leslie matrix. There is no autocovariance over time. Equations 19 and 20 of Sykes (1969) were used to compute the mean final population vector and the variance-covariance matrix for the final population vector, respectively.

From these quantities the mean and variance of the total population can be computed. The expected value for the total population is the sum of the expected values of each age class, while the variance of the total population is the sum of the entries in the variance-covariance matrix. The distribution of the final total population is not normal, but confidence limits can be computed from the realized factor of increase. Mathematical details of the procedure are given in Gerrodette et al. (1983).[1]

FORTRAN program SLT (Stochastic Life Table simulator) simulates the growth of an age-structured population under fluctuating environmental conditions. At each time period, a new set of fecundity and survival rates (the elements of the Leslie matrix) are chosen and used to project the population. Each fecundity and survival rate is a normally distributed random variable with a mean, a variance, and covariances with every other fecundity and survival rate specified by the user. Distributions other than normal could also be used. The projection, starting from the same initial population vector, may be replicated a given number of times. From these results, the mean, variance, and covariances of the population vector are computed, together with statistics on a variety of other demographic parameters, including the distributions of the total population and the realized factor of increase.

The two programs have been written in similar, easy-to-use formats. Each requires only an input file in which the data and control parameters are given; after that, each program runs automatically. Guides to the use of these two programs are given in Appendices 1 and 2. Source code listings are provided in Appendices 3 and 4.

NUMERICAL EXAMPLE

We illustrate the application of programs SPP and SLT by a projection of the female population of the United States in 1940, an example introduced by Keyfitz (1964) and used by Sykes (1969). Each age-class (and time step) in this example represents a span of 15 years. The mean vital rates and the covariance matrix for the vital rates over time are given in Table 1; we project the initial population vector through 7 time steps, a total of 105 years, as Sykes does. The results of program SPP presented in Table 2 differ slightly from Sykes' (1969) results. After carefully checking our results and after programming Sykes' formula in two different ways with identical answers, we believe that the differences can be ascribed to round-off errors owing to

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[1] Gerrodette, T., D. Goodman, and J. P. Barlow. 1983. Population projection when vital rates vary randomly. In prep.

differences in computational hardware. In addition, independent programming of Sykes' formula by T. Polacheck of the University of Oregon has given results identical to ours (T. Polacheck, Univ. of Oregon, Eugene, OR. pers. comm.).

In Table 2, the expected population vector approaches the stable age distribution with each time step, as follows from Sykes' Equation 19. The covariance matrix for the population vector gives, on the diagonal, the variances of each age-class and, above the diagonal, the covariances of each age-class with every other age-class. The 95% confidence interval for the total population and for the factor of increase realized relative to the initial population are shown for each time step. After 30 years, for example, we can be 95% confident that the size of the population will be between 34.733 and 60.276 million females, with a mean of 46.858 million. Because the distribution of final population size is skewed in a stochastic projection (Gerrodette et al. 1982), the mean will not, in general, lie midway between the lower and upper confidence limits. The probability that the total population will have increased over its initial value is also shown for each time step. Due to transients in the population vector, this probability decreases at first and then increases in this example.

The results of the Monte Carlo simulation of the same population projection (program SLT) are given in Table 3. By comparing the results in Table 3, Section 2, with those of time step 7 in Table 2, we see that the results of the simulation (SLT) and the analytic solution (SPP) agree closely.

As a check on the confidence limits for the total population computed by program SPP, program SLT tallies the proportion of final populations which fall above and below the computed upper and lower confidence limits. We expect that 2.5% of the cases should fall above the upper limit and 2.5% below the lower limit if the 95% confidence interval has been correctly estimated. The last part of Section 2 of Table 3 shows that 2.25% of the cases fell above the upper limit and 2.57% below the lower limit on this particular run. Other runs gave similar results. Program SLT also computes the proportion of cases in which the final population was greater than the initial population, and this answer (0.8664, Table 3) is close to the probability computed analytically by program SPP assuming that the realized factor of increase is normally distributed (0.8697, Table 2). Thus, if vital rates are variable but follow the distributions given in Table 1, we can say that the probability that the population will be larger 105 years in the future is 0.87, while the probability that it will be smaller is 0.13.

The third section of Table 3 shows that the distributions of the vital rates achieved on this run were close to the "target" rates specified in Table 1 (and repeated in the first parts of Sections 1 and 2 of Table 3). Section 4 of Table 3 shows the results of a deterministic projection using the achieved mean values of the vital rates. For reasonably large simulation runs, Section 4 should be similar to Section 1, which carries out a deterministic projection using the specified input values of vital rates.

LITERATURE CITED

Keyfitz, N. 1964. The population projection as a matrix operator. Demography 1:56-73.

Leslie, P. H. 1945. On the use of matrices in certain population mathematics. Biometrika 33:183-212.

Sykes, Z. M. 1969. Some stochastic versions of the matrix model for population dynamics. J. Amer. Stat. Assn. 64:111-130.

Table 1. Mean vital rates and covariance matrix of vital rates for the female population of the United States in 1940 (from Sykes 1969). M refers to effective fecundity, P to survival rate, and numbers to 15-year age-classes.

	Mean	Covariance Matrix					
		M1	M2	M3	P1	P2	P3
M1	0.32167	.01021	.01392	-.00052	.00111	.00188	0.00000
M2	0.68154		.02194	.00128	.00091	.00160	0.00000
M3	0.12110			.00149	-.00053	-.00084	0.00000
P1	0.98610				.00029	.00047	0.00000
P2	0.97203					.00076	0.00000
P3	0.00000						0.00000

Table 2. Output of program SPP for the projection of the 1940 U.S. female population for 7 time steps, using the vital rates given in Table 1.

```
              * * * * STOCHASTIC POPULATION PROJECTION * * * *

      GIVEN THE FOLLOWING INITIAL POPULATION AND VITAL RATES -
         AGE CLASS   POPULATION SIZE   FECUNDITY (M)   SURVIVAL RATE (P)
             1            14459.00           .32167           .98610
             2            15264.00           .68154           .97203
             3            11346.00           .12110          0.00000
                      ------------
           TOTAL =        41069.00

      WITH THE FOLLOWING COVARIANCE STRUCTURE AMONG THE VITAL RATES -
                   1         2         3         1         2         3
      M( 1)    .01021    .01392   -.00052    .00111    .00188   0.00000
      M( 2)   0.00000    .02194    .00128    .00091    .00160   0.00000
      M( 3)   0.00000   0.00000    .00149   -.00053   -.00084   0.00000
      P( 1)   0.00000   0.00000   0.00000    .00029    .00047   0.00000
      P( 2)   0.00000   0.00000   0.00000   0.00000    .00076   0.00000
      P( 3)   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000


      THE EXPECTED POPULATION VECTOR AFTER  1 T'ME INTERVALS IS -
         AGE CLASS   POPULATION SIZE
             1            16428.05
             2            14258.02
             3            14837.07
                      ------------
           TOTAL =        45523.14

      WITH THE COVARIANCE MATRIX -
                  1           2           3
         1  .13855E+08  .34595E+06  .64223E+06
         2  .00000E+01  .60628E+05  .10373E+06
         3  .00000E+01  .00000E+01  .17707E+06

      THE 95% CONFIDENCE LIMITS ARE -
          FOR THE FINAL POPULATION:     .37614E+05   TO  .53432E+05
          FOR THE FACTOR OF INCREASE:         .9159   TO       1.3010

      THE PROBABILITY THAT THE POPULATION IS INCREASING IS  .8652


      THE EXPECTED POPULATION VECTOR AFTER  2 TIME INTERVALS IS -
         AGE CLASS   POPULATION SIZE
             1            16798.59
             2            16199.71
             3            13859.22
                      ------------
           TOTAL =        46857.52
```

```
WITH THE COVARIANCE MATRIX -
          1           2           3
  1  .16188E+08 .51029E+07 .74913E+06
  2  .00000E+01 .13555E+08 .44185E+06
  3  .00000E+01 .00000E+01 .21183E+06

THE 95% CONFIDENCE LIMITS ARE -
    FOR THE FINAL POPULATION:     .34733E+05  TO  .60276E+05
    FOR THE FACTOR OF INCREASE:        .9196  TO      1.2115

THE PROBABILITY THAT THE POPULATION IS INCREASING IS  .8108



THE EXPECTED POPULATION VECTOR AFTER  3 TIME INTERVALS IS -
   AGE CLASS   POPULATION SIZE
       1           18122.70
       2           16565.09
       3           15746.60
               ------------
     TOTAL =       50434.39

WITH THE COVARIANCE MATRIX -
          1           2           3
  1  .27782E+08 .91136E+07 .11401E+08
  2  .00000E+01 .15828E+08 .50216E+07
  3  .00000E+01 .00000E+01 .13017E+08

THE 95% CONFIDENCE LIMITS ARE -
    FOR THE FINAL POPULATION:     .32172E+05  TO  .72725E+05
    FOR THE FACTOR OF INCREASE:        .9218  TO      1.2098

THE PROBABILITY THAT THE POPULATION IS INCREASING IS  .8150



THE EXPECTED POPULATION VECTOR AFTER  4 TIME INTERVALS IS -
   AGE CLASS   POPULATION SIZE
       1           19026.21
       2           17870.80
       3           16101.77
               ------------
     TOTAL =       52998.78

WITH THE COVARIANCE MATRIX -
          1           2           3
  1  .35508E+08 .16819E+08 .14749E+08
  2  .00000E+01 .27119E+08 .88809E+07
  3  .00000E+01 .00000E+01 .15175E+08

THE 95% CONFIDENCE LIMITS ARE -
    FOR THE FINAL POPULATION:     .31573E+05  TO  .80725E+05
    FOR THE FACTOR OF INCREASE:        .9364  TO      1.1841
```

```
THE PROBABILITY THAT THE POPULATION IS INCREASING IS  .8298


THE EXPECTED POPULATION VECTOR AFTER  5 TIME INTERVALS IS -
   AGE CLASS   POPULATION SIZE
       1           20249.75
       2           18761.75
       3           17370.95
               ------------
     TOTAL =       56382.45

WITH THE COVARIANCE MATRIX -
          1           2           3
  1  .48911E+08 .24923E+08 .25246E+08
  2  .00000E+01 .34643E+08 .16289E+08
  3  .00000E+01 .00000E+01 .25886E+08

THE 95% CONFIDENCE LIMITS ARE -
    FOR THE FINAL POPULATION:    .30847E+05  TO  .91465E+05
    FOR THE FACTOR OF INCREASE:        .9444  TO      1.1737

THE PROBABILITY THAT THE POPULATION IS INCREASING IS  .8436


THE EXPECTED POPULATION VECTOR AFTER  6 TIME INTERVALS IS -
   AGE CLASS   POPULATION SIZE
       1           21404.24
       2           19968.28
       3           18236.98
               ------------
     TOTAL =       59609.51

WITH THE COVARIANCE MATRIX -
          1           2           3
  1  .62527E+08 .35957E+08 .33752E+08
  2  .00000E+01 .47694E+08 .24080E+08
  3  .00000E+01 .00000E+01 .33026E+08

THE 95% CONFIDENCE LIMITS ARE -
    FOR THE FINAL POPULATION:    .30618E+05  TO  .10131E+06
    FOR THE FACTOR OF INCREASE:        .9522  TO      1.1624

THE PROBABILITY THAT THE POPULATION IS INCREASING IS  .8575


THE EXPECTED POPULATION VECTOR AFTER  7 TIME INTERVALS IS -
   AGE CLASS   POPULATION SIZE
       1           22702.78
       2           21106.72
       3           19409.77
               ------------
```

```
     TOTAL =        63219.27

WITH THE COVARIANCE MATRIX -
           1           2           3
  1  .80587E+08 .48805E+08 .46933E+08
  2  .00000E+01 .60952E+08 .34684E+08
  3  .00000E+01 .00000E+01 .45403E+08

THE 95% CONFIDENCE LIMITS ARE -
    FOR THE FINAL POPULATION:     .30432E+05  TO  .11252E+06
    FOR THE FACTOR OF INCREASE:         .9581  TO      1.1549

THE PROBABILITY THAT THE POPULATION IS INCREASING IS  .8697
```

Table 3. Output of program SLT for the projection of the 1940 U.S. female population for 7 time steps. Sample size for the simulation was 10,000 trials. The results in section 2 of this table should be compared with the "predicted" values for time step 7 in Table 2.

```
    * * * * * STOCHASTIC LIFE TABLE SIMULATION * * * * *


1.  INPUT

THE FOLLOWING VITAL PARAMETERS HAVE BEEN SPECIFIED:


 AGE     BIRTH      SURVIVAL  REPRODUCTIVE
CLASS     RATE        RATE        VALUE      DER/DM      DER/DP

  1      .32167      .98610    1.00000      .55104      .41227
  2      .68154      .97203     .74817      .51289      .05863
  3      .12110     0.00000     .11431      .47057     0.00000

ASYMPTOTIC FACTOR OF INCREASE      1.0594
NET REPLACEMENT RATE               1.1098
GENERATION TIME                    1.8147
VAR. IN AGE OF REPRODUCTION         .3601


USING THESE VITAL RATES, THE INITIAL POPULATION IS PROJECTED
FOR  7 TIME STEPS AS FOLLOWS:


     AGE       INITIAL          FINAL
    CLASS    POPULATION      POPULATION

      1       14459.00        22702.78
      2       15264.00        21106.72
      3       11346.00        19409.77

FINAL/INITIAL POPULATION           1.5393
REALIZED FACTOR OF INCREASE        1.0636


2.  STOCHASTIC POPULATION PROJECTION

THE FOLLOWING COVARIANCE MATRIX HAS BEEN SPECIFIED:
(FIRST  3 COLUMNS ARE BIRTH RATES, SECOND  3 ARE SURVIVAL RATES)

            1          2          3          4          5          6
         .01021     .01392    -.00052     .00111     .00188    0.00000
        0.00000     .02194     .00128     .00091     .00160    0.00000
        0.00000    0.00000     .00149    -.00053    -.00084    0.00000
```

```
          0.00000   0.00000   0.00000    .00029    .00047   0.00000
          0.00000   0.00000   0.00000   0.00000    .00076   0.00000
          0.00000   0.00000   0.00000   0.00000   0.00000   0.00000


USING THE MEAN VITAL RATES WITH THIS COVARIANCE STRUCTURE,
THE INITIAL POPULATION IS PROJECTED FOR  7 TIME STEPS 10000 TIMES.

     AGE       INITIAL      (MEAN) FINAL
    CLASS    POPULATION      POPULATION

      1       14459.00        22682.34
      2       15264.00        21085.40
      3       11346.00        19283.76

COVARIANCE MATRIX FOR FINAL POPULATION VECTOR

          1          2          3
     .7952E+08 .4756E+08 .4588E+08
     .0000E+01 .5922E+08 .3300E+08
     .0000E+01 .0000E+01 .4415E+08

                                          MEAN     VARIANCE

FINAL/INITIAL POPULATION                 1.5353     .25836
REALIZED FACTOR OF INCREASE              1.0561     .00251

FREQUENCY DISTRIBUTION FOR FACTOR OF INCREASE

                                                          REALIZED
         LOWER BOUND  FINAL/INITIAL     LOWER BOUND  FACTOR OF
         OF INTERVAL   POPULATION       OF INTERVAL   INCREASE

 1       < .2500           0            < .8250          0
 2         .2500          10              .8250          0
 3         .5000         262              .8500          0
 4         .7500        1064              .8750          5
 5        1.0000        1803              .9000         41
 6        1.2500        2095              .9250        130
 7        1.5000        1779              .9500        374
 8        1.7500        1296              .9750        786
 9        2.0000         791             1.0000       1329
10        2.2500         457             1.0250       1782
11        2.5000         224             1.0500       1970
12        2.7500         107             1.0750       1657
13        3.0000          63             1.1000       1098
14        3.2500          31             1.1250        543
15        3.5000          11             1.1500        199
16        3.7500           4             1.1750         71
17        4.0000           2             1.2000         13
18        4.2500           0             1.2250          2
```

```
19        4.5000        1            1.2500        0
20        4.7500        0            1.2750        0

PROPORTION OF PROJECTIONS WITH FACTOR OF INCREASE > 1.0      IS  .8664
PROPORTION OF FINAL/INITIAL POPULATION RATIOS BELOW   .7410 IS  .0257
PROPORTION OF FINAL/INITIAL POPULATION RATIOS ABOVE   2.740 IS  .0225


3.  STATISTICAL PROFILES OF DEMOGRAPHIC PARAMETERS USED IN STOCHASTIC
    POPULATION PROJECTIONS

     THESE STATISTICS ARE COMPUTED ON THE BASIS OF 70,000 VALUES
     ( 7 TIME STEPS, 10000 REPETITIONS)


 AGE     BIRTH RATE      SURVIVAL RATE  REPRODUCTIVE VALUE DER/DM DER/DP
CLASS    MEAN    VAR       MEAN    VAR        MEAN     VAR     MEAN   MEAN

  1    .32106  .01055   .98369  .00020  1.00000  .00000  .54893  .41021
  2    .68236  .02252   .96907  .00058   .74775  .00296  .51855  .06034
  3    .12231  .00145  0.00000 0.00000   .11708  .00143  .49059 0.00000

                                      MEAN     VARIANCE

ASYMPTOTIC FACTOR OF INCREASE       1.0564   .01917
NET REPLACEMENT RATE                1.1086   .06611
GENERATION TIME                     1.8250   .00627
VAR. IN AGE OF REPRODUCTION          .3558   .00060

***** WARNING:  FACTOR OF INCREASE COULD NOT BE COMPUTED   14 TIMES


THE FOLLOWING COVARIANCE STRUCTURE WAS ACHIEVED:

          1          2          3          4          5          6
       .01055     .01438    -.00054     .00092     .00165    0.00000
      0.00000     .02252     .00122     .00079     .00146    0.00000
      0.00000    0.00000     .00145    -.00041    -.00070    0.00000
      0.00000    0.00000    0.00000     .00020     .00034    0.00000
      0.00000    0.00000    0.00000    0.00000     .00058    0.00000
      0.00000    0.00000    0.00000    0.00000    0.00000    0.00000


4.  DETERMINISTIC POPULATION PROJECTION


THE ACHIEVED MEAN VITAL RATES PRODUCE THE FOLLOWING ASSOCIATED
PARAMETERS:


 AGE     BIRTH      SURVIVAL   REPRODUCTIVE
CLASS     RATE        RATE         VALUE       DER/DM       DER/DP
```

```
  1     .32106     .98369   1.00000     .55078     .41314
  2     .68236     .96907    .75009     .51165     .05910
  3     .12231    0.00000    .11550     .46823    0.00000

ASYMPTOTIC FACTOR OF INCREASE      1.0589
NET REPLACEMENT RATE               1.1089
GENERATION TIME                    1.8156
VAR. IN AGE OF REPRODUCTION         .3607


USING THESE ACHIEVED MEAN VITAL RATES, THE INITIAL POPULATION
IS PROJECTED FOR  7 TIME STEPS AS FOLLOWS:


    AGE      INITIAL        FINAL
   CLASS   POPULATION    POPULATION

     1      14459.00      22647.48
     2      15264.00      21013.83
     3      11346.00      19275.70

FINAL/INITIAL POPULATION           1.5325
REALIZED FACTOR OF INCREASE        1.0629
```

APPENDIX 1. User's guide to program SPP.

A. General

The program was written in Microsoft F80 FORTRAN. This FORTRAN is compatible with almost any machine except for machine specificity of the I/O (READ, WRITE, and OPEN statements). The random number generator (subroutine RANDM) may also be machine-specific.

B. Input

The input file, called 'INPUT.SPP', contains the control parameters and data in the following order and format:

Record 1:	N	The number of age classes in the population to be projected. As currently dimensioned, the program handles up to 25 age classes (I2)
Record 2:	NT	The number of time steps the population is to be projected (I2)
Record 3:	ICONF	The percentage confidence limits desired; (1.0-alpha error) x 100; thus if 95 is specified here, the 95% confidence limits will be computed (I2)
Record 4:	IOP1	An option (implemented by IOP1>0) which allows the initial population vector to be the stable age vector; otherwise (IOP1=0) the initial population vector is specified in the input file - see Record 10 below (I1)
Record 5:	IOP2	An option (implemented by IOP2>0) which allows a simplified covariance structure to be specified; otherwise (IOP2=0) covariances between each fecundity and survival rate must be specified in the input file - see Records 11-14 below (I1)
Record 6:	IOP3	An output option (implemented by IOP3>0) which lists the input data (Records 8-11); otherwise (IOP3=0) only the results are output (I1)
Record 7:	IOP4	An output option (implemented by IOP4>0) which displays the results of the computations after each time step; otherwise (IOP4=0) only the final results after NT time steps are shown (I1)
Record 8:	EMX	Effective fecundity rate vector; the first row of the Leslie matrix (NF10.6, where N is Record 1)
Record 9:	PX	Survival rate vector; the subdiagonal of the Leslie matrix (NF10.6, where N is Record 1)
Record 10:	X	If IOP1=0, the initial population vector (NF10.2); if IOP1>0, the total initial population (F10.2)
Record 11:	COVAR	If IOP2=0, the elements of the covariance matrix of the fecundity and survival rates. COVAR is visualized as an upper triangular matrix, but entered as a vector by reading from left to right across each row, thus:

$$\begin{array}{c|cccc} & M1 & M2 & P1 & P2 \\ \hline M1 & 1 & 2 & 3 & 4 \\ M2 & & 5 & 6 & 7 \\ P1 & & & 8 & 9 \\ P2 & & & & 10 \end{array}$$

(N(2N+1)F10.6)

If IOP2>0, COVAR contains the variances of the fecundity and survival rates (elements 1,5,8,10 in the example above) (2NF10.6)

(Records 12-14 are read only if IOP2>0)
Record 12: C1 — The correlation among all fecundity rates (F10.6)
Record 13: C2 — The correlation between all fecundity and all survival rates (F10.6)
Record 14: C3 — The correlation among all survival rates (F10.6)

The following diagram will clarify the use of IOP2, the simplified covariance option:

```
     M1   M2 . . . Mn   P1   P2 . . . Pn
M1   V1               .
M2        V2     C1   .
:              .      .       C2
:                 .   .
Mn                 Vn ................
P1                      Vn+1
P2                           Vn+2     C3
:                                 .
:                                     .
Pn                                     V2n
```

where the V's are the variances specified in Record 11.

C. Execution

Once the input file is specified, the program is executed by the command SPP. The input file and program SPP must be on the 'current disk drive.'

D. Output

The output is written to the console screen and to a diskfile called 'OUTPUT.SPP'.

E. Other comments

Caution should be exercised in specifying the variances and covariances of the vital rates. Large variances may cause the vital rates to assume unreasonable values (e.g., a survival rate greater than 1.0 or a negative fecundity rate). The program does not detect this. Variances must be positive, while covariances may be either negative or positive. Zeros are permissible.

APPENDIX 2. User's guide to program SLT.

A. General

The program was written in Microsoft F80 FORTRAN. This FORTRAN is compatible with almost any machine except for machine specificity of the I/O (READ, WRITE, and OPEN statements). The random number generator (subroutine RANDM) may also be machine-specific.

B. Input

The input file, called 'INPUT.SLT', contains the control parameters and data in the following order and format:

Record 1:	N	The number of age classes in the population to be projected. As currently dimensioned, the program handles up to 20 age classes (I2)
Record 2:	NT	The number of time steps the population is to be projected (I2)
Record 3:	NR	The number of times the simulation is to be repeated; the "sample size" of the stochastic simulation (I5)
Record 4:	IOP1	An option (implemented by IOP1>0) which allows the initial population vector to be the stable age vector; otherwise (IOP1=0) the initial population vector is specified in the input file - see Record 17 below (I1)
Record 5:	IOP2	An option (implemented by IOP2>0) which allows a simplified covariance structure to be specified; otherwise (IOP2=0) covariances between each fecundity and survival rate must be specified in the input file - see Records 18-21 below (I1)
Record 6:	LUN	The logical unit number to which the output is directed during execution (e.g., console), other than 1 or 6, the input and output LUNs (I1)
Record 7:	ISEED	The seed for the random number generator; a positive integer in the range 1-32767 (I5)
Record 8:	KK	The number of intervals in the frequency distribution to be computed (I2)
Record 9:	WIDTH1	The width of the interval for the frequency distribution of the factor of increase (F10.6)
Record 10:	BL1	The lower bound of the frequency distribution of the factor of increase (F10.6)
Record 11:	WIDTH2	The width of the interval for the frequency distribution of the final/initial population ratio (F10.6)
Record 12:	BL2	The lower bound of the frequency distribution of the final/initial population ratio (F10.6)
Record 13:	RL	Estimated lower final/initial population ratio (F10.6)
Record 14:	RU	Estimated upper final/initial population ratio (F10.6)

Record 15: BX	Effective fecundity rate vector; the first row of the Leslie matrix (NF10.6, where N is Record 1)
Record 16: PX	Survival rate vector; the subdiagonal of the Leslie matrix (NF10.6, where N is Record 1)
Record 17: RNX	If IOP1=0, the initial population vector (NF10.2); if IOP1>0, the total initial population (F10.2)
Record 18: S	If IOP2=0, the elements of the covariance matrix of the fecundity and survival rates. S is visualized as an upper triangular matrix, but entered as a vector by reading from left to right across each row, thus:

$$\begin{array}{c|cccc}
 & M1 & M2 & P1 & P2 \\
M1 & 1 & 2 & 3 & 4 \\
M2 & & 5 & 6 & 7 \\
P1 & & & 8 & 9 \\
P2 & & & & 10
\end{array} \qquad (N(2N+1)F10.6)$$

If IOP2>0, S contains the variances of the fecundity and survival rates (elements 1,5,8,10 in the example above) (2NF10.6)

(Records 19-21 are read only if IOP2>0)

Record 19: C1	The correlation among all fecundity rates (F10.6)
Record 20: C2	The correlation between all fecundity and all survival rates (F10.6)
Record 21: C3	The correlation among all survival rates (F10.6)

The following diagram will clarify the use of IOP2, the simplified covariance option:

$$\begin{array}{c|cccccccc}
 & M1 & M2 & \ldots & Mn & P1 & P2 & \ldots & Pn \\
M1 & V1 & & & & . & & & \\
M2 & & V2 & & C1 & . & & & \\
: & & & . & & . & & C2 & \\
: & & & & . & . & & & \\
Mn & & & & Vn & \ldots & \ldots & \ldots & \ldots \\
P1 & & & & & Vn+1 & & & \\
P2 & & & & & & Vn+2 & & C3 \\
: & & & & & & & . & \\
: & & & & & & & & . \\
Pn & & & & & & & & V2n
\end{array}$$

where the V's are the variances specified in Record 18.

C. Execution

Once the input file is specified, the program is executed by the command SLT. The input file and program SLT must be on the 'current disk drive.'

D. Output

The output is written to the device specified by LUN and to a diskfile called 'OUTPUT.SLT'. The output is in 4 parts. First, the vital rates specified as input are listed, together with associated demographic parameters such as reproductive value and asymptotic factor of increase. The population structure which results from projecting with these vital rates is shown.

Second, the results of the stochastic population projections are shown. The results are summarized in terms of the mean and variance for each parameter. The frequency distributions of the ratio of the final to the initial population and of the realized factor of increased are given. Also shown are the proportions of the projections which fall below and above the estimated confidence limits given on input, and the proportion of the final population ratios which are larger than the initial population.

Third, a statistical summary of the vital rates and associated parameters achieved in the projections is displayed. These statistics indicate how closely the simulation was able to match the specified vital rates and their covariance structure. If all has gone well, the values in this section should closely match the values in Section 1 of the output. Because the vital rates are randomly generated, it is possible that the algorithm to compute the asymptotic factor of increase will not converge. A message will appear if this has happened.

Fourth, a deterministic population projection is carried out using the mean values actually obtained in the stochastic projection These results allow a direct comparison with the results in Section 2.

E. Other comments

In specifying the vital rates, variances must be positive, while covariances may be either negative or positive. Zeros are permissible. Unless these variances and covariances are carefully specified, the vital rates may assume unrealistic values such as negative survival rates. The program truncates the realized distribution so that the survival rates lie between 0 and 1 (inclusive) and the fecundities are equal to or greater than 0. This truncation may cause the achieved means to deviate systematically from the specified values.

APPENDIX 3. Listing of program SPP.

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      PROGRAM SPP          - - STOCHASTIC POPULATION PROJECTION
C
C...THIS PROGRAM ESTIMATES THE EXPECTED POPULATION VECTOR AND ASSOCIATED
C     STATISTICS AFTER ITERATIVE APPLICATIONS OF A STOCHASTIC LESLIE MATRIX.
C     THE VARIANCE-COVARIANCE STRUCTURE OF THE EXPECTED POPULATION VECTOR IS
C     BASED ON THE SYKES ALGORITHM (1969,MODEL 3). THE CONFIDENCE LIMITS FOR
C     THE FACTOR OF INCREASE AND THE PROBABILITY THAT THE POPULATION IS
C     INCREASING ARE COMPUTED BY A FORMULA DEVELOPED FOR THIS PROGRAM.
C...INPUT FILE:
C     RECORD 1:  N=NUMBER OF AGE CLASSES (I2)
C     RECORD 2:  NT=NUMBER OF TIME STEPS TO BE PROJECTED (I2)
C     RECORD 3:  ICONF=CONFIDENCE LIMITS DESIRED (I2)
C     RECORD 4:  IOP1=OPTION FOR INITIAL STABLE AGE DISTRIBUTION (I1)
C     RECORD 5:  IOP2=OPTION FOR SIMPLIFIED COVARIANCE STRUCTURE (I1)
C     RECORD 6:  IOP3=OPTION FOR LISTING OF INPUT DATA (I1)
C     RECORD 7:  IOP4=OPTION FOR OUTPUT AFTER EACH TIME STEP (I1)
C     RECORD 8:  EMX=VECTOR OF FECUNDITIES        (NF10.6)
C     RECORD 9:  PX=VECTOR OF SURVIVAL RATES      (NF10.6)
C     RECORD 10: X=INITIAL POPULATION VECTOR      (NF10.2)
C     RECORD 11: COVAR=COVARIANCE MATRIX FOR VITAL RATES. COVAR IS VISUALIZED
C                 AS AN UPPER TRIANGULAR MATRIX, BUT ENTERED AS A VECTOR BY
C                 READING FROM LEFT TO RIGHT ACROSS EACH ROW, E.G.,
C                             M1   M2   P1   P2
C                       M1     1    2    3    4
C                       M2          5    6    7
C                       P1               8    9          (N(2N+1)F10.6)
C                       P2                   10
C                 IF THE SIMPLIFIED COVARIANCE OPTION IS USED, COVAR IS
C                 ENTERED IN A DIFFERENT FASHION. REFER TO FILE SPPGUIDE.
C...OUTPUT IS WRITTEN TO THE CRT (LUN=1) AND TO DISKFILE 'OUTPUT.SPP'.
C...REQUIRED SUBROUTINES: SYKES,LOAD,PTOL,MMNT,ERNWT,STABL,LMPHI,PRJCT,DS2,
C                         DU,CLPOP,MOMENT,PMPY,PSUB,PILD,PQSD,NRINV,CUMPR.
      DIMENSION X(25),EMX(25),PX(25),V(25,25),COVAR(1275)
      DIMENSION CC(100),W(100),C(325)
C
C...STEP ONE - - INPUT PARAMETERS AND COMPUTE STABLE AGE DISTRIBUTION
      CALL OPEN (10,'INPUT   SPP',0)
      CALL OPEN (2,'OUTPUT  SPP',0)
      CALL LOAD (N,NT,ICONF,IOP1,IOP2,IOP3,IOP4,EMX,PX,X,COVAR)
      N2=N*2
      IF (IOP1) 25,25,10
 10   POP=X(1)
      CALL PTOL (PX,X,N)
      CALL LMPHI (X,EMX,W,N)
      CALL MMNT (W,RZ,GT,S2,N)
      CALL ERNWT (W,RZ,GT,S2,ER,N)
      CALL STABL (ER,X,X,N)
      DO 15 I=1,N
 15   X(I)=X(I)*POP
C
C...STEP TWO - - INITIALIZE PARAMETERS
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 25   ENO=0.0
      DO 30 I=1,N
      ENO=ENO+X(I)
      DO 30 J=1,N
 30   V(I,J)=0.0
C
C...STEP THREE - - WRITE HEADING AND ECHO INPUT DATA
      IF (IOP3) 70,70,35
 35   WRITE (2,200)
      DO 40 I=1,N
 40   WRITE (2,201) I,X(I),EMX(I),PX(I)
      WRITE (2,205) ENO
      IF (PX(N)) 45,45,42
 42   WRITE (2,202)
 45   WRITE (2,203) (J,J=1,N),(J,J=1,N)
      JSTP=0
      DO 60 I=1,N2
      JSTRT=JSTP+1
      JSTP=JSTRT+N2-I
      W(I)=0.0
      K=I-1
      IF (K) 50,50,55
 50   WRITE (2,213) I,(COVAR(J),J=JSTRT,JSTP)
      GO TO 60
 55   IF (I-N) 56,56,57
 56   WRITE (2,213) I,(W(J),J=1,K),(COVAR(J),J=JSTRT,JSTP)
      GO TO 60
 57   IN=I-N
      WRITE (2,214) IN,(W(J),J=1,K),(COVAR(J),J=JSTRT,JSTP)
 60   CONTINUE
C
C...STEP FOUR - - PROJECT COVARIANCES USING THE SYKES ALGORITHM
 70   DO 180 IT=1,NT
      CALL SYKES (N,X,V,25,EMX,PX,COVAR,C,W)
C
C...STEP FIVE - - PROJECT THE MEAN POPULATION VECTOR
      CALL PRJCT (X,PX,EMX,N)
      IF (IOP4) 145,145,150
 145  IF (IT-NT) 180,150,150
C
C...STEP SIX - - COMPUTE CONFIDENCE LIMITS
 150  CALL CLPOP (X,V,25,N,IT,ICONF,ENO,ENT,W,C,CC,CLL,CLU,POPL,POPU,PR)
C
C...STEP SEVEN - - OUTPUT RESULTS
      WRITE (1,207) IT
      WRITE (2,207) IT
      DO 165 I=1,N
      WRITE (1,201) I,X(I)
 165  WRITE (2,201) I,X(I)
      WRITE (1,205) ENT
      WRITE (2,205) ENT
      WRITE (1,210) (J,J=1,N)
      WRITE (2,210) (J,J=1,N)
      DO 170 I=1,N
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      W(I)=0.0
      K=I-1
      IF (K) 168,168,169
  168 WRITE (1,211) I,(V(I,J),J=1,N)
      WRITE (2,211) I,(V(I,J),J=1,N)
      GO TO 170
  169 WRITE (1,211) I,(W(J),J=1,K),(V(I,J),J=I,N)
      WRITE (2,211) I,(W(J),J=1,K),(V(I,J),J=I,N)
  170 CONTINUE
      IF (PR) 174,175,175
  174 WRITE (1,208)
      WRITE (2,208)
      GO TO 180
  175 WRITE (1,215) ICONF,POPL,POPU,CLL,CLU,PR
      WRITE (2,215) ICONF,POPL,POPU,CLL,CLU,PR
  180 CONTINUE
C
C...OUTPUT FORMATS
  200 FORMAT (10X,'* * * * STOCHASTIC POPULATION PROJECTION * * * *'//
     #1X,'GIVEN THE FOLLOWING INITIAL POPULATION AND VITAL RATES -'/
     #4X,'AGE CLASS',3X,'POPULATION SIZE',3X,'FECUNDITY (M)',3X,'SURVIVA
     #L RATE (P)')
  201 FORMAT (I9,F19.2,2F16.5)
  202 FORMAT (/5X,'(YOU HAVE SPECIFIED A NON-ZERO SURVIVAL RATE FOR',
     #' THE LAST AGE CLASS.'/6X,'IF IT IS TRULY THE LAST AGE CLASS, NO',
     #' SURVIVORS ARE PERMITTED.)')
  203 FORMAT (/1X,'WITH THE FOLLOWING COVARIANCE STRUCTURE AMONG THE',
     #' VITAL RATES -'/5(4X,10I10/))
  205 FORMAT (16X,12('-')/6X,'TOTAL =',F15.2)
  207 FORMAT (//1X,'THE EXPECTED POPULATION VECTOR AFTER',I3,' TIME',
     #' INTERVALS IS -'/4X,'AGE CLASS',3X,'POPULATION SIZE')
  208 FORMAT (/1X,'YOUR SPECIFIED COVARIANCE STRUCTURE AMONG THE VITAL',
     #' RATES HAS LED TO '/1X,'AN ANAMOLOUS RESULT FOR THE EXPECTED',
     #' VARIANCE IN THE FACTOR OF INCREASE.'/1X,'FURTHER STATISTICS',
     #' CANNOT BE COMPUTED.'/)
  210 FORMAT (/1X,'WITH THE COVARIANCE MATRIX -'/3(1X,10I11/))
  211 FORMAT (I4,1X,3(10E11.5/5X))
  213 FORMAT (' M(',I2,')',5(10F10.5/6X))
  214 FORMAT (' P(',I2,')',5(10F10.5/6X))
  215 FORMAT (/1X,'THE 'I2'% CONFIDENCE LIMITS ARE -'
     #/5X,'FOR THE FINAL POPULATION:',E14.5,'  TO',E12.5
     #/5X,'FOR THE FACTOR OF INCREASE:',F12.4,'  TO',F12.4
     #//1X,'THE PROBABILITY THAT THE POPULATION IS INCREASING IS',F7.4/)
      CALL EXIT
      END

C*****SUBROUTINES FOR PROGRAM SPP
C
      SUBROUTINE SYKES (N,X,V,NR,EMX,PX,COVAR,C,W)
C.....RETURNS COVARIANCE MATRIX V OF AN N-ORDER POPULATION VECTOR X
C     WHOSE DYNAMICS ARE GOVERNED BY THE FECUNDITY VECTOR EMX AND THE
C     SURVIVAL RATE VECTOR PX WITH COVARIANCES AMONG VITAL RATES
C     SPECIFIED IN COVAR. NR IS THE ROW DIMENSION OF V IN THE CALLING
C     PROGRAM; W AND C ARE WORK ARRAYS WITH DIMENSIONS AT LEAST N AND
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C.....N(N+1)/2, RESPECTIVELY. THE DIMENSION OF COVAR IS N(2N+1).
C.....REFERENCE: Z.M.SYKES (1969), J.AM.STAT.ASSN. 64:111-130, EQ. 20.
      DIMENSION V(NR,1),X(1),EMX(1),PX(1),COVAR(1),C(1),W(1)
      SCRPT(I,J,N)=(I-1)*N-((I-3)*I)/2-I+J
      N2=N*2
      SUM1=0.0
      DO 90 I=1,N
      SUM2=0.0
      DO 80 J=1,N
      IF (I-J) 50,50,55
 50   IJ=SCRPT(I,J,N2)
      GO TO 60
 55   IJ=SCRPT(J,I,N2)
 60   SUM1=SUM1+COVAR(IJ)*(V(I,J)+X(I)*X(J))
      IF (I-1) 80,80,65
 65   IJ=SCRPT(J,I+N-1,N2)
      SUM2=SUM2+COVAR(IJ)*(V(I-1,J)+X(I-1)*X(J))
      IF (J-1) 80,80,70
 70   IF (I-J) 75,75,80
 75   IJ=SCRPT(I-1+N,J-1+N,N2)
      K=SCRPT(I,J,N)
      C(K)=COVAR(IJ)*(V(I-1,J-1)+X(I-1)*X(J-1))
 80   CONTINUE
      C(I)=SUM2
 90   CONTINUE
      C(1)=SUM1
      DO 110 J=1,N
      DO 100 I=1,N
 100  W(I)=V(I,J)
      CALL PRJCT (W,PX,EMX,N)
      DO 110 I=1,N
 110  V(I,J)=W(I)
      DO 130 J=1,N
      DO 120 I=1,N
 120  W(I)=V(J,I)
      CALL PRJCT (W,PX,EMX,N)
      DO 130 I=1,N
 130  V(J,I)=W(I)
      DO 140 I=1,N
      DO 140 J=I,N
      IJ=SCRPT(I,J,N)
      V(I,J)=V(I,J)+C(IJ)
 140  V(J,I)=V(I,J)
      RETURN
      END

      SUBROUTINE LOAD (N,NT,ICONF,IOP1,IOP2,IOP3,IOP4,EMX,PX,X,COVAR)
C.....READS INPUT DATA FOR PROGRAM SPP, INCLUDING REPACKING OF COVARIANCE
C.....MATRIX GIVEN SIMPLIFIED COVARIANCE STRUCTURE.
      DIMENSION X(1),EMX(1),PX(1),COVAR(1)
   1  FORMAT (I1)
   2  FORMAT (I2)
   3  FORMAT (8F10.2)
   4  FORMAT (8F10.6)
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      READ (10,2) N
      N2=N*2
      NCV=N*(N2+1)
      READ (10,2) NT
      READ (10,2) ICONF
      READ (10,1) IOP1
      READ (10,1) IOP2
      READ (10,1) IOP3
      READ (10,1) IOP4
      READ (10,4) (EMX(I),I=1,N)
      READ (10,4) (PX(I),I=1,N)
      IF (IOP1) 15,15,10
 10   READ (10,3) X(1)
      GO TO 20
 15   READ (10,3) (X(I),I=1,N)
C.....COVARIANCE VECTOR
 20   IF (IOP2) 25,25,30
 25   READ (10,4) (COVAR(I),I=1,NCV)
      GO TO 2200
 30   READ (10,4) (COVAR(I),I=1,N2)
      READ (10,4) C1
      READ (10,4) C2
      READ (10,4) C3
      IF (ABS(C1)-1.) 40,40,90
 40   IF (ABS(C2)-1.) 50,50,90
 50   IF (ABS(C3)-1.) 60,60,90
 60   DO 70 I=1,N2
      IF (COVAR(I)) 80,70,70
 70   CONTINUE
      GO TO 1000
 80   WRITE (1,89)
 89   FORMAT (/' ERROR -- VARIANCES IN COVARIANCE MATRIX CANNOT'/
     #9X'HAVE NEGATIVE VALUES ')
      GO TO 2200
 90   WRITE (1,99)
 99   FORMAT (/' ERROR -- ELEMENTS OF CORRELATION MATRIX CANNOT'/
     #9X'HAVE ABSOLUTE VALUE GREATER THAN 1.0')
      GO TO 2200
C.....REPACK
 1000 MID=NCV-N*(N+1)/2
      NDX=NCV+1
      K=NCV
      I=N2
      M=1
      DO 1200 NNN=1,NCV
      NDX=NDX-1
      IF (NDX-K) 1100,1160,1100
 1100 IF (NDX-MID) 1110,1110,1120
 1110 IF (NDX-L) 1150,1140,1140
 1120 COVAR(NDX)=C3*SQRT(COVAR(I)*COVAR(J))
      GO TO 1200
 1140 COVAR(NDX)=C2*SQRT(COVAR(I)*COVAR(J))
      GO TO 1200
 1150 COVAR(NDX)=C1*SQRT(COVAR(I)*COVAR(J))
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      GO TO 1200
 1160 COVAR(NDX)=COVAR(I)
      L=K-N
      M=M+1
      K=K-M
      I=I-1
      J=N2+1
 1200 J=J-1
C.....ZERO-MEAN CASES
 2000 DO 2100 I=1,N
      IF (EMX(I)-.0000001) 2030,2030,2010
 2010 IF (PX(I)-.0000001) 2020,2020,2100
 2020 IR=I+N
      GO TO 2040
 2030 IR=I
 2040 DO 2070 J=1,N2
      IF (J-IR) 2050,2050,2060
 2050 IJ=(J-1)*N2-((J-3)*J)/2-J+IR
      GO TO 2070
 2060 IJ=(IR-1)*N2-((IR-3)*IR)/2-IR+J
 2070 COVAR(IJ)=0.0
      IF (IR-I-N) 2010,2100,2100
 2100 CONTINUE
 2200 RETURN
      END

      SUBROUTINE PRJCT(ENX,PX,EMX,N)
C*****ADVANCES A POPULATION VECTOR ONE TIME UNIT FORWARD BY APPLICATION
C     OF A LESLIE MATRIX DEFINED BY N-ORDER VECTORS OF FECUNDITY AND
C.....SURVIVAL RATES.
      DIMENSION ENX(1),PX(1),EMX(1)
      SUM=0.0
      IX=N+1
      IXM1=N
      DO 1050 I=2,N
      IX=IX-1
      IXM1=IXM1-1
      SUM=SUM+ENX(IX)*EMX(IX)
      ENX(IX)=ENX(IXM1)*PX(IXM1)
 1050 CONTINUE
      ENX(1)=SUM+ENX(1)*EMX(1)
      RETURN
      END

      SUBROUTINE PTOL(PX,ELX,N)
C*****CONVERTS AN N-ORDER VECTOR OF SURVIVAL RATES TO SURVIVORSHIPS.
      DIMENSION PX(1),ELX(1)
      ELX(1)=1.0
      ELAST=1.0
      IM1=0
      DO 1050 I=2,N
      IM1=IM1+1
      ELAST=ELAST*PX(IM1)
      ELX(I)=ELAST
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```
 1050 CONTINUE
      RETURN
      END

      SUBROUTINE LMPHI(ELX,EMX,PHIX,N)
C*****CONVERTS AN N-ORDER VECTOR OF SURVIVORSHIPS AND A VECTOR OF
C.....FECUNDITIES TO A NET MATERNITY FUNCTION.
      DIMENSION ELX(1),EMX(1),PHIX(1)
      DO 1050 I=1,N
      PHIX(I)=ELX(I)*EMX(I)
 1050 CONTINUE
      RETURN
      END

      SUBROUTINE MMNT(PHIX,RZ,T,S2,N)
C*****CALCULATES THE NET REPLACEMENT RATE, THE MEAN T, AND THE VARIANCE S2
C     IN AGES OF REPRODUCTION FOR A COHORT.
      DIMENSION PHIX(1)
      RZ=0.0
      X1PHI=0.0
      X2PHI=0.0
      DO 1000 I=1,N
      PHI=PHIX(I)
      FI=FLOAT(I)
      RZ=RZ+PHI
      X1PHI=X1PHI+PHI*FI
      X2PHI=X2PHI+PHI*FI*FI
 1000 CONTINUE
      T=X1PHI/RZ
      S2=X2PHI/RZ-T*T
      RETURN
      END

      SUBROUTINE ERNWT (PHIX,RZ,T,S2,ER,N)
C*****CALCULATES THE ASYMPTOTIC FACTOR OF INCREASE (E RAISED TO THE POWER
C     OF THE MALTHUSIAN PARAMETER) BY NEWTON'S METHOD.  THE INITIAL VALUE
C.....IS CALCULATED FROM INPUT MOMENTS OF THE NET MATERNITY FUNCTION.
      DIMENSION PHIX(1)
      ER=EXP((T/S2)*(RZ**(S2/T**2)-1.0))
      CT=0.0001
      DO 2000 J=1,20
      ERX=1.0
      FR=-1.0
      DFDR=0.0
      DO 1050 I=1,N
      ERX=ERX*ER
      TRM=PHIX(I)/ERX
      FR=FR+TRM
      DFDR=DFDR-FLOAT(I)*TRM
 1050 CONTINUE
      DELT=FR/DFDR
      ABFR=ABS(FR)
      IF(ABFR-CT) 1060,1060,1090
 1060 ABDLT=-ABFR/DFDR
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      IF (ABDLT-CT) 2050,2050,1090
 1090 ER=ER-DELT
 2000 CONTINUE
      WRITE (2,9000)
 9000 FORMAT (/5X,'*****  WARNING: NO CONVERGENCE IN SUBROUTINE ERNWT'/)
      ER=-100.0
 2050 ER=ER-DELT
      RETURN
      END

      SUBROUTINE STABL (ER,ELX,CX,N)
C*****CALCULATES THE STABLE AGE DISTRIBUTION CX, GIVEN THE ASYMPTOTIC
C*****FACTOR OF INCREASE AND THE SURVIVORSHIP VECTOR OF ORDER N.
      DIMENSION ELX(1),CX(1)
      SUM=0.0
      EMRX=1.0
      EMR=1.0/ER
      DO 1000 I=1,N
      EMRX=EMRX*EMR
      CX(I)=ELX(I)*EMRX
 1000 SUM=SUM+CX(I)
      DO 1100 I=1,N
 1100 CX(I)=CX(I)/SUM
      RETURN
      END

      SUBROUTINE MOMENT (N,C,NTERMS)
C.....COMPUTES THE VECTOR OF COEFFICIENTS C OF THE N-TH MOMENT OF A
C.....NORMALLY DISTRIBUTED VARIABLE (THE N-TH DERIVATIVE OF THE MOMENT
C.....GENERATING FUNCTION).  IF U AND S2 ARE THE MEAN AND VARIANCE OF
C.....THE NORMAL VARIABLE, VECTOR C WILL CONTAIN COEFFICIENTS OF A
C.....POLYNOMIAL IN U WITH EXPONENTS N,N-2,N-4,... AND IN S2 WITH
C.....EXPONENTS 0,1,2,3,... AND WHERE NTERMS IS THE NUMBER OF TERMS
C.....IN THE POLYNOMIAL (DIMENSION OF C).
      DIMENSION C(1)
      NTERMS=IFIX (FLOAT(N)/2.0+1.001)
      IF (NTERMS-1) 600,600,100
  100 DO 200 K=2,NTERMS
  200 C(K)=0.0
      DO 500 NN=2,N
      SAVE1=1.0
      DO 400 K=2,NTERMS
      L=NN+3-K*2
      IF (L) 500,500,300
  300 IF (SAVE1-.1E+30) 350,500,500
  350 SAVE2=C(K)
      C(K)=SAVE2+SAVE1*FLOAT(L)
  400 SAVE1=SAVE2
  500 CONTINUE
  600 C(1)=1.0
      RETURN
      END

      SUBROUTINE CLPOP (X,V,NR,N,IT,ICONF,ENO,ENT,W,C,CC,CLL,CLU,POPL,
```

```
     #     POPU,PR)
C.....RETURNS UPPER AND LOWER CONFIDENCE LIMITS CLU AND CLL ON THE
C     FACTOR OF INCREASE AND POPU AND POPL ON THE TOTAL POPULATION
C     ENT FOR PROGRAM SPP.  THE PROBABILITY PR THAT THE FACTOR OF
C     INCREASE IS GREATER THAN 1.0 IS ALSO COMPUTED.
C.....REQUIRED SUBROUTINES: MOMENT,DU,DS2,PMPY,PSUB,PILD,PQSD,CUMPR,NRINV
      DIMENSION X(1),V(NR,1),W(1),C(1),CC(1)
C.....COMPUTE TOTAL POPULATION EXPECTATION AND VARIANCE
      ENT=0.0
      VAR=0.0
      DO 1000 I=1,N
      ENT=ENT+X(I)
      DO 1000 J=1,N
 1000 VAR=VAR+V(I,J)
      IF (VAR) 1020,1020,1040
 1020 PR=-100.0
      RETURN
 1040 VAR=VAR/(ENO*ENO)
      FPR=ENT/ENO
      IF (FPR) 1020,1020,1060
C.....COMPUTE COEFFICIENTS OF MOMENT POLYNOMIALS
 1060 IT2=IT*2
      CALL MOMENT (IT,C,IDIM1)
      CALL PMPY (W,IDIMW,C,IDIM1,C,IDIM1)
      CALL MOMENT (IT2,CC,IDIM2)
      CALL PSUB (CC,IDIM2,CC,IDIM2,W,IDIMW)
C.....INITIAL GUESS AT ROOTS BY TRUNCATED TAYLOR SERIES
      FT=FLOAT(IT)
      R=FPR**(1.0/FT)
      D=FT*FPR/R
      S2=VAR/(D*D)
      T=FPR-(FT-1.0)*FT/2.0*S2*R**(IT-2)
      IF (T) 60,60,65
   60 U=1.0
      GO TO 70
   65 U=T**(1.0/FT)
C.....IMPROVE ESTIMATES OF ROOTS BY ITERATION
C.....SEARCH IS TERMINATED WHEN PERCENTAGE CHANGE BETWEEN SUCCESSIVE
C.....ESTIMATIONS IS LESS THAN EPS FOR BOTH VARIABLES.
   70 EPS=.000001
      IFLAG=0
      UST=U
      S2ST=S2
   80 DO 300 K=1,20
C.....FIRST FUNCTION
      ICUT=IDIM1
      CALL DU (IT,ICUT,U,S2,C,W,FPR,FU,DFDU)
      IF (FU) 90,200,90
   90 IF (DFDU) 120,100,120
  100 SLOPE1=0.0
      B1=S2
      GO TO 200
  120 D=FU/DFDU
      U1=U-D
```

```
      IF (D) 130,140,130
  130 CALL DS2 (IT,ICUT,IDIM1,U,S2,C,W,FPR,FS2,DFDS2)
      IF (DFDS2) 160,140,160
  140 SLOPE1=10.**10
      GO TO 180
  160 DD=FS2/DFDS2
      SLOPE1=DD/(-D)
  180 B1=S2-SLOPE1*U1
C.....SECOND FUNCTION
  200 ICUT=IDIM2
      CALL DU (IT2,ICUT,U,S2,CC,W,VAR,GU,DGDU)
      IF (GU) 220,290,220
  220 IF (DGDU) 240,230,240
  230 SLOPE2=0.0
      B2=S2
      GO TO 290
  240 D=GU/DGDU
      U2=U-D
      IF (D) 245,250,245
  245 CALL DS2 (IT2,ICUT,IDIM2,U,S2,CC,W,VAR,GS2,DGDS2)
      IF (DGDS2) 260,250,260
  250 SLOPE2=10.**10
      GO TO 270
  260 DD=GS2/DGDS2
      SLOPE2=DD/(-D)
  270 B2=S2-SLOPE2*U2
C.....SOLVE FOR NEW VALUES AND TEST FOR CONVERGENCE
  290 DIFF=SLOPE1-SLOPE2
      IF (DIFF) 295,310,295
  295 U=(B2-B1)/DIFF
      IF (U) 296,310,296
  296 DELTU=ABS((U-ULAST)/U)
      S2=SLOPE2*U+B2
      IF (S2) 297,310,297
  297 DELTS2=ABS((S2-S2LAST)/S2)
      IF (DELTU.LT.EPS .AND. DELTS2.LT.EPS) GO TO 320
      ULAST=U
      S2LAST=S2
  300 CONTINUE
C.....FAILURE TO CONVERGE
  310 IF (IFLAG) 315,315,1020
  315 U=UST-.01*UST
      S2=S2ST+.01*S2ST
      IFLAG=1
      GO TO 80
C.....COMPUTE CONFIDENCE LIMITS
  320 IF (S2) 1020,1020,340
  340 STD=SORT(S2)
      CONF=FLOAT(ICONF)/100.
      P=(1.0-CONF)/2.0+CONF
      CALL NRINV (P,Z)
      CLU=U+Z*STD
      CLL=U-Z*STD
      POPU=ENO*CLU**IT
```

```
      POPL=ENO*CLL**IT
      CALL CUMPR (1.0,U,STD,PR)
      PR=1.0-PR
      RETURN
      END

      SUBROUTINE DU (IT,ICUT,U,S2,C,W,CONST,FU,DFDU)
C.....RETURNS THE VALUE OF THE FUNCTION FU AND THE PARTIAL DERIVATIVE
C.....DFDU EVALUATED AT U AND S2 FOR THE MOMENT POLYNOMIAL, WHOSE
C.....COEFFICIENTS ARE GIVEN IN VECTOR C.  W IS A WORK VECTOR.
C.....TO AVOID ARITHMETIC OVERFLOW, HIGHER ORDER TERMS IN S2 ARE
C.....TRUNCATED WHEN THEY BECOME VERY SMALL (ICUT IS CUTOFF INDEX).
      DIMENSION C(1),W(1)
      IDIM=IT+1
      J=IDIM+2
      I=0
      T=1.0
  100 I=I+1
      J=J-2
      K=J+1
      W(K)=0.0
      IF (J) 300,300,200
  200 W(J)=C(I)*T
      T=T*S2
      IF (I-1) 220,220,240
  220 F=W(J)/10000.
      GO TO 100
  240 IF (W(J)-F) 260,260,100
  260 ICUT=I
      J=J-1
  280 W(J)=0.0
      J=J-1
      IF (J) 300,300,280
  300 W(1)=W(1)-CONST
      CALL PILD (FU,DFDU,U,W,IDIM)
      RETURN
      END

      SUBROUTINE DS2 (IT,ICUT,IDIM,U,S2,C,W,CONST,FS2,DFDS2)
C.....RETURNS THE VALUE OF THE FUNCTION FS2 AND THE PARTIAL DERIVATIVE
C.....DFDS2 EVALUATED AT U AND S2 FOR THE MOMENT POLYNOMIAL, WHOSE
C.....COEFFICIENTS ARE GIVEN IN VECTOR C.  W IS A WORK VECTOR.
C.....IDIM IS ONE MORE THAN THE DEGREE OF THE POLYNOMIAL IN S2.
C.....TO AVOID ARITHMETIC OVERFLOW, HIGHER ORDER TERMS IN S2 ARE
C.....TRUNCATED WHEN THEY BECOME VERY SMALL (ICUT IS CUTOFF INDEX).
      DIMENSION C(1),W(1)
      U2=U*U
      UIT=U**IT
      DO 200 I=1,ICUT
      W(I)=C(I)*UIT
  200 UIT=UIT/U2
  300 IF (I-IDIM) 400,500,500
  400 I=I+1
      W(I)=0.0
```

```
      GO TO 300
  500 W(1)=W(1)-CONST
      CALL PILD (FS2,DFDS2,S2,W,IDIM)
      RETURN
      END

      SUBROUTINE CUMPR (X,XBAR,STD,PR)
C.....RETURNS THE AREA PR UNDER THE NORMAL PROBABILITY CURVE WITH GIVEN
C.....MEAN AND STANDARD DEVIATION LYING BELOW THE SPECIFIED VALUE X.
C.....THIS VERSION USES HASTING'S APPROXIMATION FOR THE INTEGRAL
C.....OF A NORMAL PROBABILITY DENSITY FUNCTION.
      DATA A/0.3989423/P/0.2316419/B1/0.319381530/B2/-0.356563782/
     *     B3/1.781477937/B4/-1.821255978/B5/1.330274429/
      Y=X-XBAR
      Z=Y/STD
      T=1.0/(1.0+P*ABS(Z))
      PR=A*EXP(-Z*Z/2.0)*T*(B1+T*(B2+T*(B3+T*(B4+T*B5))))
      IF (Y) 1000,1000,900
  900 PR=1.0-PR
 1000 RETURN
      END

      SUBROUTINE NRINV (P,Z)
C.....INVERSE OF CUMULATIVE DISTRIBUTION FUNCTION FOR GAUSSIAN WITH
C     ZERO MEAN AND UNIT VARIANCE.  USES HASTING APROXIMATION TO
C     FIND Z FOR THE FUNCTION Z=G(P), WHERE P=F(Z) IS THE INTEGRAL
C     OF THE NORMAL CURVE FROM MINUS INFINITY TO Z.
C.....THE TRANSFORMATION AT STATEMENT 1070 AVOIDS AN ILLEGAL ARGUMENT
C     CONDITION (LOG OF ZERO) IN THE FOLLOWING STATEMENT FOR VALUES
C.....OF P NEAR ZERO OR ONE (SPECIFIC FOR IMS 5000).
      DATA AZ/2.515517/A1/0.802853/A2/0.010328/B1/1.432788/B2/0.189269/
     #     B3/0.001308/
      PM5=P-0.5
      IF (PM5) 1060,1050,1040
 1040 Q=1.0-P
      GO TO 1070
 1050 Z=0.0
      GO TO 1090
 1060 Q=P
 1070 Q=(Q+0.0000001)/1.0000001
      T=SQRT(-2.0*ALOG(Q))
      T2=T*T
      T3=T2*T
      Z=T-(AZ+A1*T+A2*T2)/(1.0+B1*T+B2*T2+B3*T3)
      IF (PM5) 1080,1050,1090
 1080 Z=-Z
 1090 RETURN
      END

      SUBROUTINE PMPY (Z,IDIMZ,X,IDIMX,Y,IDIMY)
C.....MULTIPLIES POLYNOMIAL X BY POLYNOMIAL Y.  RESULTANT VECTOR Z
C.....CANNOT BE IN THE SAME LOCATION AS EITHER X OR Y.  DIMENSIONS
C.....OF VECTORS OF COEFFICIENTS ARE ONE MORE THAN THE DEGREE OF THE
C.....POLYNOMIAL.  COEFFICIENTS ARE ORDERED FROM SMALLEST TO LARGEST.
```

```
C.....THIS VERSION IS MODIFIED TO AVOID ARITHMETIC OVERFLOW WITH
C.....VERY LARGE COEFFICIENTS COMPUTED WITH SUBROUTINE MOMENT.
      DIMENSION Z(1),X(1),Y(1)
      IF (IDIMX*IDIMY) 10,10,20
   10 IDIMZ=0
      GO TO 50
   20 IDIMZ=IDIMX+IDIMY-1
      KMAX=IDIMZ
      DO 30 I=1,IDIMZ
   30 Z(I)=0.0
      DO 32 I=1,IDIMX
      IF (X(I)-.1E+18) 32,32,34
   32 CONTINUE
      GO TO 35
   34 KMAX=2*I-2
   35 DO 40 I=1,IDIMX
      DO 40 J=1,IDIMY
      K=I+J-1
      IF (K-KMAX) 38,38,36
   36 Z(K)=.1E+20
      GO TO 40
   38 Z(K)=Z(K)+X(I)*Y(J)
   40 CONTINUE
   50 RETURN
      END

      SUBROUTINE PSUB (Z,IDIMZ,X,IDIMX,Y,IDIMY)
C.....SUBTRACTS POLYNOMIAL Y FROM POLYNOMIAL X.  RESULTANT VECTOR Z
C.....MAY BE IN THE SAME LOCATION AS EITHER VECTOR X OR Y ONLY IF
C.....THE DIMENSION OF THAT VECTOR IS NOT LESS THAN THE OTHER INPUT
C.....VECTOR. COEFFICIENTS ARE ORDERED FROM SMALLEST TO LARGEST.
C.....DIMENSIONS OF X AND Y ARE ONE MORE THAN THE DEGREE OF THE
C.....POLYNOMIAL.
      DIMENSION Z(1),X(1),Y(1)
      NDIM=IDIMX
      IF (IDIMX-IDIMY) 10,20,20
   10 NDIM=IDIMY
   20 IF (NDIM) 90,90,30
   30 DO 80 I=1,NDIM
      IF (I-IDIMX) 40,40,60
   40 IF (I-IDIMY) 50,50,70
   50 Z(I)=X(I)-Y(I)
      GO TO 80
   60 Z(I)=-Y(I)
      GO TO 80
   70 Z(I)=X(I)
   80 CONTINUE
   90 IDIMZ=NDIM
      RETURN
      END

      SUBROUTINE PILD (POLY,DER,ARGUM,X,IDIMX)
C.....EVALUATES POLYNOMIAL AND ITS FIRST DERIVATIVE FOR A GIVEN
C.....ARGUMENT BY MEANS OF QUADRATIC SYNTHETIC DIVISION (PQSD).
```

```
C.....VECTOR X CONTAINS THE COEFFICIENTS OF THE POLYNOMIAL.
      DIMENSION X(1)
      P=ARGUM+ARGUM
      Q=-ARGUM*ARGUM
      CALL POSD (DER,POLY,P,Q,X,IDIMX)
      POLY=ARGUM*DER+POLY
      RETURN
      END

      SUBROUTINE POSD (A,B,P,Q,X,IDIMX)
C.....QUADRATIC SYNTHETIC DIVISION
      DIMENSION X(1)
      A=0.
      B=0.
      J=IDIMX
    1 IF (J) 3,3,2
    2 Z=P*A+B
      B=Q*A+X(J)
      A=Z
      J=J-1
      GO TO 1
    3 RETURN
      END
```

APPENDIX 4. Listing of program SLT.

```
      PROGRAM SLT        --  STOCHASTIC LIFE TABLE SIMULATOR
C
C**** POPULATION PROJECTION USING STOCHASTIC LESLIE MATRICES ***********
C     INPUT DATA AND CONTROL PARAMETERS ARE SPECIFIED ON INPUT FILE
C     'INPUT.SLT'.  FOR INSTRUCTIONS,  REFER TO FILE SLTGUIDE.
C.....THE MEAN (DETERMINISTIC) VALUES FOR PROJECTION ARE GIVEN BY THE
C     SCHEDULES OF FECUNDITY (BX) AND SURVIVAL RATES (PX). THE STOCHASTIC
C     VERSIONS ADD RANDOMNESS IN ACCORDANCE WITH VARIANCES AND COVARIANCES
C     SPECIFIED IN THE COVARIANCE MATRIX S.
C.....SPECIAL OPTIONS ARE:
C          IOP1=0  INITIAL POPULATION VECTOR (RNX) IS SPECIFIED IN DATA
C          IOP1>0  INITIAL POPULATION VECTOR IS CALCULATED AS THE STABLE AGE
C                  DISTRIBUTION; THE TOTAL INITIAL POPULATION MUST BE SPECIFIED
C          IOP2=0  FULL COVARIANCE STRUCTURE
C          IOP2>0  SIMPLIFIED COVARIANCE STRUCTURE; A SINGLE COVARIANCE BETWEEN
C                  ALL FECUNDITIES AND ALL SURVIVAL RATES IS SPECIFIED
C
      DIMENSION RNX(20),ENX(20),XBAR(20),XCVAR(210),BX(20),PX(20),S(820)
      DIMENSION BT(20),PT(20),BTBAR(20),PTBAR(20),BTVAR(20),PTVAR(20)
      DIMENSION VX(20),VXBAR(20),VXVAR(20),DERDM(20),DRDMM(20),DRDMV(20)
      DIMENSION DERDP(20),DRDPM(20),DRDPV(20),ELX(20),PHIX(20)
      DIMENSION BPT(40),BPTBAR(40),BPCV(820),W1(40),W2(40),V(1600)
      DIMENSION ITAB1(50),ITAB2(50)
      COMMON ZERO(40)
      DATA MM/0/NBAD/0/ITAB1/50*0/ITAB2/50*0/NOBS/0/ZERO/40*0.0/
      DATA IP/0/ICLL/0/ICLU/0/
      CALL OPEN (1,'INPUT   SLT',0)
      CALL OPEN (6,'OUTPUT  SLT',0)
C.....SET PARAMETERS AND OPTIONS
      CALL LOAD (N,NT,NR,IOP1,IOP2,LUN,ISEED,RNX,BX,PX,S,BPCV,KK,WIDTH1,
     #           BL1,WIDTH2,BL2,RL,RU)
      NRNT=NR*NT
      N2=N*2
      ICNT=20/(N*NT)*100
      IF (ICNT .EQ. 0) ICNT=10
C.....COMPUTE LIFE TABLE PARAMETERS
      CALL PTOL (PX,ELX,N)
      CALL LMPHI (ELX,BX,PHIX,N)
      CALL MMNT (PHIX,RZ,T,S2,N)
      CALL ERNWTA (PHIX,RZ,T,S2,ER,N)
      CALL FISH (PHIX,ELX,VX,ER,N)
      CALL DERV (ELX,VX,T,ER,DERDM,DERDP,N)
C.....COMPUTE THE STABLE AGE DISTRIBUTION
      IF (IOP1) 1140,1140,1100
 1100 P=RNX(1)
      CALL STABL (ER,ELX,RNX,N)
      DO 1120 I=1,N
      RNX(I)=RNX(I)*P
 1120 CONTINUE
C.....INITIALIZE POPULATION VECTOR AND PROJECT
 1140 DO 1200 I= 1,N
```

```
      ENX(I)=RNX(I)
 1200 CONTINUE
      DO 1210 NTT=1,NT
      CALL PRJCT (ENX,PX,BX,N)
 1210 CONTINUE
      CALL EREAL (RNX,ENX,N,NT,FI,ERT)
C.....OUTPUT OF DETERMINISTIC PROJECTION AND ECHO OF INPUT
      CALL OUT (1,N,RNX,ENX,ZERO,BX,ZERO,PX,ZERO,VX,ZERO,DERDM,DERDP,
     *     S,BPCV,ER,0.,RZ,0.,T,0.,S2,0.,FI,0.,ERT,0.,IP,NR,NT,1,0,
     *     LUN,IOP1,KK,WIDTH1,BL1,WIDTH2,BL2,ITAB1,ITAB2,RL,RU,0,0)
C.....EIGENANALYSIS OF COVARIANCE MATRIX (REVERSE PCA)
      CALL EIGEN (S,V,N2,0)
C.....MESSAGE TO OPERATOR
      WRITE (LUN,13)
   13 FORMAT (//5X,'BEGINNING STOCHASTIC PROJECTIONS NOW . . .'/)
C.....STOCHASTIC PROJECTIONS
      NDX=0
      DO 2000 NRR=1,NR
C.....INITIALIZE POPULATION VECTOR
      DO 1800 I=1,N
      ENX(I)=RNX(I)
 1800 CONTINUE
C.....GENERATE STOCHASTIC VECTORS, ANALYZE AND PROJECT
      DO 1840 NTT=1,NT
 1820 CALL BPIN (BX,PX,BPT,N)
      CALL XRGEN (BPT,S,V,W1,W2,N2,ISEED)
      CALL BPOUT (BPT,BT,PT,N)
      CALL PTOL (PT,ELX,N)
      CALL LMPHI (ELX,BT,PHIX,N)
      CALL MMNT (PHIX,RZ,T,S2,N)
      CALL ERNWTA (PHIX,RZ,T,S2,ER,N)
      IF (ER+90.0) 1825,1825,1830
 1825 NBAD=NBAD+1
      GO TO 1820
 1830 CALL FISH (PHIX,ELX,VX,ER,N)
      CALL DERV (ELX,VX,T,ER,DERDM,DERDP,N)
      CALL PRJCT (ENX,PT,BT,N)
C.....ACCUMULATE SUMS FOR MEANS AND VARIANCES
      NDX=NDX+1
      CALL MVAC (ER,ERBAR,ERVAR,NDX,NRNT)
      CALL MVAC (T,TBAR,TVAR,NDX,NRNT)
      CALL MVAC (S2,S2BAR,S2VAR,NDX,NRNT)
      CALL MVAC (RZ,RZBAR,RZVAR,NDX,NRNT)
      CALL MVVAC (VX,VXBAR,VXVAR,N,NDX,NRNT)
      CALL MVVAC (DERDM,DRDMM,DRDMV,N,NDX,NRNT)
      CALL MVVAC (DERDP,DRDPM,DRDPV,N,NDX,NRNT)
      CALL CVAC (BPT,BPTBAR,BPCV,N2,NDX,NRNT)
 1840 CONTINUE
      CALL CVAC (ENX,XBAR,XCVAR,N,NRR,NR)
      CALL EREAL (RNX,ENX,N,NT,FI,ERT)
      CALL MVAC (FI,FIBAR,FIVAR,NRR,NR)
      CALL MVAC (ERT,ERTBR,ERTVR,NRR,NR)
      CALL TALLY (FI,KK,WIDTH1,BL1,NOBS,ITAB1)
      CALL TALLY (ERT,KK,WIDTH2,BL2,NOBS,ITAB2)
```

```
      CALL COUNT (NRR,ICNT,MM,LUN)
      IF (ERT-1.0) 1870,1870,1860
 1860 IP=IP+1
 1870 IF (ERT-RL) 1880,1880,1890
 1880 ICLL=ICLL+1
 1890 IF (ERT-RU) 2000,1900,1900
 1900 ICLU=ICLU+1
 2000 CONTINUE
C.....OUTPUT OF STOCHASTIC PROJECTIONS
      CALL BPOUT (BPTBAR,BTBAR,PTBAR,N)
      CALL VAROUT (BPCV,BTVAR,PTVAR,N)
      CALL OUT (2,N,RNX,XBAR,XCVAR,BTBAR,BTVAR,PTBAR,PTVAR,VXBAR,VXVAR,
     *     DRDMM,DRDPM,BPCV,S,ERBAR,ERVAR,RZBAR,RZVAR,TBAR,TVAR,S2BAR,
     *     S2VAR,FIBAR,FIVAR,ERTBR,ERTVR,IP,NR,NT,NDX,NBAD,LUN,IOP1,KK,
     *     WIDTH1,BL1,WIDTH2,BL2,ITAB1,ITAB2,RL,RU,ICLL,ICLU)
C.....DETERMINISTIC PROJECTION WITH ACHIEVED MEAN VALUES
C.....COMPUTE LIFE TABLE PARAMETERS FOR ACHIEVED MEAN VALUES
      CALL PTOL (PTBAR,ELX,N)
      CALL LMPHI (ELX,BTBAR,PHIX,N)
      CALL MMNT (PHIX,RZ,T,S2,N)
      CALL ERNWTA (PHIX,RZ,T,S2,ER,N)
      CALL FISH (PHIX,ELX,VX,ER,N)
      CALL DERV (ELX,VX,T,ER,DERDM,DERDP,N)
C.....COMPUTE NEW STABLE AGE DISTRIBUTION
      IF (IOP1) 2080,2080,2040
 2040 CALL STABL (ER,ELX,RNX,N)
      DO 2060 I=1,N
      RNX(I)=RNX(I)*P
 2060 CONTINUE
C.....INITIALIZE POPULATION VECTOR AND PROJECT
 2080 DO 2100 I=1,N
      ENX(I)=RNX(I)
 2100 CONTINUE
      DO 2120 NTT=1,NT
      CALL PRJCT (ENX,PTBAR,BTBAR,N)
 2120 CONTINUE
      CALL EREAL (RNX,ENX,N,NT,FI,ERT)
C.....OUTPUT OF SECOND DETERMINISTIC PROJECTION
      CALL OUT (3,N,RNX,ENX,ZERO,BTBAR,ZERO,PTBAR,ZERO,VX,ZERO,DERDM,
     *     DERDP,S,BPCV,ER,0.,RZ,0.,T,0.,S2,0.,FI,0.,ERT,0.,IP,1,NT,1,
     *     0,LUN,0,KK,WIDTH1,BL1,WIDTH2,BL2,ITAB1,ITAB2,RL,RU,0,0)
      CALL EXIT
      END

      SUBROUTINE OUT (ICODE,N,RNX,XBAR,XCVAR,BXBAR,BXVAR,PXBAR,PXVAR,
     *     VXBAR,VXVAR,DRDMM,DRDPM,COVAR,W,ER,ERVAR,RZ,RZVAR,T,TVAR,S2,
     *     S2VAR,FI,FIVAR,ERT,ERTVAR,IP,NR,NT,NDX,NBAD,LUN,IOP1,KK,
     *     WIDTH1,BL1,WIDTH2,BL2,ITAB1,ITAB2,RL,RU,ICLL,ICLU)
C*****OUTPUT FOR STOCHASTIC LIFE TABLE SIMULATION OF NT TIME STEPS WITH
C     NR REPETITIONS. THE VECTORS ARE OF LENGTH N WITH MEANS CALCULATED
C     OVER NDX VALUES. W IS A WORK VECTOR DIMENSIONED .GE. COVAR.
      DIMENSION RNX(1),XBAR(1),XCVAR(1),BXBAR(1),BXVAR(1),PXBAR(1),W(1)
      DIMENSION PXVAR(1),VXBAR(1),VXVAR(1),DRDMM(1),DRDPM(1),COVAR(1)
      DIMENSION ITAB1(1),ITAB2(1)
```

```
C.....HEADING FOR VITAL RATES
      GO TO (1100,2000,1150),ICODE
 1100 WRITE (LUN,10)
      WRITE (6,10)
   10 FORMAT (1H1///9X,'* * * * * STOCHASTIC LIFE TABLE SIMULATION * * *
     * * *'///5X,'1.  INPUT'//5X,'THE FOLLOWING VITAL PARAMETERS HAVE BE
     *EN SPECIFIED:'/)
      GO TO 1160
 1150 WRITE (LUN,15)
      WRITE (6,15)
   15 FORMAT (1H1///5X,'4.  DETERMINISTIC POPULATION PROJECTION'///5X,
     *'THE ACHIEVED MEAN VITAL RATES PRODUCE THE FOLLOWING ASSOCIATED'
     *1X,'PARAMETERS:'/)
C.....TABLE OF VITAL RATES
 1160 WRITE (LUN,30)
      WRITE (6,30)
   30 FORMAT (/6X,'AGE',4X,'BIRTH',5X,'SURVIVAL',2X,'REPRODUCTIVE'/5X,
     *'CLASS',4X,'RATE',7X,'RATE',7X,'VALUE',5X,'DER/DM',5X,'DER/DP'/)
      DO 1170 I=1,N
      WRITE (LUN,35) I,BXBAR(I),PXBAR(I),VXBAR(I),DRDMM(I),DRDPM(I)
      WRITE (6,35) I,BXBAR(I),PXBAR(I),VXBAR(I),DRDMM(I),DRDPM(I)
   35 FORMAT (6X,I2,5F11.5)
 1170 CONTINUE
      WRITE (LUN,40) ER,RZ,T,S2
      WRITE (6,40) ER,RZ,T,S2
   40 FORMAT (/5X,'ASYMPTOTIC FACTOR OF INCREASE',1X,F10.4/
     *5X,'NET REPLACEMENT RATE',10X,F10.4/
     *5X,'GENERATION TIME',15X,F10.4/
     *5X,'VAR. IN AGE OF REPRODUCTION',3X,F10.4)
C.....POPULATION VECTOR TABLE
      GO TO (1200,2000,1260),ICODE
 1200 WRITE (LUN,45) NT
      WRITE (6,45) NT
   45 FORMAT (///5X,'USING THESE VITAL RATES, THE INITIAL POPULATION IS
     * PROJECTED FOR',I3,' TIME STEPS AS FOLLOWS:')
      IF (IOP1) 1300,1300,1210
 1210 S=0.0
      DO 1220 I=1,N
      S=S+RNX(I)
 1220 CONTINUE
      WRITE (LUN,46) S
      WRITE (6,46) S
   46 FORMAT (9X,'(YOU HAVE SPECIFIED THAT THE INITIAL POPULATION SHOULD
     * BE IN STABLE AGE DISTRIBUTION'/9X,'WITH A TOTAL OF',F10.0,
     *' INDIVIDUALS)')
      GO TO 1300
 1260 WRITE (LUN,48) NT
      WRITE (6,48) NT
   48 FORMAT (///5X,'USING THESE ACHIEVED MEAN VITAL RATES, THE INITIAL
     * POPULATION'/5X,'IS PROJECTED FOR',I3,' TIME STEPS AS FOLLOWS:')
 1300 WRITE (LUN,50)
      WRITE (6,50)
   50 FORMAT (//10X,'AGE',6X,'INITIAL', 9X,'FINAL'/
     *9X,'CLASS',3X,'POPULATION',5X,'POPULATION'/)
```

```
      DO 1310 I=1,N
      WRITE (LUN,52) I,RNX(I),XBAR(I)
      WRITE (6,52) I,RNX(I),XBAR(I)
   52 FORMAT (10X,I2,2F15.2)
 1310 CONTINUE
      WRITE (LUN,53) ERT,FI
      WRITE (6,53) ERT,FI
   53 FORMAT (/5X,'FINAL/INITIAL POPULATION',6X,F10.4/
     *5X,'REALIZED FACTOR OF INCREASE',3X,F10.4)
      GO TO (1400,2000,3000),ICODE
C.....LIST SPECIFIED COVARIANCE MATRIX
 1400 N2=N*2
      WRITE (LUN,60) N,N
      WRITE (6,60) N,N
   60 FORMAT (1H1///5X,'2.  STOCHASTIC POPULATION PROJECTION'//
     *5X,'THE FOLLOWING COVARIANCE MATRIX HAS BEEN SPECIFIED:'/
     *10X,'(FIRST',I3,' COLUMNS ARE BIRTH RATES, SECOND',I3,
     *' ARE SURVIVAL RATES)'/)
      CALL WTRI (N2,COVAR,W,LUN,2)
      CALL WTRI (N2,COVAR,W,6,2)
      GO TO 3000
C.....STOCHASTIC PROJECTION OUTPUT
C.....POPULATION VECTOR TABLE
 2000 WRITE (LUN,70) NT,NR
      WRITE (6,70) NT,NR
   70 FORMAT (///5X,'USING THE MEAN VITAL RATES WITH THIS COVARIANCE'
     *1X,'STRUCTURE,'/5X,'THE INITIAL POPULATION IS PROJECTED FOR',I3,
     *' TIME STEPS',I6,' TIMES.')
      WRITE (LUN,72)
      WRITE (6,72)
   72 FORMAT (/10X,'AGE',6X,'INITIAL',5X,'(MEAN) FINAL'/
     *9X,'CLASS',3X,'POPULATION',5X,'POPULATION'/)
      DO 2220 I=1,N
      WRITE (LUN,52) I,RNX(I),XBAR(I)
      WRITE (6,52) I,RNX(I),XBAR(I)
 2220 CONTINUE
C.....POPULATION VECTOR COVARIANCE MATRIX
      WRITE (LUN,74)
      WRITE (6,74)
   74 FORMAT (//5X,'COVARIANCE MATRIX FOR FINAL POPULATION VECTOR'/)
      CALL WTRI (N,XCVAR,W,LUN,1)
      CALL WTRI (N,XCVAR,W,6,1)
      WRITE (LUN,75) ERT,ERTVAR,FI,FIVAR
      WRITE (6,75) ERT,ERTVAR,FI,FIVAR
   75 FORMAT (/40X,'MEAN',4X,'VARIANCE'//
     *5X,'FINAL/INITIAL POPULATION',6X,F10.4,F10.5/
     *5X,'REALIZED FACTOR OF INCREASE',3X,F10.4,F10.5)
      WRITE (LUN,77)
      WRITE (6,77)
   77 FORMAT (//5X,'FREQUENCY DISTRIBUTION FOR FACTOR OF INCREASE'//
     *57X,'REALIZED'/14X,'LOWER BOUND',2X,'FINAL/INITIAL', 3X,'LOWER'
     *1X,'BOUND',2X,'FACTOR OF'/4X,'INTERVAL',2X,'OF INTERVAL',3X,
     *'POPULATION', 5X,'OF INTERVAL',3X,'INCREASE'/)
      B1=BL1
```

```
      B2=BL2
      DO 2300 I=1,KK
      IF (I-1) 2240,2240,2260
 2240 WRITE (LUN,78) I,B2,ITAB2(I),B1,ITAB1(I)
      WRITE (6,78) I,B2,ITAB2(I),B1,ITAB1(I)
   78 FORMAT (5X,I2,5X,'<',F8.4,I10,10X,'<',F8.4,I10)
      GO TO 2300
 2260 WRITE (LUN,79) I,B2,ITAB2(I),B1,ITAB1(I)
      WRITE (6,79) I,B2,ITAB2(I),B1,ITAB1(I)
   79 FORMAT (5X,I2,6X,F8.4,I10,11X,F8.4,I10)
      B1=B1+WIDTH1
      B2=B2+WIDTH2
 2300 CONTINUE
      FNR=FLOAT(NR)
      PR1=FLOAT(IP)/FNR
      PR2=FLOAT(ICLL)/FNR
      PR3=FLOAT(ICLU)/FNR
      WRITE (LUN,76) PR1,RL,PR2,RU,PR3
      WRITE (6,76) PR1,RL,PR2,RU,PR3
   76 FORMAT (/5X,'PROPORTION OF PROJECTIONS WITH FACTOR OF INCREASE',
     *' GREATER THAN 1.0 IS',F7.4/5X,'PROPORTION OF FINAL/INITIAL',
     *' POPULATION RATIOS BELOW',F8.4,' IS',F7.4/5X,'PROPORTION OF',
     *' FINAL/INITIAL POPULATION RATIOS ABOVE',F8.3,' IS',F7.4)
C.....HEADING FOR PART 3 - STATISTICAL SUMMARY
      WRITE (LUN,80) NDX,NT,NR
      WRITE (6,80) NDX,NT,NR
   80 FORMAT (1H1///5X,'3.  STATISTICAL PROFILES OF DEMOGRAPHIC PARAMETE
     *RS USED IN STOCHASTIC POPULATION PROJECTIONS'//
     *10X,'THESE STATISTICS ARE COMPUTED ON THE BASIS OF',I6,' VALUES'/
     *10X,'(',I2,' TIME STEPS,',I5,' REPETITIONS)')
      WRITE (LUN,81)
      WRITE (6,81)
   81 FORMAT (//6X,'AGE',10X,'BIRTH RATE',8X,'SURVIVAL RATE',4X,'REPRODU
     *CTIVE VALUE',4X,'DER/DM',3X,'DER/DP'/
     *5X,'CLASS',3(7X,'MEAN',6X,'VAR'),7X,'MEAN',5X,'MEAN'/)
      DO 2310 I=1,N
      WRITE (LUN,82) I,BXBAR(I),BXVAR(I),PXBAR(I),PXVAR(I),VXBAR(I),
     *     VXVAR(I),DRDMM(I),DRDPM(I)
      WRITE (6,82) I,BXBAR(I),BXVAR(I),PXBAR(I),PXVAR(I),VXBAR(I),
     *     VXVAR(I),DRDMM(I),DRDPM(I)
   82 FORMAT (6X,I2,3X,4(F11.5,F9.5))
 2310 CONTINUE
C.....WRITE OTHER STATISTICS
      WRITE (LUN,85) ER,ERVAR,RZ,RZVAR,T,TVAR,S2,S2VAR
      WRITE (6,85) ER,ERVAR,RZ,RZVAR,T,TVAR,S2,S2VAR
   85 FORMAT (/40X,'MEAN',4X,'VARIANCE'//
     *5X,'ASYMPTOTIC FACTOR OF INCREASE',1X,F10.4,F10.5/
     *5X,'NET REPLACEMENT RATE',10X,F10.4,F10.5/
     *5X,'GENERATION TIME',15X,F10.4,F10.5/
     *5X,'VAR. IN AGE OF REPRODUCTION',3X,F10.4,F10.5)
      IF (NBAD) 2400,2400,2315
 2315 WRITE (LUN,86) NBAD
      WRITE (6,86) NBAD
   86 FORMAT (/10X,'***** WARNING:  FACTOR OF INCREASE COULD NOT BE COMP
```

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     *UTED',I5,' TIMES')
C.....CALCULATED COVARIANCE MATRIX FOR VITAL RATES
 2400 WRITE (LUN,87)
      WRITE (6,87)
   87 FORMAT (//5X,'THE FOLLOWING COVARIANCE STRUCTURE WAS ACHIEVED:'/)
      CALL WTRI (N2,COVAR,W,LUN,2)
      CALL WTRI (N2,COVAR,W,6,2)
 3000 RETURN
      END

      SUBROUTINE WTRI (N,COVAR,W,LUN,IOP)
C.....WRITES THE CONTENTS OF A SINGLE-SUBSCRIPTED, UPPER TRIANGULAR
C.....MATRIX COVAR IN N BY N TABLE FORM ON DEVICE LUN.  W IS A WORK
C.....VECTOR DIMENSIONED AT LEAST AS GREAT AS COVAR.  IOP=1 FOR E
C.....FORMAT, IOP=2 FOR F FORMAT (F10.5).
      DIMENSION COVAR(1),W(1)
      COMMON ZERO(40)
      WRITE (LUN,75) (J,J=1,N)
   75 FORMAT (6X,10I10)
C.....REPACK MATRIX
      L=0
      DO 2250 I=1,N
      DO 2240 J=I,N
      L=L+1
      IJ=(J*J-J)/2+I
      W(L)=COVAR(IJ)
 2240 CONTINUE
 2250 CONTINUE
C.....LIST
      JSTP=0
      DO 2300 I=1,N
      ZERO(I)=0.0
      JSTRT=JSTP+1
      JSTP=JSTRT+N-I
      K=I-1
      IF (IOP-1) 2280,2280,2255
 2255 IF (K) 2260,2260,2270
 2260 WRITE (LUN,77) (W(J),J=JSTRT,JSTP)
      GO TO 2300
 2270 WRITE (LUN,77) (ZERO(J),J=1,K),(W(J),J=JSTRT,JSTP)
   77 FORMAT (9X,10F10.5)
      GO TO 2300
 2280 IF (K) 2285,2285,2290
 2285 WRITE (LUN,78) (W(J),J=JSTRT,JSTP)
      GO TO 2300
 2290 WRITE (LUN,78) (ZERO(J),J=1,K),(W(J),J=JSTRT,JSTP)
   78 FORMAT (9X,10E10.4)
 2300 CONTINUE
      RETURN
      END

      SUBROUTINE EIGEN(A,R,N,MV)
C*****EIGENANALYSIS OF REAL SYMMETRIC MATRIX, A, OF ORDER N,
C     RECEIVED IN UPPER TRIANGULAR SINGLE SUBSCRIPT NOTATION.
```

```
C.....RETURNS EIGENVALUES, LARGEST FIRST, IN FIRST N ELEMENTS OF A.
C     OPTIONALLY, RETURNS EIGENVECTORS AS COLUMNS OF MATRIX R, STORED
C     IN SINGLE SUBSCRIPT NOTATION, IN ORDER CORRESPONDING TO THE
C     ORDER OF THE EIGENVALUES.
C.....IF THE CONTROL PARAMETER MV EQUALS ONE, EIGENVECTORS ARE
C     NOT COMPUTED, AND R NEED NOT BE DIMENSIONED LARGER THAN 1.
C.....THE INPUT MATRIX IS DESTROYED IN COMPUTATION.
C     THIS SUBROUTINE IS SLOWER THAN THE HOUSEHOLDER ALGORITHM FOR
C     LARGE MATRICES, BUT IT IS REPORTED TO BE MORE ACCURATE WHEN THERE
C     SMALL EIGENVALUES.
C.....THE UPPER TRIANGULAR ARRAY MUST BE DIMENSIONED AT LEAST N*(N+1)/2
C     IN THE MAIN PROGRAM
C.....MODIFIED FROM SSP TO RETURN EIGENVALUES IN FIRST N ELEMENTS OF A.
      DIMENSION A(1),R(1)
C        GENERATE IDENTITY MATRIX
      IF(MV-1) 10,25,10
   10 IQ=-N
      DO 20 J=1,N
      IQ=IQ+N
      DO 20 I=1,N
      IJ=IQ+I
      R(IJ)=0.0
      IF(I-J) 20,15,20
   15 R(IJ)=1.0
   20 CONTINUE
C        COMPUTE INITIAL AND FINAL NORMS (ANORM AND ANORMX)
   25 ANORM=0.0
      DO 35 I=1,N
      DO 35 J=I,N
      IF(I-J) 30,35,30
   30 IA=I+(J*J-J)/2
      ANORM=ANORM+A(IA)*A(IA)
   35 CONTINUE
      IF(ANORM) 165,165,40
   40 ANORM=1.414*SQRT(ANORM)
      ANRMX=ANORM*0.000001/FLOAT(N)
C        INITIALIZE INDICATORS AND COMPUTE THRESHOLD, THR
      IND=0
      THR=ANORM
   45 THR=THR/FLOAT(N)
   50 L=1
   55 M=L+1
C        COMPUTE SIN AND COS
   60 MQ=(M*M-M)/2
      LQ=(L*L-L)/2
      LM=L+MQ
   62 IF( ABS(A(LM))-THR) 130,65,65
   65 IND=1
      LL=L+LQ
      MM=M+MQ
      X=0.5*(A(LL)-A(MM))
   68 Y=-A(LM)/ SQRT(A(LM)*A(LM)+X*X)
      IF(X) 70,75,75
   70 Y=-Y
```

```
   75 SINX=Y/ SQRT(2.0*(1.0+( SQRT(1.0-Y*Y))))
      SINX2=SINX*SINX
   78 COSX= SQRT(1.0-SINX2)
      COSX2=COSX*COSX
      SINCS =SINX*COSX
C        ROTATE L AND M COLUMNS
      ILQ=N*(L-1)
      IMQ=N*(M-1)
      DO 125 I=1,N
      IQ=(I*I-I)/2
      IF(I-L) 80,115,80
   80 IF(I-M) 85,115,90
   85 IM=I+MQ
      GO TO 95
   90 IM=M+IQ
   95 IF(I-L) 100,105,105
  100 IL=I+LQ
      GO TO 110
  105 IL=L+IQ
  110 X=A(IL)*COSX-A(IM)*SINX
      A(IM)=A(IL)*SINX+A(IM)*COSX
      A(IL)=X
  115 IF(MV-1) 120,125,120
  120 ILR=ILQ+I
      IMR=IMQ+I
      X=R(ILR)*COSX-R(IMR)*SINX
      R(IMR)=R(ILR)*SINX+R(IMR)*COSX
      R(ILR)=X
  125 CONTINUE
      X=2.0*A(LM)*SINCS
      Y=A(LL)*COSX2+A(MM)*SINX2-X
      X=A(LL)*SINX2+A(MM)*COSX2+X
      A(LM)=(A(LL)-A(MM))*SINCS+A(LM)*(COSX2-SINX2)
      A(LL)=Y
      A(MM)=X
C        TESTS FOR COMPLETION
C        TEST FOR M = LAST COLUMN
  130 IF(M-N) 135,140,135
  135 M=M+1
      GO TO 60
C        TEST FOR L = SECOND FROM LAST COLUMN
  140 IF(L-(N-1)) 145,150,145
  145 L=L+1
      GO TO 55
  150 IF(IND-1) 160,155,160
  155 IND=0
      GO TO 50
C        COMPARE THRESHOLD WITH FINAL NORM
  160 IF(THR-ANRMX) 165,165,45
C        SORT EIGENVALUES AND EIGENVECTORS
  165 IQ=-N
      DO 185 I=1,N
      IQ=IQ+N
      LL=I+(I*I-I)/2
```

```
      JQ=N*(I-2)
      DO 185 J=I,N
      JQ=JQ+N
      MM=J+(J*J-J)/2
      IF(A(LL)-A(MM)) 170,185,185
  170 X=A(LL)
      A(LL)=A(MM)
      A(MM)=X
      IF(MV-1) 175,185,175
  175 DO 180 K=1,N
      ILR=IQ+K
      IMR=JQ+K
      X=R(ILR)
      R(ILR)=R(IMR)
  180 R(IMR)=X
  185 CONTINUE
C.....REPACK EIGENVALUE ARRAY
      J=0
      DO 190 I=1,N
      J=J+I
      A(I)=A(J)
  190 CONTINUE
      RETURN
      END

      SUBROUTINE MVAC (X,XBAR,XVAR,NTT,NT)
C*****CALCULATES THE MEAN AND VARIANCE OF A SCALAR X
C     BY ACCUMULATING THE SUMS AND SUMS OF SQUARES EACH TIME THE SUBROUTINE
C     IS CALLED. THE CALL TO THIS SUBROUTINE SHOULD BE INSIDE A LOOP INDEXED
C.....BY NTT. AFTER THE LAST PASS (NTT=NT), MEAN AND VARIANCE ARE COMPUTED.
      IF (NTT-1) 4900,4900,5000
 4900 XBAR=0.0
      XVAR=0.0
 5000 XBAR=XBAR+X
      XVAR=XVAR+X*X
      IF (NTT-NT) 5200,5100,5100
 5100 FNT=FLOAT(NT)
      XBAR=XBAR/FNT
      XVAR=XVAR/FNT-XBAR*XBAR
 5200 RETURN
      END

      SUBROUTINE NRMUL (X,XBAR,STD,ISEED)
C*****RETURNS NORMALLY DISTRIBUTED RANDOM NUMBER, X, WITH MEAN XBAR
C     AND STANDARD DEVIATION STD AT EACH CALL.
C     REF: BOX AND MULLER (1958) ANN. MATH. STAT. 29: 610-611.
C          MULLER (1959) J. ASSOC. COMPUTING MACHINERY 6: 376-383.
      DATA TPI/6.2831853072/
      CALL RANDM (U1,ISEED)
      U1=(U1+0.0000001)/1.0000001
      CALL RANDM (U2,ISEED)
      X=XBAR+STD*COS(TPI*U2)*SQRT(-2.0*ALOG(U1))
      RETURN
      END
```

```
      SUBROUTINE RANDM (X,ISEED)
C.....RANDOM NUMBER GENERATOR. RETURNS A RANDOM NUMBER X WITH A UNIFORM
C     DISTRIBUTION BETWEEN ZERO AND UNITY, INCLUSIVE. ISEED MUST BE
C     INITIALIZED WITH A POSITIVE INTEGER IN THE RANGE 1-32767, BUT THE
C     VALUE OF ISEED MUST NOT BE RESET BETWEEN CALLS.
C.....THIS GENERATOR IS SPECIFIC FOR THE IMS 5000 AND IBM 1800 (6/1/81).
      COMMON MARKR,MULT,DIV
      IF (ISEED) 1040,1040,1030
 1030 MULT=12781
      DIV=-1.0/32767.0
      ISEED=ISEED*(-1)
      MARKR=ISEED
 1040 ISEED=ISEED*MULT-2797
      IF (ISEED) 1060,1060,1050
 1050 ISEED=ISEED-32767-1
 1060 X=FLOAT(ISEED)*DIV
      IF (ISEED-MARKR) 1100,1080,1100
 1080 MULT=MULT+8
 1100 RETURN
      END

      SUBROUTINE MTPRD (A,B,C,NA,MA,MB)
C*****MATRIX MULTIPLICATION.  APPLIES MATRIX A (NA BY MA) TO MATRIX
C     B (MA BY MB) TO FORM THE MATRIX C (NA BY MB).
C.....THE MATRICES ARE STORED IN SINGLE SUBSCRIPT NOTATION.
      DIMENSION A(1),B(1),C(1)
      IR=0
      IK=-MA
      DO 1050 K=1,MB
      IK=IK+MA
      DO 1040 J=1,NA
      IR=IR+1
      JI=J-NA
      IB=IK
      C(IR)=0.0
      DO 1030 I=1,MA
      JI=JI+NA
      IB=IB+1
      C(IR)=C(IR)+A(JI)*B(IB)
 1030 CONTINUE
 1040 CONTINUE
 1050 CONTINUE
      RETURN
      END

      SUBROUTINE FISH(PHIX,ELX,VX,ER,N)
C*****CALCULATES VECTOR OF REPRODUCTIVE VALUES GIVEN N-ORDER NET MATERNITY
C.....FUNCTION, SURVIVORSHIP SCHEDULE, AND ER, THE FACTOR OF INCREASE.
      DIMENSION PHIX(1),ELX(1),VX(1)
      SUM=0.0
      ERX=ER**N
      IX=N+1
      DO 1050 I=1,N
```

```
      IX=IX-1
      SUM=SUM+PHIX(IX)/ERX
      ERX=ERX/ER
      IF (ELX(IX)) 1030,1030,1040
 1030 VX(IX)=0.0
      GO TO 1050
 1040 VX(IX)=SUM*ERX/ELX(IX)
 1050 CONTINUE
      RETURN
      END

      SUBROUTINE PRJCT(ENX,PX,EMX,N)
C*****ADVANCES A POPULATION VECTOR ONE TIME UNIT FORWARD BY APPLICATION
C     OF A LESLIE MATRIX DEFINED BY N-ORDER VECTORS OF FECUNDITY AND
C.....SURVIVAL RATES.
      DIMENSION ENX(1),PX(1),EMX(1)
      SUM=0.0
      IX=N+1
      IXM1=N
      DO 1050 I=2,N
      IX=IX-1
      IXM1=IXM1-1
      SUM=SUM+ENX(IX)*EMX(IX)
      ENX(IX)=ENX(IXM1)*PX(IXM1)
 1050 CONTINUE
      ENX(1)=SUM+ENX(1)*EMX(1)
      RETURN
      END

      SUBROUTINE PTOL(PX,ELX,N)
C*****CONVERTS AN N-ORDER VECTOR OF SURVIVAL RATES TO SURVIVORSHIPS.
      DIMENSION PX(1),ELX(1)
      ELX(1)=1.0
      ELAST=1.0
      IM1=0
      DO 1050 I=2,N
      IM1=IM1+1
      ELAST=ELAST*PX(IM1)
      ELX(I)=ELAST
 1050 CONTINUE
      RETURN
      END

      SUBROUTINE DERV(ELX,VX,G,ER,DERDM,DERDP,N)
C*****CALCULATES THE N-ORDER VECTORS OF DERVIATIVES OF ER WITH RESPECT TO M(X)
C     AND P(X) FOR ALL X.
C     REQUIRES INPUT OF THE SURVIVORSHIP VECTOR, REPRODUCTIVE VALUE VECTOR,
C.....THE POPULATION GENERATION TIME, AND THE FACTOR OF INCREASE.
      DIMENSION ELX(1),VX(1),DERDM(1),DERDP(1)
      ER1MX=ER
      IXP1=1
      DO 1050 I=1,N
      IXP1=IXP1+1
      ER1MX=ER1MX/ER
```

```
      ELERG=ER1MX*ELX(I)/G
      DERDM(I)=ELERG
      IF (I-N) 1040,1060,1040
 1040 DERDP(I)=ELERG*VX(IXP1)
 1050 CONTINUE
 1060 DERDP(N)=0.0
      RETURN
      END

      SUBROUTINE MVVAC (X,XBAR,XVAR,N,NTT,NT)
C*****COMPUTES THE VECTORS OF MEANS AND VARIANCES OF THE N-ORDER VECTOR X
C     BY ACCUMULATING THE SUMS AND SUMS OF SQUARES EACH TIME THE SUBROUTINE
C     IS CALLED. THE CALL TO THIS SUBROUTINE SHOULD BE INSIDE A LOOP INDEXED
C.....BY NTT. AFTER THE LAST PASS (NTT=NT), MEANS AND VARIANCES ARE COMPUTED.
      DIMENSION X(1),XBAR(1),XVAR(1)
      IF (NTT-1) 4900,4900,5060
 4900 DO 5000 I=1,N
      XBAR(I)=0.0
      XVAR(I)=0.0
 5000 CONTINUE
 5060 DO 5100 I=1,N
      A=X(I)
      XBAR(I)=XBAR(I)+A
      XVAR(I)=XVAR(I)+A*A
 5100 CONTINUE
      IF (NTT-NT) 5300,5200,5200
 5200 FNT=FLOAT(NT)
      DO 5300 I=1,N
      S=XBAR(I)
      XBAR(I)=S/FNT
      XVAR(I)=(XVAR(I)-S*S/FNT)/FNT
 5300 CONTINUE
      RETURN
      END

      SUBROUTINE CVAC (X,XBAR,C,N,NTT,NT)
C**** CALCULATES THE COVARIANCE MATRIX C OF THE N-ORDER VECTOR X BY
C     ACCUMULATING SUMS OF CROSS PRODUCTS EACH TIME SUBROUTINE IS CALLED.
C     THE CALL TO THIS SUBROUTINE SHOULD BE INSIDE A LOOP INDEXED BY NTT.
C     WHEN NTT=NT, MEANS, VARIANCES AND COVARIANCES ARE COMPUTED.
C.... THE TRIANGULAR COVARIANCE MATRIX C IS STORED IN SINGLE SUBSCRIPT
C     NOTATION. ELEMENT (I,J) OF THE MATRIX IS ELEMENT IJ OF THE VECTOR,
C.... WHERE IJ=(J*J-J)/2+I AND I.LE.J. THE DIMENSION OF C IS N*(N+1)/2.
      DIMENSION X(1),XBAR(1),C(1)
      IF (NTT-1) 6000,6000,6200
 6000 NDX=0
      DO 6100 J=1,N
      XBAR(J)=0.0
      DO 6080 I=1,J
      NDX=NDX+1
      C(NDX)=0.0
 6080 CONTINUE
 6100 CONTINUE
 6200 NDX=0
```

```
      DO 6240 J=1,N
      A=X(J)
      XBAR(J)=XBAR(J)+A
      DO 6220 I=1,J
      NDX=NDX+1
      C(NDX)=C(NDX)+A*X(I)
 6220 CONTINUE
 6240 CONTINUE
      IF (NTT-NT) 7000,6300,6300
 6300 FNT=FLOAT(NT)
      NDX=0
      DO 6340 J=1,N
      XBAR(J)=XBAR(J)/FNT
      DO 6320 I=1,J
      NDX=NDX+1
      C(NDX)=C(NDX)/FNT-XBAR(I)*XBAR(J)
 6320 CONTINUE
 6340 CONTINUE
 7000 RETURN
      END

      SUBROUTINE STABL (ER,ELX,CX,N)
C*****CALCULATES THE STABLE AGE DISTRIBUTION CX, GIVEN THE ASYMPTOTIC
C*****FACTOR OF INCREASE AND THE SURVIVORSHIP VECTOR OF ORDER N.
      DIMENSION ELX(1),CX(1)
      SUM=0.0
      EMRX=1.0
      EMR=1.0/ER
      DO 1000 I=1,N
      EMRX=EMRX*EMR
      SUM=SUM+ELX(I)*EMRX
 1000 CONTINUE
      EMRX=1.0
      DO 1100 I=1,N
      EMRX=EMRX*EMR
      CX(I)=ELX(I)*EMRX/SUM
 1100 CONTINUE
      RETURN
      END

      SUBROUTINE ERNWTA (PHIX,RZ,T,S2,ER,N)
C.....THIS VERSION IS MODIFIED TO ACCEPT UNUSUAL VITAL RATES GENERATED
C     RANDOMLY IN PROGRAM SLT.
      DIMENSION PHIX(1)
      IF (S2) 1000,1000,1020
 1000 ER=RZ**(1.0/T)
      GO TO 1030
 1020 ER=EXP((T/S2)*(RZ**(S2/T**2)-1.0))
 1030 CT=0.0001
      DO 2000 J=1,20
      ERX=1.0
      FR=-1.0
      DFDR=0.0
      DO 1050 I=1,N
```

```
      ERX=ERX*ER
      IF (ERX-0.00001) 2010,2010,1040
 1040 TRM=PHIX(I)/ERX
      FR=FR+TRM
      DFDR=DFDR-FLOAT(I)*TRM
 1050 CONTINUE
      DELT=FR/DFDR
      ABFR=ABS(FR)
      IF(ABFR-CT) 1060,1060,1090
 1060 ABDLT=-ABFR/DFDR
      IF (ABDLT-CT) 2050,2050,1090
 1090 ER=ER-DELT
 2000 CONTINUE
 2010 ER=-100.0
 2050 ER=ER-DELT
 2100 RETURN
      END

      SUBROUTINE LMPHI(ELX,EMX,PHIX,N)
C*****CONVERTS AN N-ORDER VECTOR OF SURVIVORSHIPS AND A VECTOR OF
C     FECUNDITIES TO A NET MATERNITY FUNCTION.
      DIMENSION ELX(1),EMX(1),PHIX(1)
      DO 1050 I=1,N
      PHIX(I)=ELX(I)*EMX(I)
 1050 CONTINUE
      RETURN
      END

      SUBROUTINE MMNT(PHIX,RZ,T,S2,N)
C*****CALCULATES THE NET REPLACEMENT RATE, THE MEAN T, AND THE VARIANCE
C     S2 IN AGES OF REPRODUCTION FOR A COHORT.
      DIMENSION PHIX(1)
      RZ=0.0
      X1PHI=0.0
      X2PHI=0.0
      DO 1050 I=1,N
      PHI=PHIX(I)
      FI=FLOAT(I)
      RZ=RZ+PHI
      X1PHI=X1PHI+PHI*FI
      X2PHI=X2PHI+PHI*FI*FI
 1050 CONTINUE
      IF (RZ) 1060,1060,1070
 1060 T=1.0
      S2=0.0
      GO TO 1100
 1070 T=X1PHI/RZ
      S2=X2PHI/RZ-T*T
 1100 RETURN
      END

      SUBROUTINE COUNT (N,M,I,LUN)
C.....DISPLAYS ON THE CRT THE PROGRESS OF A LONG PROGRAM.  THE CALL TO
C     SUBROUTINE COUNT IS INSIDE A LOOP INDEXED BY N, AND THE VALUE OF
```

```
C     N IS DISPLAYED ON THE SCREEN EVERY M LOOPS.  I IS A COUNTER WHOSE
C     VALUE MUST NOT BE RESET BETWEEN CALLS.
      I=I+1
      IF (I-M) 1080,1060,1060
 1060 WRITE (LUN,10) N
   10 FORMAT (1X,I8,' PROJECTIONS COMPLETED.')
      I=0
 1080 RETURN
      END

      SUBROUTINE VAROUT (VCV,BTVAR,PTVAR,N)
      DIMENSION VCV(1),BTVAR(1),PTVAR(1)
      DO 1000 I=1,N
      J=N+I
      II=I*(I+1)/2
      JJ=J*(J+1)/2
      BTVAR(I)=VCV(II)
      PTVAR(I)=VCV(JJ)
 1000 CONTINUE
      RETURN
      END

      SUBROUTINE TALLY (X,K,WIDTH,BL,NOBS,ITAB)
C*****TALLIES THE VARIABLE X INTO K-2 CLASSES OF WIDTH WIDTH, BEGINNING
C     WITH LOWER BOUND BL.  ON OUTPUT, NOBS IS THE TOTAL NUMBER OF OBSER-
C     VATIONS TALLIED INTO THE K-ORDER VECTOR ITAB. NOBS AND ITAB MUST
C     BE SET TO ZERO BEFORE THE FIRST CALL.
      DIMENSION ITAB(1)
      KM1=K-1
      B=BL-WIDTH
      DO 2000 I=1,KM1
      B=B+WIDTH
      IF (X-B) 2100,2000,2000
 2000 CONTINUE
      I=K
 2100 ITAB(I)=ITAB(I)+1
      NOBS=NOBS+1
      RETURN
      END

      SUBROUTINE XRGEN (X,S,V,W1,W2,N,ISEED)
C*****GIVEN AN N-ORDER VECTOR OF MEANS X, AN N BY N MATRIX V, THE INVERTED
C     TRANSPOSE OF THE MATRIX OF EIGENVECTORS OF THE COVARIANCE MATRIX,
C     AND A VECTOR S OF EIGENVALUES (VARIANCES), THIS SUBROUTINE GENERATES
C     A RANDOM VECTOR, RETURNED AS X, WITH THE SPECIFIED MEAN, VARIANCE
C     AND COVARIANCE PROPERTIES.
C.....CALLS SUBROUTINES MTPRD AND NRMUL. ISEED IS THE SEED FOR THE RANDOM
C     NUMBER GENERATOR. WORKSPACES W1 AND W2 MUST BE DIMENSION AT LEAST N
C     IN THE MAIN PROGRAM.
      DIMENSION X(1),S(1),V(1),W1(1),W2(1)
C.....GENERATE RANDOM VECTOR WITH NO COVARIANCE
 1000 DO 1100 I=1,N
      STD=SQRT(ABS(S(I)))
      CALL NRMUL (Z,0.0,STD,ISEED)
```

```
      W1(I)=Z
 1100 CONTINUE
C.....ADD COVARIANCE AND MEANS
      L=1
      CALL MTPRD (V,W1,W2,N,N,L)
      DO 1200 I=1,N
      X(I)=X(I)+W2(I)
 1200 CONTINUE
C.....TRUNCATE VALUES FOR DEMOGRAPHIC APPLICATIONS
      NH=N/2
      DO 2200 K=1,N
      XK=X(K)
      IF (XK) 2110,2120,2120
 2110 X(K)=0.0
 2120 IF (K-NH) 2200,2200,2130
 2130 IF (XK-1.0) 2200,2200,2140
 2140 X(K)=1.0
 2200 CONTINUE
      RETURN
      END

      SUBROUTINE LOAD (N,NT,NR,IOP1,IOP2,LUN,ISEED,X,BX,PX,COVAR,W,KK,
     #                 WIDTH1,BL1,WIDTH2,BL2,RL,RU)
C*****READS THE CONTROL PARAMETERS AND INPUT VECTORS OF INITIAL POPULATION,
C     FECUNDITY, SURVIVAL AND COVARIANCE FOR PROGRAM SLT.
C.....IF IOP1>0, THE TOTAL INITIAL POPULATION SHOULD BE SPECIFIED ON THE
C     DATA CARD. THE INITIAL POPULATION VECTOR WILL BE COMPUTED FROM THE
C     STABLE AGE DISTRIBUTION.
C.....IF IOP2>0, THE COVAR DATA CARD SHOULD CONTAIN THE 2*N VARIANCES,
C     FOLLOWED BY THE CORRELATIONS OF FECUNDITY RATES, BETWEEN FECUNDITY
C     AND SURVIVAL RATES, AND AMONG SURVIVAL RATES.
C.....THE COVARIANCE MATRIX (IN SINGLE SUBSCRIPT FORM) IS FIRST READ INTO
C     WORK VECTOR W, THEN REPACKED IN STANDARD UPPER TRIANGULAR FORM IN
C.....COVAR.
      DIMENSION X(1),BX(1),PX(1),COVAR(1),W(1)
C.....LOAD
   10 FORMAT (8F10.6)
   11 FORMAT (8F10.2)
   12 FORMAT (I1)
   13 FORMAT (I2)
   14 FORMAT (I5)
      READ (1,13) N
      READ (1,13) NT
      READ (1,14) NR
      READ (1,12) IOP1
      READ (1,12) IOP2
      READ (1,12) LUN
      READ (1,14) ISEED
      READ (1,13) KK
      READ (1,10) WIDTH1
      READ (1,10) BL1
      READ (1,10) WIDTH2
      READ (1,10) BL2
      READ (1,10) RL
```

```
      READ (1,10) RU
      READ (1,10) (BX(I),I=1,N)
      READ (1,10) (PX(I),I=1,N)
      IF (IOP1) 1000,1000,1020
 1000 READ (1,11) (X(I),I=1,N)
      GO TO 1040
 1020 READ (1,11) X(1)
 1040 N2=N*2
      NCV=N*(N2+1)
C.....COVARIANCE VECTOR
      IF (IOP2) 1060,1060,1080
 1060 READ (1,10) (W(I),I=1,NCV)
      GO TO 2110
 1080 READ (1,10) (W(I),I=1,N2)
      READ (1,10) C1
      READ (1,10) C2
      READ (1,10) C3
      IF (ABS(C1)-1.) 40,40,90
 40   IF (ABS(C2)-1.) 50,50,90
 50   IF (ABS(C3)-1.) 60,60,90
 60   DO 70 I=1,N2
      IF (W(I)) 80,70,70
 70   CONTINUE
      GO TO 100
 80   WRITE (LUN,89)
 89   FORMAT (/' ERROR -- VARIANCES IN COVARIANCE MATRIX CANNOT'/
     #9X'HAVE NEGATIVE VALUES ')
      GO TO 2200
 90   WRITE (LUN,99)
 99   FORMAT (/' ERROR -- ELEMENTS OF CORRELATION MATRIX CANNOT'/
     #9X'HAVE ABSOLUTE VALUE GREATER THAN 1.0')
      GO TO 2200
C.....REPACK
 100  MID=NCV-N*(N+1)/2
      NDX=NCV+1
      K=NCV
      I=N2
      M=1
      DO 1200 NNN=1,NCV
      NDX=NDX-1
      IF (NDX-K) 1100,1160,1100
 1100 IF (NDX-MID) 1110,1110,1120
 1110 IF (NDX-L) 1150,1140,1140
 1120 W(NDX)=C3*SQRT(W(I)*W(J))
      GO TO 1200
 1140 W(NDX)=C2*SQRT(W(I)*W(J))
      GO TO 1200
 1150 W(NDX)=C1*SQRT(W(I)*W(J))
      GO TO 1200
 1160 W(NDX)=W(I)
      L=K-N
      M=M+1
      K=K-M
      I=I-1
```

```
      J=N2+1
 1200 J=J-1
C.....ZERO-MEAN CASES
 2000 DO 2100 I=1,N
      IF (BX(I)-.0000001) 2030,2030,2010
 2010 IF (PX(I)-.0000001) 2020,2020,2100
 2020 IR=I+N
      GO TO 2040
 2030 IR=I
 2040 DO 2070 J=1,N2
      IF (J-IR) 2050,2050,2060
 2050 IJ=(J-1)*N2-((J-3)*J)/2-J+IR
      GO TO 2070
 2060 IJ=(IR-1)*N2-((IR-3)*IR)/2-IR+J
 2070 W(IJ)=0.0
      IF (IR-I-N) 2010,2100,2100
 2100 CONTINUE
C.....REPACK AGAIN TO STANDARD UPPER TRIANGULAR FORM
 2110 NDX=0
      DO 2140 I=1,N2
      DO 2120 J=I,N2
      IJ=(J*J-J)/2+I
      NDX=NDX+1
      COVAR(IJ)=W(NDX)
 2120 CONTINUE
 2140 CONTINUE
 2200 RETURN
      END

      SUBROUTINE EREAL (RNX,ENX,N,NT,ER,ERT)
C*****COMPUTES THE REALIZED FACTOR OF INCREASE FOR AN INITIAL POPULATION
C     VECTOR RNX AND A FINAL VECTOR ENX OF ORDER N OVER NT TIME STEPS.
      DIMENSION RNX(1),ENX(1)
      TI=0.0
      TF=0.0
      DO 1000 I=1,N
      TI=TI+RNX(I)
      TF=TF+ENX(I)
 1000 CONTINUE
      ERT=TF/TI
      ER=ERT**(1.0/FLOAT(NT))
      RETURN
      END

      SUBROUTINE BPIN (BX,PX,W,N)
C*****LOADS VECTORS BX AND PX INTO W.
      DIMENSION BX(1),PX(1),W(1)
      NDX=N
      DO 1000 I=1,N
      NDX=NDX+1
      W(I)=BX(I)
      W(NDX)=PX(I)
 1000 CONTINUE
      RETURN
```

```
      END

      SUBROUTINE BPOUT (W,BT,PT,N)
C*****UNLOADS THE CONTENTS OF W INTO BT AND PT.
      DIMENSION W(1),BT(1),PT(1)
      NDX=N
      DO 1000 I=1,N
      NDX=NDX+1
      BT(I)=W(I)
      PT(I)=W(NDX)
 1000 CONTINUE
      RETURN
      END
```

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